Bull. Astr. Soc. India (1996) **24**, 309–315

Long-Period Oscillations of the Sun's Interior

Joseph M. Davila

NASA-Goddard Space Flight Center, Greenbelt MD USA

S. M. Chitre

Tata Institute of Fundamental Research, Homi Bhabha Rd., Bombay INDIA

Abstract. The long-period modes of a differentially rotating, solar-type star are consided. Two modes are found. The non-axisymmetric modes are Rossby waves, also known as r-modes, with the Coriolis force as the dominant restoring force. These oscillations have periods of order a month to a few years for solar parameters. An axisymmetric mode with purely magnetic restoring force is also found, which we call the b-mode. The fundamental period of the b-mode is large, of order 300 years. Higher harmonics are shown to have periods of order $10 - 20$ years. The dispersion relation for depends on the variation of the magnetic field strength and the mass density with depth through the convection zone. We suggest that if these modes could be observed information on the structure of the magnetic field below the surface could be deduced. Accounting for differential rotation does not affect the period of either mode, however the radial and lattitudinal profiles of the wave amplitude are slightly modified when compared with the previous solutions assuming solid body rotation.

Key words: solar oscillations, magnetic field, torsional oscillations

1. Introduction

For the most part, the forces generated by the solar magnetic field can be neglected in the interior of the Sun. The long-period modes of a uniformly rotating, field-free star have been considered by a number of authors (Provost etal 1981, Saio, 1982, Wolff and Blizzard 1986). These studies have demonstrated that Rossby waves, r-modes, with periods of order a few months could exist on the Sun. A univerally accepted excitation mechanism has not been identified and there has been no observational detection of the r-mode.

The structure of the sub-surface magnetic field would lead one to expect the existence of another long-period mode with the magnetic field providing the restoring force. Dynamo models with field generation distributed throughout the convection zone cannot account for the at least some of the observed properties of the solar cycle (Gilman 1983). This has led to the suggestion that dynamo processes are confined primarily to the thin convective overshoot region at the lower boundary of the convection zone (Schussler 1980). In this scenario, a large toroidal field is generated which sporadically erupts and rises, resulting in the observed field at the surface of the Sun. This process is shown schematically in Figure 1. The core of a solar-type star is effectively field-free. This can lead to the presence of a class of trapped magnetic torsional modes of oscillation which we refer to as b-modes. Similar motions have been considered before by Vandakurov (1988).

In this paper, we consider these two possible modes for a differentally rotating solar type star. The expected periods of the r-mode and b-mode waves are estimated for representative solar parameters.

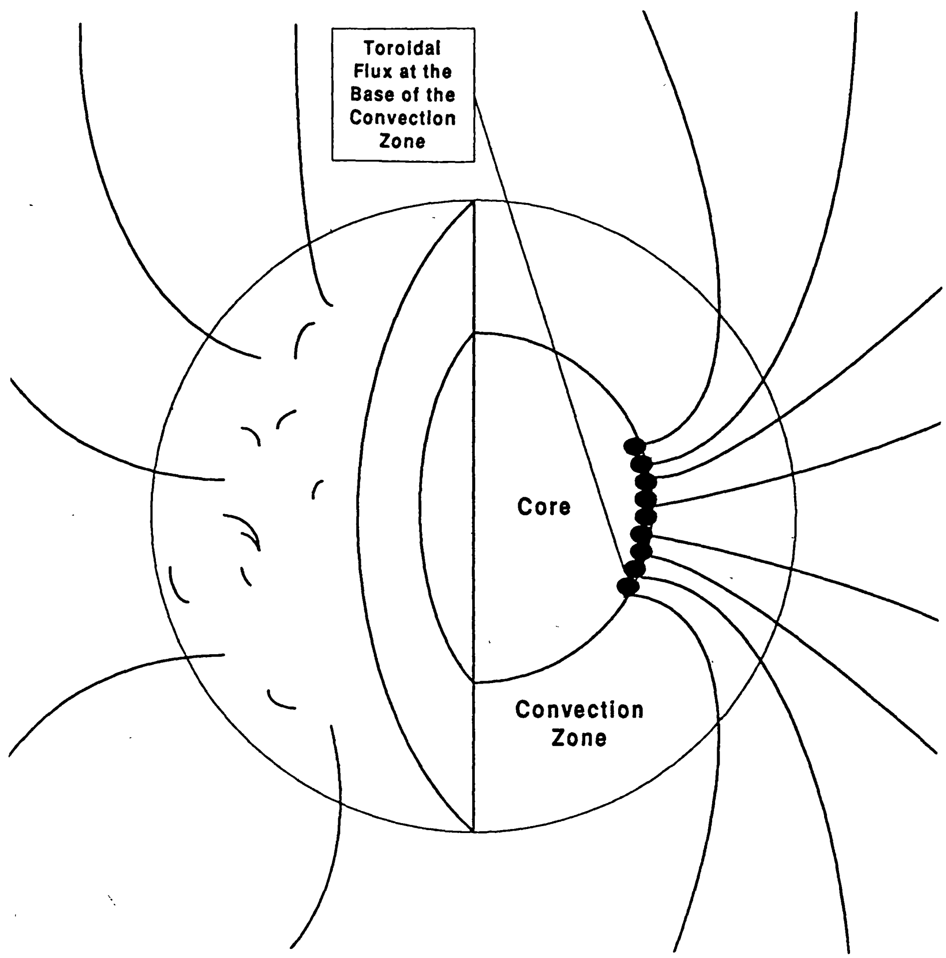

Figure 1. The magnetic field of the Sun is thought to be generated at the base of the convection zone. There a strong toroidal field occasionally erupts resulting in the observed surface field.

2. The Basic Model

The exact spatial variation of the solar magnetic field through the convection zone is not known. For this calculation we will assume the simplest representation of the field possible which can still capture the essence of the complex emerging field configuration shown in Figure 1. In particular, we will assume that the equilibrium magnetic field can be represented by $\vec{B}_0 = B_0(r,\theta)\ \hat{r}$. The wave amplitude , $\vec{v}$, is assumed to be transverse, but otherwise allowed to be arbitrary.

The linearized equations for an ideal fluid with no time averaged background flow are then written as

$$\frac{\partial \vec{B}}{\partial t} = \nabla \times (\vec{v} \times \vec{B}_0) \tag{1}$$

$$\frac{\partial \rho}{\partial t} + \rho_0 \nabla \cdot \vec{v} + v_r \frac{\partial \rho_0}{\partial r} + \frac{v_\theta}{r}\frac{\partial \rho_0}{\partial \theta} = 0 \tag{2}$$

$$\rho_0 \frac{\partial \vec{v}}{\partial t} = -\nabla(p + \frac{B_0 B_r}{4\pi}) + \frac{1}{4\pi}\left[(\vec{B}\cdot\nabla)\vec{B}\right]_1 + \rho\vec{g} - 2\rho_0\vec{\Omega}\times\vec{v} \tag{3}$$

$$\frac{\partial p}{\partial t} - c_s^2(\frac{\partial \rho}{\partial t} + v_r\frac{\partial \rho_0}{\partial r} + \frac{v_\theta}{r}\frac{\partial \rho_0}{\partial \theta}) + v_r\frac{\partial p_0}{\partial r} + \frac{v_\theta}{r}\frac{\partial p_0}{\partial \theta} = 0 \tag{4}$$

where

$$c_s^2 = \frac{\Gamma_1 p_0}{\rho_0} \tag{5}$$

and the subscript 1 indicates that only terms first order in the wave amplitude are considered. These equations are correct only to lowest order in the ratio of centrifugal to gravitational force. The oscillations of a non-spherical star will be considered in a subsequent paper.

Consider the force terms on the right hand side of this equation. The two largest forces are due to the pressure gradient and gravity. These forces give rise to the observed spectrum of p-modes and g-modes with periods of order 300 s and a few hours in the solar convection zone. These forces are the dominant restoring force for all compressible perturbations.

Of the remaing two force terms the Coriolis force is by far the largest. It is of order of 10^4 times larger than the magnetic force, and hence dominates the dynamics of all non-axisymmetric incompressible perturbations. The resulting waves are known as r-modes.

However, as we show below, for axisymmetric incompressible perturbations, the Coriolis force is ineffective, leaving only the magnetic term to balance the inertal term on the left hand side of the momentum equation. This leads to an incompressible magnetic torsional wave which we refer to as the b-mode.

Since we are interested in long-period motions, we will consider only incompressible motions of the plasma. With this assumption, the gravitational and pressure forces both vanish, and the equation for displacement vector $\vec{\xi}$ can be written as

$$\vec{\xi} = (0, R(r)\frac{1}{\sin\theta}\frac{\partial Y(\theta,\varphi)}{\partial \varphi}, -R(r)\frac{\partial Y(\theta,\varphi)}{\partial \theta}) \tag{6}$$

With this assumption for the displacement, the equation of continuity and the energy equation are automatically satisfied for any arbitrary scalar function $Y(\theta, \varphi)$.

By combining the displacement with the momentum and induction equation and assuming all perturbation quanities vary as $e^{-i\omega t}$, and $Y(\theta, \varphi) = Y(\theta)e^{im\varphi}$, one arrives at the following relationship for $Y(\theta)$

$$\overbrace{-\omega^2 R(r)\left[\frac{\partial}{\partial\theta}\sin\theta\frac{\partial Y(\theta)}{\partial\theta} - \frac{m^2}{\sin\theta}Y(\theta)\right]}^{\text{Inertial Terms}} = \overbrace{2m\omega\Omega_0 R(r)\sin\theta\left[(\frac{\Omega}{\Omega_0}) - \frac{\cot\theta}{\Omega_0}\frac{\partial\Omega}{\partial\theta}\right]Y(\theta)}^{\text{Coriolis Force Terms}} \overbrace{+ v_A^2\frac{1}{rB_0}\frac{\partial^2}{\partial r^2}rB_0 R(r)\left[\frac{\partial}{\partial\theta}\sin\theta\frac{\partial Y(\theta)}{\partial\theta} - \frac{m^2}{\sin\theta}Y(\theta)\right]}^{\text{Magnetic Force Terms}} \quad (7)$$

Again one can identify the left side of this equation with the inertia of the plasma, the first term on the right is due to the magnetic field and the second term on the right is due to the Coriolis force. All of the angular dependence is contained in the terms enclose in square brackets.

3. The r-Modes

The Coriolis force is much larger than the magnetic force, so initially we simply neglect magnetic forces. One can see from Eqn. 7 that the angular dependence of the wave amplitude can be removed if the angular eigenfunctions satisfy the relation

$$\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\sin\theta\frac{\partial Y}{\partial\theta} - \frac{m^2}{\sin^2\theta}Y = -\ell(\ell+1)\left[(\frac{\Omega}{\Omega_0}) - \frac{\cot\theta}{\Omega_0}\frac{\partial\Omega}{\partial\theta}\right]Y(\theta) \quad (8)$$

where ℓ is the eigenvalue. For solid body rotation, $Y(\theta) = P_\ell^m cos(\theta)$ is the Associated Legendre polynomial. The observed solar differential rotation rate can be well approximated by the function $\Omega(\theta) = \Omega_0 + \Omega_2 cos^2(\theta) + \Omega_4 cos^4(\theta)$ (e.g. see Komm etal 1994). Using this form for the rotation velocity, the angular equation can be solved using the method of Frobenius to obtain a generalized polynomial solution for $Y(\theta)$. The details of this solution will be left for a later paper.

With Equation (8) satisfied, the frequency of the wave motion can be obtained as

$$\omega = \frac{2m\Omega_0}{\ell(\ell+1)} \quad (9)$$

The radial dependence of the wave amplitude must be determined by solving the equations to first order in the centrifugal force. This has been done for solid body rotation by Wolff and Blizzard (1986). The expression for the frequency obtained here is the same, thus demonstrating that diffential rotation does not affect the r-mode frequency.

4. The b-Modes

From (7) it is clear that when $m = 0$ the Coriolis force term vanishes. Therefore for axisymmetric modes, only the magnetic force is available to balance the inertia terms on the left. In this circumstance the angular dependence divides out of both terms in the zero order equations, and one is left with an equation for the radial variation of the wave amplitude.

$$\frac{\partial^2 U}{\partial r^2} + \frac{\omega^2}{v_A^2(r)} U = 0 \tag{10}$$

where $U(r) = rB_0(r)R(r)$. The angular dependence for the b-mode must be obtained by solving the equations that are first order in the centrifugal force term. This is analagous to the r-mode case where the radial dependence of the wave amplitude must be determined by going to the first order equations. Again this solution will be left for a future study.

The solutions to Equation (10) depend on the variation of $v_A(r) = B_0(r)/\sqrt{4\pi\rho_0(r)}$ with radius, r. To approximate the variation of the Alfven speed with radius, we obtained density measurements obtained from the latest p-mode inversions (Antia, private communication). These density measurements were combined with the assumption of a radial field to obtain $v_A(r)$. A plot of the result is shown in Figure 2. Near the solar surface the Aflven speed increases rapidly with radius as the density decreases. Just below the base of the convection zone the magnetic field strength increases rapidly at the seat of the solar dynamo. This also results in an increase in v_A. The result is that a radial cavity is formed where Alfven waves can be trapped in the convection zone.

A solution for the wave period was obtained in two ways. The Alfven speed profile shown in Figure 2 was used to numerically obtain the period of standing waves trapped in the convection zone. The results of these calculations are shown in Table 1. In addition, the "exact" Alfen speed profile was approximated by assuming an exponential variation with constant scale height of the form

$$v_A(r) = v_{A0} exp \frac{(r - r_0)}{\lambda} \tag{11}$$

where $r_0 = 0.7\ R_{sun}$ is the radius at the base of the convection zone. The resulting equation has the solution

$$U(r) = A[J_0(\alpha e^{\frac{-(r-r_0)}{\lambda}})Y_0(\alpha) - Y_0(\alpha e^{-(r-r_0)/over\lambda})J_0(\alpha)] \tag{12}$$

where $\alpha = R_{sun}\omega\lambda/v_{A0}$. The wave amplitude vanishes at the base of the convection zone, since the core is assumed to be field-free and cannot therefore support magnetic oscillations. At the surface the sharp increase in the Alfven speed reflects most of the wave energy causing the amplitude to vanish there also. This leads to a dispersion relation to determine the parameter alpha, and hence the period of the oscillation. The periods obtained, using a scale height of $\lambda = 0.7\ R_{sun}$, and an Alfven speed at the base of the convection zone of $v_{A0} = 3.2$ cm/s, are shown in Table 1. Comparison shows reasonable good agreement between the analytic and the numerical solutions.

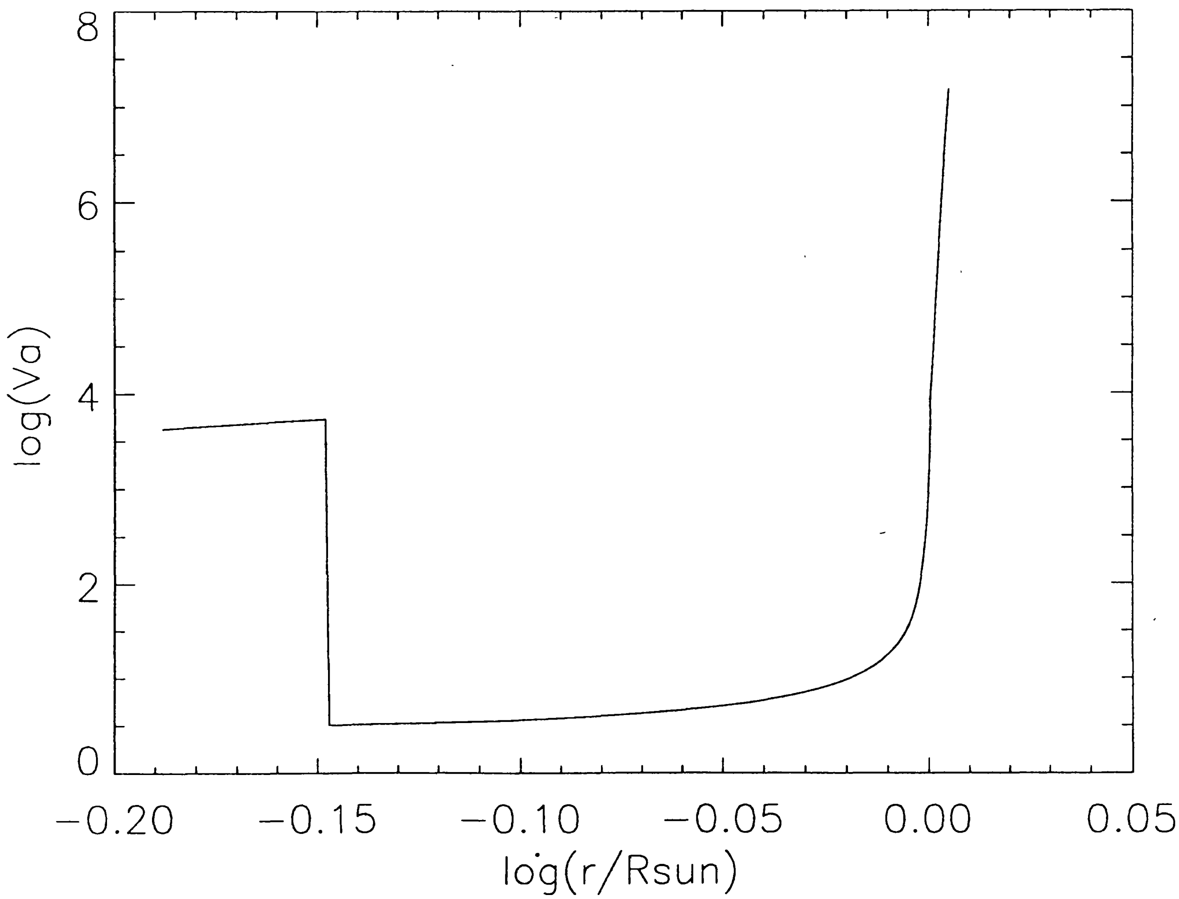

Figure 2. The Alfven speed varies with radius because both the magnetic field strength and the density are variable. At the surface the Alfven speed increases rapidly because the density rapidly drops. At the base of the convection zone, the Alfven speed is assumed to increase because of the sharp increase in magnetic field strength due to the flux tube dynamo. This causes a cavity where magnetic modes can be trapped.

5. Discussion and Conclusions

The p-modes and g-modes of the solar interior are well known. These motions have periods of order a few minutes to a few hours. The restoring force for these modes is proportional to the local gravity and pressure gradient, the two strongest forces in the solar interior. It is perhaps less well known that two longer-period motions of the solar interior can exist which are restored by the Coriolis and magnetic forces, two much weaker forces.

Both modes are incompressible. For non-axisymmetric perturbations the Coriolis force is the dominant. The resulting wave modes are Rossby waves also known as r-modes. These modes have periods of order the solar rotation period (or longer), and the period is not affected by the presence of differential rotation.

Table 1. Period in years for the magnetic b-mode assuming a 3 Gauss average field at the surface of the Sun.

Radial Order	Period (YR) Exponential v_A	Period (YR) "Exact" v_A
1	337	397
2	168	174
3	112	108
4	84	78
5	67	61
6	56	50
7	48	42
8	42	36
9	37	32
10	--	28
15	--	18
20	--	13
25	--	11
30	--	9

For axisymmetric perturbations, the Coriolis force is ineffective. These motions are only opposed by the (much weaker) magnetic force. The resulting waves, which we call b-modes, have a period even longer than the r-modes. The fundamental period is of order 350 years. Higher harmonics have periods of order the length of the solar cycle. It is not known whether these modes play a role in the 11-year magnetic variablity of the Sun. The expected dependence of the Alfven speed with depth in the convection zone is favorable for trapping these waves. An exitation mechanism for these waves has not been identified. Since they correspond to large scale torsional motions at the solar surface, perhaps they could be excited by the torque the variable solar wind exerts on the photosphere.

If b-modes were to be observed, they could provide a unique view of the average magnetic field below the photosphere. The phase speed depends only on the the mass density and the magnetic field strength. The density can be determined from p-mode observations, leaving only the magnetic field to be determined. Therefore periods could be used to determine strength and structure of the average sub- surface field. It remains to be seen whether r-modes and b-modes will provide us yet another window on the Sun's interior.

References

Komm R.W., Howard R.F., Harvey J.W., 1994, Sol Phys, 151, 15.
Provost J., Berthomieu G., Rocca A., 1981, A&A, 94, 126.
Saio H., 1982, ApJ, 256, 717.
Vandakurov Y.V., 1988, Sov Astron Lett, 14, 141.
Wolff C.L., Blizzard J.B., 1986, Sol Phys 105, 1.