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Asteroseismological calibration of open clusters

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Abstract. Combining classical and seismological data permits one to perform a calibration of stars, and therefore to improve the estimate of their fundamental parameters. We present preliminary results of the calibration of stars in the Hyades open cluster, using the SVD formalism. This permits us to see which are the most important observables for estimating stellar parameters. The analysis will help to optimize possible experiments and to improve the stellar modelling. It will also be useful for the choice of targets for the future seismic space mission STARS.

Key words: stars: oscillations, open clusters: Hyades

1. Introduction

Asteroseismology provides different kinds of information from classical astronomical data, and can therefore be very useful in determining fundamental stellar parameters. Combining classical and seismic data, improvement is obtained in the estimate of the mass, the age, the mixing-length parameter, etc. Previous studies have been carried out by Gough and Novotny (1993a) and Brown et al. (1994), and a review given by Gough (1995).

2. Calibration of open clusters

The open clusters are particularly interesting targets for testing stellar modelling since more constraints are available. Calibration of open clusters has been studied by Gough and Novotny (1993b) and Fridlund et al. (1994). Here we apply the SVD formalism, described by Brown et al. (1994), in the case of the Hyades cluster. Having chosen a set of M observables B_i, we construct a model whose N parameters P_j allow the calculation of these observables. We then assume that the model may be linearized in the neighborhood of a reference model characterized by parameters P_{j0}. If we denote by β_i the values of the actual observables and by σ_i their standard errors, the problem we wish to solve is to find the parameter corrections $P_j - P_{j0}$ such that

$$\chi^2 = \sum_{i=1}^{M} \frac{(B_i - \beta_i)^2}{\sigma_i^2} \tag{1}$$

is minimum. Assuming that the corrections are small, we obtain a linear relation between changes in the parameters and the resulting changes in the observables. This enables us to relate errors on the observables to errors on the inferred parameters. The SVD technique leads to a precise analysis of the transformation and facilitates the interpretation of the results.

We must point out that we are discussing only the accuracy of the parameter-fitting procedure, and our models are supposed to be adequate to reproduce the observations. Thus we do not consider the systematic errors which would arise from inadequacies of the models themselves, e.g. in the assumed physics.

Table 1. Observables B_i and their assumed errors σ_i. The four last lines correspond to the pseudo-observables added when no oscillation data are available.

B_i	σ_i
m_V	0.01 mag
$b-y$	0.01 mag
$\log Z$	0.1
π	0".004
D_0	0.05 μHz
d_0	0.3 μHz
$\nu_{n,\ell}$	0.4 μHz
$\log M$	0.20
$\log Y$	0.12
$\log \alpha$	0.40
$\log \tau$	1.20

We shall consider 6 stars of mass $M_j = 0.9, 1.0, 1.1, 1.5, 2.0$, and $2.2 M_\odot$, which are characteristic of stars belonging to the Hyades open cluster. In agreement with the literature, we assumed the age τ to be 700 million years, the distance D is 45 pc, the helium and heavy-element contents are 0.282 and 0.03 respectively, the mixing-length parameter α_j is 1.83 (the same value was used to compute all of the models, but α_j was taken as a free parameter for each star j individually when solving the least-squares problem). We use the EFF equation of state, with the OPAL opacities. The parameters we wish to infer are M_j, Y, Z, τ, α_j and D (with $j = 1$ to 6). The observables are the magnitudes $m_{V,i}$, the color indices $(b-y)_i$, the heavy-element abundance Z, the parallax π (with $i = 1$ to 6), and the seismic data. Each observable has associated with it an uncertainty σ_i (see Table 1). The uncertainties adopted for the seismic data correspond to the one-month observations projected for the STARS mission. We consider three cases for the available seismic observations:

1 - no oscillation data,

2 - the large and small splittings D_0 and d_0 are observed ($D_0 = \langle \nu_{n,\ell} - \nu_{n-1,\ell} \rangle_{n,\ell}$ and $d_0 = \langle \nu_{n,0} - \nu_{n-1,2} \rangle_n$, where n is the radial order and ℓ the degree),

3 - individual frequencies $\nu_{n,\ell}$

In the first case, there are fewer observables than parameters, and the problem is underconstrained. In order to have a well-posed problem in this case, we add pseudo-observables, consisting of $\log M_i$, $\log Y$, $\log \alpha_i$ and $\log \tau$ (see Brown et al., 1994). They are assigned standard errors σ_i which represent the entire plausible range of the corresponding parameters for our choosen stars (see the last lines of Table 1). We must, however, be aware that these pseudo-observables are only prejudices.

Since solar-like stars pulsate in modes of high radial order, while δ Scuti-like stars pulsate in modes of low radial order, we have selected modes in the range $n = 17 - 29$ for the stars of mass 0.9, 1.0 and 1.1 $M_\odot$, and modes with $n = 1 - 10$ for the three other stars. The degree $\ell = 0, 1, 2$, as expected for the STARS mssion. No mixed mode is present in the high-mass stars due to their young evolutionary state.

Figure 1 shows the logarithm of the relative errors in the parameters P_j, in the 3 different cases of seismic data described above. Each of these steps involves additional information, so the errors derived from the case 1 are larger than those of the two other cases. An exception occurs for the mixing-length parameter of the stars of mass 2 and 2.2 $M_\odot$, where case 2 gives worse results than case 1. Here we do not constrain α beyond the prejudices, mainly because the stellar structure is insensitive to the value of α for high-mass stars. Seismic observations greatly improve the knowledge of all the other parameters.

We have assumed that our models were adequate to reproduce the observations. Looking at the very small formal errors, one can expect much larger discrepancies due to deficiencies in the modelling of the stars, and due to practical difficulties in using real data. Errors in assumptions about the physics of the models should easily show up (Brown et al., 1994).

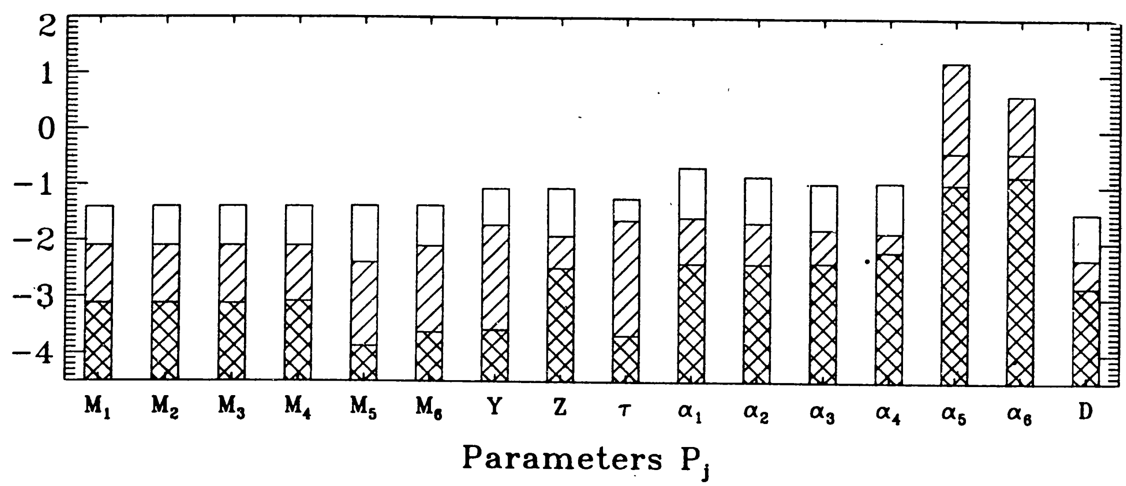

Figure 1. Logarithm of the relative errors in the parameters P_j, for 6 stars of mass 0.9, 1.0, 1.1, 1.5, 2.0, and 2.2 $M_\odot$. Three different cases are considered for the available seismic observations: no oscillation data (open boxes), the large and small splittings D_0 and d_0 (hatched boxes), and individual frequencies (cross-hatched boxes). Seismic data greatly improve the estimate of most of the parameters.

3. Conclusions

We shall extend this work to many more stars, in order to provide more constraints on the system. We can also consider other kinds of observations, such as the gravity or the masses for binary systems. Precise photometric and spectroscopic data are also required; the upcoming HIPPARCOS data will provide us with accurate value of the parallax.

In this study we analyse the accuracy of the knowledge of the parameters. Our next step is to infer the values of these stellar parameters from observations.

Our work will make it possible to optimize experiments and to help in the choice of their targets, especially within the STARS project.

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