1996BASI...24..317H

Bull. Astr. Soc. India (1996) **24,** 317–320

On the Accuracy of Opacity Interpolation Schemes

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Abstract. We have investigated some of the properties and consequences of interpolation in opacity tables. In particular, we were interested in the estimate of the maximum interpolation errors, where they occur in the tables, and how large are the changes in model results when using different interpolation schemes, in particular for the temperature gradient and the velocity of sound.

Key words: Opacity, Interpolation

1. Introduction

The opacity calculation is far too complicated to be performed within stellar evolution programmes. Consequently, the opacity is found by interpolation in precomputed tables; hence, we are faced with the choice of a suitable interpolation procedure. In this contribution we discuss the accuracy of interpolation methods and its influence on the results of stellar model calculations.

In our survey, we consider three interpolation schemes. The first method uses splines under tension (Cline 1974), which has been used so far in several evolution and envelope calculation programmes (e.g. Christensen-Dalsgaard 1982). The second scheme is based on birational splines which we have implemented according to Späth (1991). And finally the third method uses a C^1 interpolant defined over triangles, which enables us to interpolate data given on an irregular grid (Montefusco & Casciola 1989).

In a first step we compared these methods with an analytical formula derived by Stellingwerf (1975). This formula is a fit to the Cox-King tables using proper exponential functions and rational fractions. In order to emphasize the impact of the table grid-spacing on the interpolation errors, the comparison has been carried out with different mesh points in log(T) according to the OPAL92 (Iglesias, Rogers & Wilson 1992) and OPAL95 tables (Iglesias & Rogers 1995). In a next step we compared the opacity between model envelopes, calculated with the above introduced interpolation schemes for the solar case and a 1.5 $M_\odot$ ZAMS star. The resulting changes in the temperature gradient and velocity of sound of the model envelopes will be discussed.

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2. Interpolation methods

In the OPAL tables the opacity is tabulated as a function of temperature, log(R), (where $R=\rho^3/T_6$, ρ being the density and $T_6 = 10^{-6}T$), and the chemical composition which can be specified by the mass-fractions of hydrogen (X) and of the heavy elements (Z). The opacity (κ) is found by interpolation in the four-dimensional space, normally using a bivariate method in the log(T)–log(R) plane and applying a univariate scheme in X and Z, respectively. The following methods have been considered for interpolating the opacity in the temperature–density space:

2.1 Splines under tension

The geometry of the log(κ)–surface is often such that use of the commonly applied cubic spline interpolation may give rise to large artificial variations between the grid points. Cline (1974) was guided by the physical notion of an elastic band passing through rings at the interpolation points, and which can be pulled by its ends to eliminate all unnecessary wiggles. This introduces the additional requirement that the quantity $f'' - \sigma^2 f$ varies linearly between the given grid points, where f is the resulting piecewise interpolant with continuous first and second derivatives. Here the parameter σ determines the amount of the "tension". A small value of σ gives essentially a cubic spline behaviour, whereas a large value results in a linear interpolant f between the mesh points.

2.2 Rational splines

In this method rational fractions are used for the nonlinear terms of the piecewise interpolating function defined as

$$f_k(x) = a_k u + b_k t + c_k \frac{u^3}{pt+1} + d_k \frac{t^3}{pu+1}; \quad t = \frac{x - x_k}{\Delta x_k}; \quad u = 1 - t, \tag{1}$$

where a_k, b_k, c_k, d_k are the spline-coefficients. The coefficients are determined for the given function values defined at the grid points x_k. The parameter p specifies the pole of the rational function and defines the curvature of $f_k(x)$. The value $p = 0$ leads to the well-known cubic spline interpolant.

2.3 Minimum-norm interpolation in triangles

One of the disadvantages of using spline-based interpolation methods is that the given table points have to be defined on a rectangular and regular grid. Here we introduce a scheme which interpolates arbitrarely scattered data in the plane. The interpolated value is computed in three steps. First a triangulation of the given table points is carried out, using the max–min angle optimization criterion due to Lawson (1977). In the next step, the first partial derivatives are evaluated, solving the following minimization problem

$$\mathrm{Min} \sum_n \int_{e_i} \left[\frac{\partial^2 F}{\partial e_i^2}\right]^2 ds_i, \tag{2}$$

where F is an element from the set of interpolating cubic Hermite polynomials, defined on all edges e_i, $i = 1 \ldots n$, of the triangulation, and ds_i being the element of arc-length along e_i. The interpolated value is then found by the triangular blending method due to Nielson (1983) using certain minimum-norm properties, which results in an interpolant of second-order accuracy with continuous first derivatives.

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3. Results and conclusion

The relative opacity differences between the minimum-norm interpolation scheme and Stellingwerf's analytical fit to the Cox-Tabor tables are depicted in Fig. 1 as a contour plot in the log(R)–log(T) space for different grid-spacings in temperature. Using the OPAL92 grid spacing (Fig. 1a) we encounter a local maximum of the interpolation errors in the order of 1% at a temperature of log(T) $\approx$ 5.2. They occur at a point where there is a local maximum in the stepsize of the temperature grid which coincides with the falling edge of the opacity–bump at this particular temperature. The steepness of this edge increases with density or R, resulting in an absolute maximum error in the opacity of 3.4% at log(R) $\approx$ 0.8. The opacity tracks of model envelopes for the Sun and a 1.5 $M_\odot$ ZAMS star indicate their expected magnitude and location of interpolation errors.

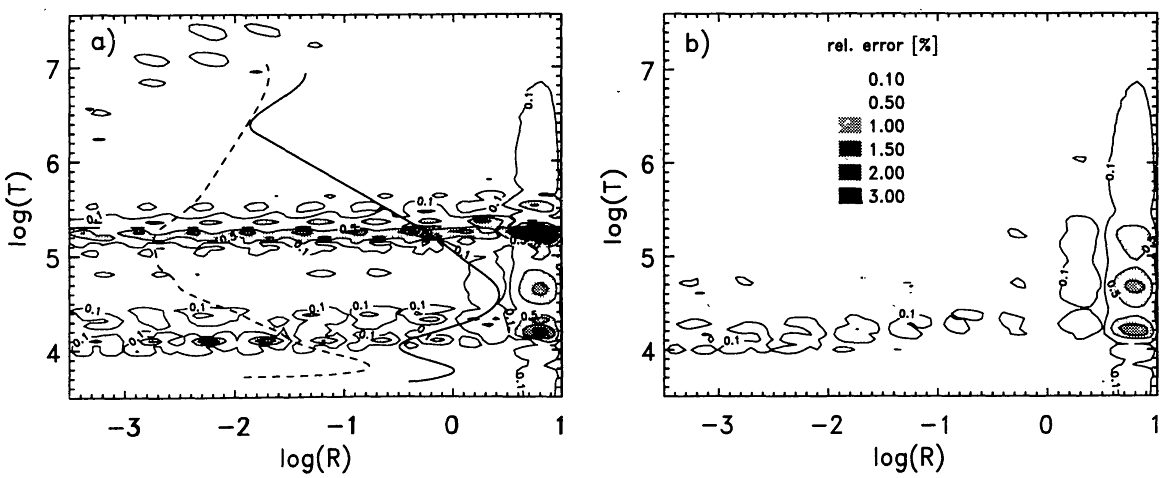

Figure 1. Relative interpolation errors between the minimum-norm scheme and Stellingwerf's analytical formula, using the grid spacing of the OPAL92 (a) and OPAL95 (b) tables. In panel (a) the opacity tracks of model envelopes for the Sun (solid line) and a 1.5 $M_\odot$ ZAMS star (dashed line) are depicted.

Suggestions have been made (e.g. Moskalik & Dziembowski 1992) that interpolation might be improved by the inclusion of additional grid points. In Fig. 1b the results are displayed for the grid used in the OPAL95 tables, which do have a finer mesh in the temperature, revealing substantially smaller errors. The maximum value has been reduced to 1.5%, and occurs at the same R-value previously found in the OPAL92 tables.

In Fig. 2 the opacity differences are depicted as a function of temperature between model envelopes calculated with the three interpolation schemes for the Sun and a 1.5 $M_\odot$ ZAMS star. The calculations have been carried out with a programme by Balmforth (1992) using the OPAL92 tables, supplemented at low temperature by tables from Kurucz (1991). For the solar case (Fig. 2a) we encounter at log(T) $\approx$ 5.1 a difference in the opacity of 1.5% between the rational splines method and Cline's splines under tension (R-C). Fortunately this occurs in the convection zone, indicated by the ratio of the convective to the total flux (F_c/F). However in the radiative atmosphere at log(T) $\approx$ 3.7 the differences exhibit a value of up to 2% , whereas the changes between the rational splines and the minimum-norm method (R-M) are substantially smaller. At this log(T) value the temperature derivative of the opacity exhibits a rapid change in

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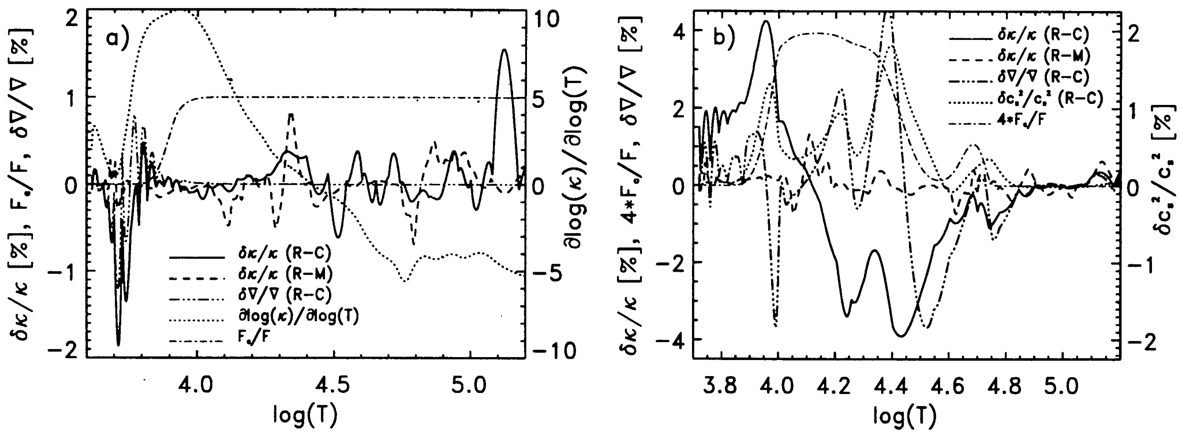

Figure 2. Relative differences of the opacity (κ) and temperature gradient (∇) between model envelopes calculated with the rational splines scheme and Cline's spline under tension (R-C), and between the rational splines and minimum-norm method (R-M) for the solar case (a) and a 1.5 $M_\odot$ ZAMS star (b). In both panels the ratio between the convective to the total flux (F_c/F) is depicted. Additionally panel (a) shows the temperature derivative of the opacity and panel (b) the differences in the velocity of sound (c_s). For the interpolation parameters σ and p values of 0.005 and 0.1 has been used, respectively.

a small interval, where Cline's interpolant seems to display artificial variations. The differences in the opacity for the R-C comparison became even more significant for the 1.5 $M_\odot$ star (Fig. 2b), where we found values larger than 4%.

In regions where radiation contributes to the total flux and coincides with the opacity changes, the differences in the temperature gradient are in the order of 1% for the solar case and larger than 4% for the 1.5 $M_\odot$ star. The resulting changes in the velocity of sound for the 1.5 $M_\odot$ star are in the order of up to 2% (Fig. 2b) and less than 0.1% for the Sun. The implications from these results on the adiabatic frequencies are rather negligible, yielding a maximum change of less then 1 μHz. However the impact on the nonadiabatic frequencies might be more severe, since nonadiabaticity is also confined in a small domain in the upper part of the convection zone (e.g. Houdek et al. 1995).

We are grateful to J. Christensen-Dalsgaard for helpful comments. This work was partly supported by the FWF (projects *P8776-AST* and *S7303-AST*).

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