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Wave leakage in a magnetized isothermal atmosphere

Dipankar Banerjee and S. S. Hasan

Indian Institute of Astrophysics, Bangalore - 560034, India

J. Christensen-Dalsgaard

Teoretisk Astrofysik Center, Danmarks Grundforskningsfond, and
Institut for Fysik og Astronomi, Aarhus Universitet, DK-8000 C, Denmark

Abstract. The present investigation is a continuation of earlier work by Hasan & Christensen-Dalsgaard (1992) and Banerjee, Hasan & Christensen-Dalsgaard (1995), where the interaction of various elementary modes in a stratified atmosphere with a vertical magnetic field was studied. In the present study, we concentrate on the behaviour of the modes near the avoided crossings in the the $k - \omega$ diagram for zero-gradient boundary conditions. We find that in such regions the frequencies of the modes become complex, whereas away from the avoided crossings the frequencies are real (in the adiabatic case) for these boundary conditions. Strong mode coupling in the avoided-crossing regions permits energy leakage for zero-gradient boundary conditions.

Key words: MHD - Sun: oscillations - Sun: magnetic fields

1. Introduction

Study of wave motions can reveal useful information on the structure of magnetic elements and thus serve as a powerful diagnostic tool. Observations of oscillations in the solar atmosphere provide a tool for understanding the solar interior as well as inferring the structure of flux tubes from their oscillations. Oscillations with periods in a fairly broad range of frequencies have been reported in magnetic elements (Moore & Rabin 1985). The aim of the present study is to contribute towards developing a theory for such wave motions, also known as magnetoatmospheric oscillations.

Recently Hasan & Christensen-Dalsgaard (1992; hereafter Paper I) and Banerjee, Hasan & Christensen-Dalsgaard (1995; Paper II) have examined the effect of a vertical magnetic field on the normal modes of an isothermal stratified atmosphere for various combinations of boundary conditions. In these studies, a semi-analytic approach based on asymptotic dispersion relations was combined with numerical solutions to examine the nature of mode interactions in a vertical magnetic field. The interaction among various modes was studied and it was shown that this interaction becomes particularly important at the locations in the diagnostic diagram where the frequencies of different elementary modes coincide. These regions, also referred to as *avoided crossings*, are characterized by a strong coupling between two modes and the character of the solution is a mixture of these two modes.

2. MAG Waves for a weak field

The asymptotic properties of the waves and the normal modes of a stratified atmosphere with a weak magnetic field were extensively studied in Papers I and II. There, we used different sets of boundary conditions and explained the behaviour of different elementary wave modes in various parts of the diagnostic diagram. A striking feature was the occurrence for certain combinations of boundary conditions of complex eigenfrequencies near avoided crossings. The mathematical reasoning behind their appearance was given in Paper II. Here we explore the implications of this result.

As before we consider a cavity of thickness d, which permits standing wave solutions. The equations are cast in dimensionless form, characterized by a dimensionless horizontal wavenumber K and frequency Ω. The behaviour of the MAG waves is reflected in their properties in the $K-\Omega$ diagram. We focus on zero gradient boundary conditions at both the ends of the cavity:

$$\frac{\mathrm{d}\xi_x}{\mathrm{d}z} = \frac{\mathrm{d}\xi_z}{\mathrm{d}z} = 0 \quad \text{at} \quad z = 0 \quad \text{and} \quad z = d\,, \tag{1}$$

where ξ_x and ξ_z are the horizontal and vertical components of the displacement. The linearized wave equations for an isothermal atmosphere with a vertical magnetic field were solved numerically, using a complex version of the Newton-Ralphson-Kantorovich scheme with the above boundary conditions, to determine Ω for different values K.

We first compare our results with those of Paper II, in which the eigenfrequencies were assumed to be purely real. Figure 2a of Paper II shows the variation of Ω with K, for a weak magnetic field. The various curves depict magnetic or slow modes m_l (l denotes order of the mode), the Lamb mode (Ω_L) and gravity modes (g_l). These results reveal that in regions where the frequencies of two different modes become close, there are large voids in the $K-\Omega$ diagram. It is instructive to consider a single mode, which we choose as the lowest order magnetic mode m_1, and follow its behaviour as it interacts with the Lamb and gravity modes, allowing Ω to be complex. Figure 1a depicts the variation with K of the real (solid line) and imaginary (dashed line) parts of Ω. The main difference with Paper II is that we find complex eigenfrequencies in the regions of avoided crossings: instead of voids in the $K-\Omega$ diagram as found earlier, the branches of the solution are connected by the real part of the complex eigenfrequencies. Also, Im(Ω) has a substantial value in the region where there is a crossing between the magnetic m_1 and the gravity g_1 mode. This behaviour is consistent with the asymptotic equations (29) and (30) of Paper II, if we note that $\delta\Omega^2_{\rm min}$ is negative.

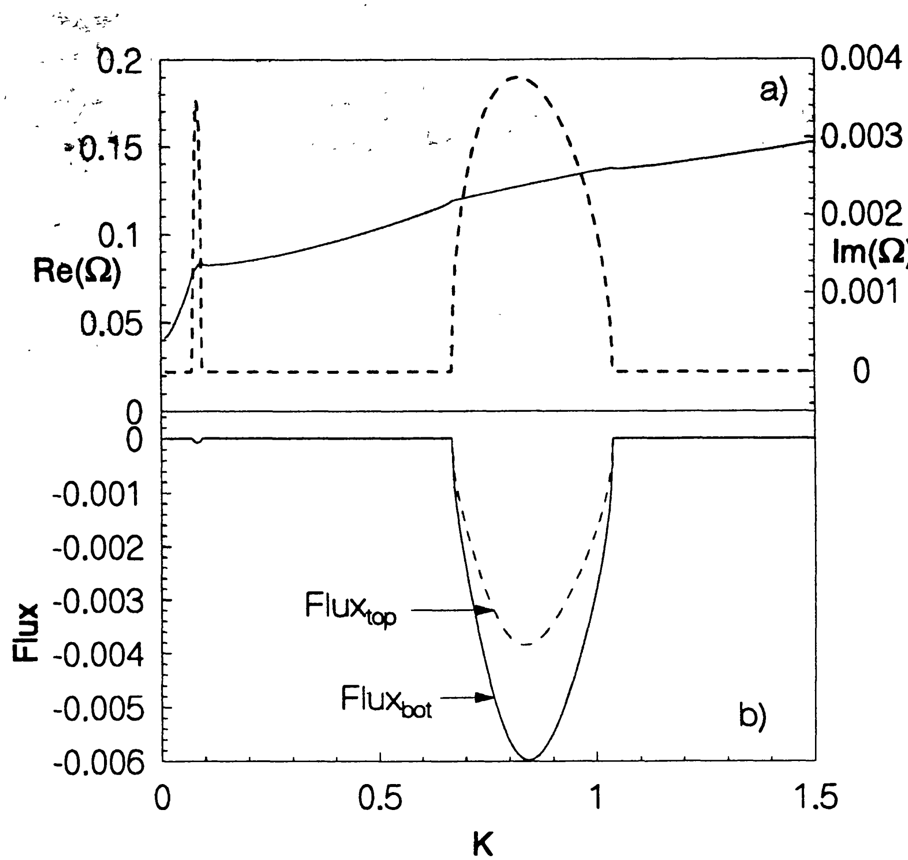

Figure 1. Expanded region of the diagnostic diagram showing the interaction between the m_1 mode with the Lamb mode and the gravity mode for zero-gradient boundary conditions.(*a*) Variation with K of the real (solid line) and the imaginary (dashed line) parts of the frequency. (*b*) Variation with K of the time-averaged net upward flux from the boundaries as indicated

We now focus our attention on the regions of avoided crossing and calculate the time-averaged net upward wave energy flux through the boundaries, following Bray and Loughead (1974). Figure 1b depicts the variation with K of the upward flux (in arbitrary units) through the boundaries. This flux is nonzero only at the avoided crossings, while elsewhere it is negligible. The negative flux at the top boundary corresponds to an energy flow into the region, but this is more than outweighed by the energy loss at the bottom boundary. Clearly this net energy transport out of the region is linked to the appearance of positive imaginary parts of the frequencies, corresponding to a decay of the modes. Since the complex frequencies come in conjugate pairs, there is evidently an additional set of growing modes, with the opposite sign of the fluxes. We neglect these from physical constraints. Finally, we note from Figure 1b that energy leakage from the cavity makes a smaller contribution for the magnetic-Lamb mode interaction than for the magnetic-gravity mode case.

3. Discussion

The aim of our study was to examine mode interaction in an isothermal atmosphere with a weak vertical magnetic field. Our results clearly demonstrate that for certain boundary conditions, the frequencies can become complex in an adiabatic system, where no explicit energy loss mechanism is taken into account. It appears that the boundary conditions permit the phase relationship between the modes to be changed in a manner so as to allow the wave to leak out from the boundary, thereby leading to a loss of energy from the cavity. This behaviour occurs only when there is a strong mode coupling for certain combination of frequencies and wave numbers. Our results complement those of Cally & Bogdan (1993) and Cally, Bogdan & Zweibel (1994), where wave-leakage through the lower boundary of a magnetized polytropic atmosphere is explicitly taken into account. In forthcoming investigations, we hope to enlarge the scope of our analysis to more realistic conditions.

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