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The Influence of Chromospheric Magnetism on Oscillation Frequencies

B. Roberts

School of Mathematical and Computational Sciences, University of St Andrews, St Andrews, Fife KY16 9SS, Scotland

Abstract. The influence of chromospheric magnetism on p- and f-modes is examined. The magnetism is modelled as a horizontal canopy field, embedded in an isothermal atmosphere, overlying a field-free polytropic convection zone. Both low and high frequency modes are considered. The recently detected solar cycle variability of p-modes is explored from the view-point of this model. The dual influence of convective motions and magnetism on the f-mode is discussed.

Key words: Magnetic effects, p- and f-mode frequency shifts, activity cycle

1. Introduction

Magnetism is all important in the Sun's atmosphere; indeed, much of what we see in the solar atmosphere – be it the spicules of the chromosphere or the high temperature of the corona – is to be understood as a consequence of magnetism, a magnetism that is rooted in the photospheric concentrations of magnetic flux in sunspots and intense flux tubes. The acoustic oscillations of the Sun, the p-modes, are basically determined by the run of sound speed in the solar interior, but the oscillations penetrate into the upper layers of the atmosphere and so are influenced by those layers. The effects are not large, because the modal structure of the oscillations is basically determined by the internal structure of the Sun, but they are distinctive. If we are to understand fully these modes – and with them the atmosphere of the Sun – we need to understand the influence of the chromosphere on acoustic oscillations, and in particular the role of magnetism.

Observations show that p-modes are significantly absorbed by sunspots (Braun et al. 1987, 1988; Braun 1995). So magnetism certainly may have an appreciable influence on p-modes, though the precise mechanism by which p-modes and sunspots interact has yet to be determined (see reviews by Bogdan 1992 and Spruit 1996).

Of particular interest here is the reported variation of p-modes with the solar cycle. The effect is evident both for low degree ($0 \leq l \leq 4$) modes (Woodard & Noyes

1985; Elsworth et al. 1990, 1994; Anguera Gubau et al. 1992; see the reviews by Pallé 1995, 1996) and intermediate degree ($5 \leq l \leq 140$) modes (Libbrecht & Woodard 1990; Woodard & Libbrecht 1991; Woodard et al. 1991; Bachmann & Brown 1993; see the review by Rhodes et al. 1995). Moreover, frequency variations are correlated with solar activity indices (Woodard et al. 1991; Bachmann & Brown 1993). Observations of high frequency ($\nu > 5$ mHz) modes, modes with frequencies above the acoustic cutoff of the chromosphere, show that they too vary with the solar cycle (Ronan et al. 1994); the effect is opposite in sign and larger in magnitude to that reported at low frequencies.

Here we consider the magnetic and thermal influences of a magnetic canopy on p- and f-modes. Our discussion is based upon the detailed investigations of Campbell & Roberts (1989), Evans & Roberts (1990, 1992), and Jain & Roberts (1993, 1994a, 1996). The influence of photospheric flux tubes is also important and offers an alternative, though complementary, view of frequency shifts to that outlined here (see Goldreich et al 1991; Rosenthal 1995). Other effects, such as non-adiabaticity or the presence of flows below the canopy, are likely to be important too. We consider briefly the influence of flows on the f-mode, following Murawski & Roberts (1993a, b).

Finally, we note that an evolving magnetic field at the base of the convection zone – the commonly supposed location for the storage and manipulation of magnetic flux – can also cause frequency shifts, especially for low degree modes; however, unless the field at the base of the convection zone is of megagauss strength (10^6 G), the frequency shifts from such a field are below those observed (Roberts & Campbell 1986). Such a megagauss field is rather stronger than is normally supposed but cannot be ruled out. Consequently, the question of the influence of buried magnetic fields on the frequencies of global oscillations remains of considerable interest.

2. The Influence of a Magnetic Canopy

We consider the effect of a magnetic chromosphere on p-mode and f-modes. In the Sun, the concentrations of magnetic field in sunspots and intense flux tubes rapidly expand outwards in response to the rapid fall-off of the confining gas pressure, stratified by gravity. The result is a complex *canopy* of magnetic field overlying essentially field-free regions (Gabriel 1976; Spruit & Roberts 1983; Solanki 1993). Observations by Giovanelli (1980) and Jones & Giovanelli (1983) indicate nearly horizontal magnetic fields by a height of about 1000 km above an optical depth of unity in the photosphere.

The true magnetic canopy of the solar chromosphere is clearly a complex three-dimensional structure, too complicated to incorporate in any analytical study of oscillations. Instead, we consider a simple planar model in which the canopy field is represented by a purely horizontal magnetic field, modelling the *mean* structure of the magnetic atmosphere, embedded in an isothermal gas with temperature T_c. We take a cartesian coodinate system x, y, z, with the z-axis pointing down (aligned with gravity). The magnetic field overlies a field-free medium, taken to represent the convection zone.

The equilibrium structure of the medium is determined by magnetohydrostatics. For a horizontal field $\mathbf{B} = B(z)\hat{\mathbf{x}}$, producing a magnetic pressure $B^2(z)/2\mu$, the gas pressure $p(z)$ and a mass density $\rho(z)$ are stratified under gravity according to:

$$\frac{d}{dz}\left(p(z) + \frac{B^2(z)}{2\mu}\right) = g\rho(z), \qquad (1)$$

where g (= 274 m s^{-2}) is the local gravitational acceleration in the Sun. Thus magnetism influences the equilibrium structure of the atmosphere, the field providing additional pressure forces. The canopy field $B(z)\hat{\mathbf{x}}$ is confined to the atmosphere ($z < 0$), the region $z > 0$ being field-free, and so the canopy base $z = 0_+$ is a current sheet across which (from equation (1)) there is total pressure balance:

$$p(z = 0_+) = p_c + \frac{B_c^2}{2\mu}, \tag{2}$$

where p_c is the gas pressure and B_c the magnetic field strength at the base ($z = 0_-$) of the canopy. The structure of the magnetic field in the chromosphere is unknown; we suppose that the field is *uniform*, so that $B(z) \equiv B_c$. Such a field provides no support for the gas, but it nonetheless exerts an influence through pressure balance.

We are particularly interested in effects that may produce the observed solar cycle variations in p-modes, and we consider the role of the magnetic canopy in producing such variations. To do this we regard the solar interior as unchanging, so that in the convection zone region $z > 0$ the equilibrium pressure $p(z)$, density $\rho(z)$ and temperature $T(z)$ are all held fixed. We then consider how changes in the magnetic atmosphere ($z < 0$), i.e. changes in the field strength B_c and temperature T_c, considered slow compared with the timescales of the p-modes, manifest themselves in frequency shifts in those modes. It is likely too that changes in the canopy height occur in response to activity changes, though we do not consider this effect here (see Evans & Roberts 1991). Nor do we consider changes in the *internal* field of the Sun (see Roberts & Campbell 1986).

Changes in either B_c or T_c result in changes in the Alfvén speed within the atmosphere. Suppose T_c is *increased*, holding B_c fixed. Then, since B_c is held fixed, pressure balance (2) means that there can be no change in the gas pressure p_c at the canopy base, so the density ρ_c at the base of the canopy is reduced; consequently, the Alfvén speed $(B_c^2/\mu\rho_c)^{1/2}$ at the canopy base is *increased*. Thus, an increase in chromospheric temperature T_c leads not only to an increase in the sound speed of the atmosphere but also to an increase in the Alfvén speed at the canopy base. An increase in Alfvén speed at the canopy base does not, however, imply a corresponding increase *everywhere* in the atmosphere; the Alfvén speed increases with height exponentially fast, on a scale that is twice the density scale height. Thus, an *increase* in T_c results in an Alfvén speed that is *higher* at the base of the canopy but *lower* at some height in the atmosphere.

An *increase* in B_c, with T_c held fixed, results in a decrease in the density (to maintain the same total pressure) and so to an *increase* in the Alfvén speed, both because the base field has increased and because the gas density has fallen. If both B_c and T_c are increased simultaneously, then the Alfvén speed at the canopy base increases but, because the density now falls off slower with height, higher in the atmosphere it may well be *lower* than for a smaller field strength. Of importance for p-modes is the behaviour of the magnetoacoustic speed $(c_s^2 + c_A^2)^{1/2}$; we see that an increase in both B_c and T_c may paradoxically lead to an increase in this speed low in the atmosphere but a decrease high in the atmosphere. Thus waves of low frequency may respond to changes in chromospheric conditions differently from waves of high frequency.

We turn now to a consideration of the dispersion relation for p- and f-modes in the presence of a magnetic canopy. Consider first the *field-free* zone ($z > 0$). Linear velocity perturbations of the form $\mathbf{v}(x,z,t) = (v_x(z), 0, v_z(z)) \exp i(\omega t - kx)$ satisfy the differential equations (see Lamb 1932)

$$(g^2k^2 - \omega^4)v_z = \omega^2 c_s^2 \frac{d\Delta}{dz} + g(\gamma\omega^2 - k^2c_s^2)\Delta, \tag{3}$$

$$\frac{dv_z}{dz} + (\frac{gk^2}{\omega^2})v_z = (1 - \frac{k^2c_s^2}{\omega^2})\Delta, \tag{4}$$

where $c_s(z) = (\gamma p(z)/\rho(z))^{\frac{1}{2}}$ is the sound speed in the gas with adiabatic index $\gamma\,(= 5/3)$, and $\Delta \equiv \operatorname{div}\mathbf{v}$. Consider a polytrope with sound speed squared linear in depth z:

$$c_s^2(z) = c_0^2(1 + \frac{z}{z_0}), \quad p(z) \propto (1 + \frac{z}{z_0})^{m+1}, \quad \rho(z) \propto (1 + \frac{z}{z_0})^m, \quad z > 0, \tag{5}$$

so that $c_s = c_0$ at $z = 0_+$; the polytropic index m satisfies $m + 1 = \gamma g/(c_s^2)' = \gamma g z_0/c_0^2$, the prime ($'$) denoting differentiation with respect to depth z. An exact solution of equations (3) and (4) is (Lamb 1932; Spiegel & Unno 1962; Campbell & Roberts 1989)

$$\Delta = \Delta_0 \exp[-k(z + z_0)]U(-a, m + 2, 2kz_0 + 2kz), \qquad z > 0, \tag{6}$$

where Δ_0 is an arbitrary constant and U is a confluent hypergeometric function. In writing (6) we have imposed the condition that the energy density in the mode declines at great depth (as $z \to \infty$). For adiabatic stratification, corresponding to a neutrally buoyant medium, $m = 1/(\gamma - 1)$ and the parameter a is given by $2a = m\Omega^2 - (m + 2)$, where $\Omega^2 \equiv \omega^2/(gk)$. The velocity component v_z follows from Eq. (3).

Consider now the motions in the *magnetic* atmosphere ($z < 0$). In the presence of a magnetic field satisfying the equilibrium (1), motions $v_z(z)$ satisfy (e.g., Roberts 1985)

$$\frac{d}{dz}\left[\rho(z)(c_s^2 + c_A^2)\left(\frac{\omega^2 - k^2c_t^2}{\omega^2 - k^2c_s^2}\right)\frac{dv_z}{dz}\right] +$$
$$\left[\rho(\omega^2 - k^2c_A^2) + gk^2\left(\frac{\rho c_s^2}{\omega^2 - k^2c_s^2}\right)' - \frac{\rho g^2k^2}{\omega^2 - k^2c_s^2}\right] v_z = 0, \tag{7}$$

where $c_t(z) = c_s(z)c_A(z)/(c_s^2(z) + c_A^2(z))^{1/2}$ denotes the MHD cusp speed and $c_A(z) = B(z)/(\mu\rho(z))^{1/2}$ is the Alfvén speed within the medium. For a uniform field in an isothermal atmosphere Eq. (7) has solutions in terms of hypergeometric functions (e.g., Adam 1977; Evans & Roberts 1990).

We may match solutions across the interface $z = 0$ by requiring that v_z is continuous. Additionally, in the presence of a magnetic field we require (from Eq. (7)) that

$$\rho(z)(c_s^2 + c_A^2)\left(\frac{\omega^2 - k^2c_t^2}{\omega^2 - k^2c_s^2}\right)\frac{dv_z}{dz} + gk^2\left(\frac{\rho c_s^2}{\omega^2 - k^2c_s^2}\right)v_z \qquad \text{be continuous.}$$

These conditions imply a dispersion relation of the form

$$2\Omega^2\frac{U'(-a, m + 2, 2kz_0)}{U(-a, m + 2, 2kz_0)} + (m + 1)\frac{\Omega^2}{kz_0} - (1 + \Omega^2) = \mathcal{F}_c, \tag{8}$$

where $U'(-a, m+2, z) \equiv dU(-a, m+2, z)/dz$. The expression $\mathcal{F}_c$ is in general complicated; in the field-free case it is simply an algebraic form (Campbell & Roberts 1989); in a uniform field it involves hypergeometric functions (Evans & Roberts 1990; Jain & Roberts 1993). More complicated atmospheres require either an approximate analytical treatment (e.g., Wright & Thompson 1992) or a numerical investigation.

It proves possible to derive approximate analytical expressions from (8) that are valid when kz_0 is small. Write $k = l/R_S$, where R_S is the solar radius and l is the degree of the mode, then $kz_0 \ll 1$ translates approximately into $l \ll 3000$, which covers all but the highest degree modes observed. With $kz_0 \ll 1$, we obtain the results

$$\Omega^2 \approx \begin{cases} \Omega_n^2 - A_n(kz_0)^{m+2}, & \text{no field,} \\ \Omega_n^2 + B_n(kz_0)^{m+1}, & \text{uniform field,} \end{cases} \tag{9}$$

where $\Omega_n^2 \equiv 1 + (2n/m)$ for p-modes of order $n = 1, 2, 3, \cdots$. The positive coefficients A_n and B_n are too complicated to give here (see Campbell & Roberts 1989; Evans & Roberts 1990, 1992; Jain & Roberts 1993, 1994a); but we note the property that $B_n = 0$ in the *absence* of a magnetic field, indicating that the next order correction is needed if the field strength B_c is too low; note too the differing powers of k that arise.

Application of Stirling's formula for the asymptotic behaviour of the gamma function allows us to simplify the coefficients A_n and B_n when $n \gg 1$ (Evans 1990; Evans & Roberts 1992). Of interest here is the *change* in cyclic frequency $\nu\,(\equiv \omega/2\pi)$ as a consequence of changes in the chromosphere. Denote by $\Delta\nu \equiv \nu(B_c', T_c') - \nu(B_c, T_c)$ the difference between a p-mode's cyclic frequency in the presence of an isothermal atmosphere with field strength B_c' and temperature T_c' and one with field strength B_c and temperature T_c. Denote by p_p and T_p the gas pressure and temperature at the top of the field-free convection zone ($z = 0_+$).

In the non-magnetic case ($B_c = B_c' = 0$), an atmospheric temperature change $\Delta T \equiv T_c' - T_c$ produces a frequency shift $\Delta\nu$ given by (Jain and Roberts 1993, 1994a)

$$\frac{1}{\nu}\Delta\nu \approx -C_m \frac{lz_0}{R_S}\frac{\Delta T}{T_p}\left(\frac{\nu}{\nu_0}\right)^{2m+2}, \tag{10}$$

where $\nu_0 = (g/mz_0)^{1/2}/2\pi$ is a base frequency. Similarly, the frequency shift as a consequence of a change in chromospheric magnetic field strength, holding fixed the temperature T_c, is given by (Evans and Roberts 1992; Jain and Roberts 1993, 1994a):

$$\frac{1}{\nu}\Delta\nu \approx D_m \frac{lz_0}{R_S}\frac{1}{p_p}\Delta(B_c^2/2\mu)\left(\frac{\nu}{\nu_0}\right)^{2m}, \tag{11}$$

where $\Delta(B_c^2/2\mu) \equiv {B_c'}^2/2\mu - B_c^2/2\mu$ denotes the change in atmospheric magnetic pressure. The coefficients C_m and D_m are given in terms of the gamma function $\Gamma(m)$:

$$C_m = \frac{1}{4\gamma}\frac{(1+m)^{2m}}{2^{2m}}D_m, \qquad D_m = \frac{1}{\Gamma(m+1)\Gamma(m+2)}.$$

Since C_m and D_m are both positive, we see that in the absence of a magnetic field an *increase* in chromospheric temperature ($T_c' > T_c$) leads to a *decrease* in frequency

($\Delta\nu < 0$), for $kz_0 \ll 1$ (Campbell & Roberts 1989; Evans & Roberts 1990; Goldreich et al. 1991; Wright & Thompson 1992; Jain & Roberts 1993; Hindman & Zweibel 1994). By contrast, in the absence of any change in chromospheric temperature, an *increase* in chromospheric magnetic field strength ($B_c' > B_c$) leads to an *increase* in frequency ($\Delta\nu > 0$), for $kz_0 \ll 1$ (Evans & Roberts 1990, 1992; Wright & Thompson 1992; Jain & Roberts 1993). Notice also that for low frequencies, $\nu \ll \nu_0$, the shift $\Delta\nu$ tends to be dominated by magnetic effects (because of the smaller power of ν^2/ν_0^2), and so is positive, whereas at high frequencies ($\nu \gg \nu_0$) the opposite occurs and shifts are negative and dominated by temperature changes. This argument has to be qualified, though, because the non-magnetic case pertains to frequencies below cutoff.

Finally, we note that a combination of the results in equations (10) and (11) *suggests* a formula of the *form* (Jain & Roberts 1993, 1994a)

$$\frac{1}{\nu}\Delta\nu \approx \frac{lz_0}{R_S}\left(\frac{\nu^2}{\nu_0^2}\right)^m \left\{\mathcal{D}_m\left(\frac{\Delta B_c^2}{2\mu p_p}\right) - \mathcal{C}_m\left(\frac{\Delta T}{T_p}\right)\left(\frac{\nu^2}{\nu_0^2}\right)\right\}, \tag{12}$$

where, however, the coefficients $\mathcal{C}_m$ and $\mathcal{D}_m$ may now be different from those in (10) and (11) (and in particular $\mathcal{C}_m$ may now depend upon field strength). A numerical solution of Eq. (8) lends some support for this suggested form (Jain & Roberts 1996).

At this point we should recall the observations of p-mode variation over the solar cycle. Considering the change in frequency from year to year, as a function of frequency in a base year, intermediate degree modes ($5 < l < 140$) have been found first to increase as a function of frequency and then to fall precipitously at about $\nu = 4\,$mHz (Libbrecht & Woodard 1990; Woodard *et al.* 1991); observed frequency changes are determined for the years 1988 and 1989, relative to the base year 1986 (1986 being solar minimum, whereas 1988 is on the rise phase of solar activity and 1989 is close to maximum). We consider the hypothesis that the observed frequency shifts are a consequence of changes in magnetic activity in the chromospheric layers, bearing in mind that other effects (e.g., vertical fields in flux tubes, non-adiabatic effects, granulation, etc.) – not included here – may also be contributory. Thus, we identify the reported changes in frequency, from one year to another, with the frequency shifts $\Delta\nu$ determined above as a consequence of magnetic and thermal changes in the chromosphere.

Figure 1 shows the application of Eq. (12) to frequency shifts as a consequence of an increase in chromospheric field strength from $B_c = 30$ G to $B_c' = 50$ G or $B_c' = 61$ G, combined with a simultaneous rise in chromospheric temperature from $T_c = 4170$ K to $T_c' = 6370$ K or $T_c' = 7770$ K. The calculated shifts compare favourably with the observations. In the absence of any temperature change, magnetic effects alone are able to match the rise phase (for $1 < \nu < 3.5$ mHz) in frequency shifts (Evans & Roberts 1992), but a temperature change seems essential for producing the observed turnover (Jain & Roberts 1994a; see also Goldreich et al. 1991; Hindman & Zweibel 1994). The reproduction of this distinctive observational feature, of a rise phase followed by a rapid downturn, offers considerable encouragement that chromospheric effects are indeed responsible to a large extent for the observed shifts. However, the illustrative temperature changes supposed by Jain & Roberts (1994a) are rather large, suggesting that perhaps

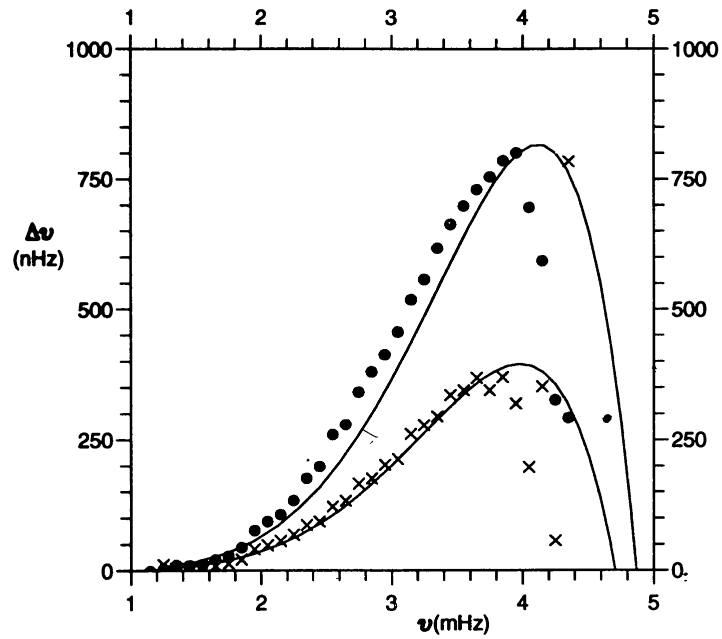

Figure 1. Calculated frequency shifts $\Delta\nu$ (in nHz) as a function of frequency ν (in mHz), according to equation (12) with $\mathcal{C}_m = C_m$ and $\mathcal{D}_m = D_m$. The curves are for *p*-modes of degree $l = 75$. The shifts arise for an increase in chromospheric field strength from $B_c = 30$ G to $B'_c = 50$ G (lower curve) or $B'_c = 61$ G (upper curve), combined with a simultaneous rise in chromospheric temperature from $T_c = 4170$ K to $T'_c = 6370$ K (lower curve) or $T'_c = 7770$ K (upper curve). Also shown are the observations of Libbrecht and Woodard (1990) for years 1988 (crosses) and 1989 (filled circles), compared with 1986. (From Jain & Roberts 1994a)

other effects, possibly masquerading as temperature changes, may also be important. The phenomenon warrants further detailed observational and theoretical study.

We turn now to a consideration of frequency shifts at high frequencies (beyond the acoustic cutoff). In a non-magnetic isothermal atmosphere frequencies above about 4.8 mHz lead to modes which *leak* from the atmosphere. However, in the presence of a uniform magnetic field, the Alfvén speed becomes arbitrarily large high in the atmosphere, dominating the structure of the motions: all modes are trapped, whatever their frequency. Dispersion relation (8) is still applicable. Johnston et al. (1995) and Jain & Roberts (1996) have examined high frequency waves using the uniform field model of the chromosphere, demonstrating that frequency shifts at high ν are *much larger in magnitude and opposite in sign* to those occurring at low ν. The shifts are largest in magnitude between $\nu = 6$ mHz and $\nu = 7$ mHz, and are comparable with those observed by Ronan et al. (1994). Figures 2 and 3 illustrate the effect. At higher degree l, the shifts are larger (see Figure 4), being approximately proportional to the degree (cf. Goldreich et al. 1991; Evans & Roberts 1992; Wright & Thompson 1992; Jain & Roberts 1993). The positive frequency shift observed at low frequencies by Libbrecht & Woodard (1990) is not apparent in Figures 2 and 3, simply because it is dominated in those figures by the negative shift at high frequency. In fact, an inspection of the results for low frequency reveals the presence of a positive shift.

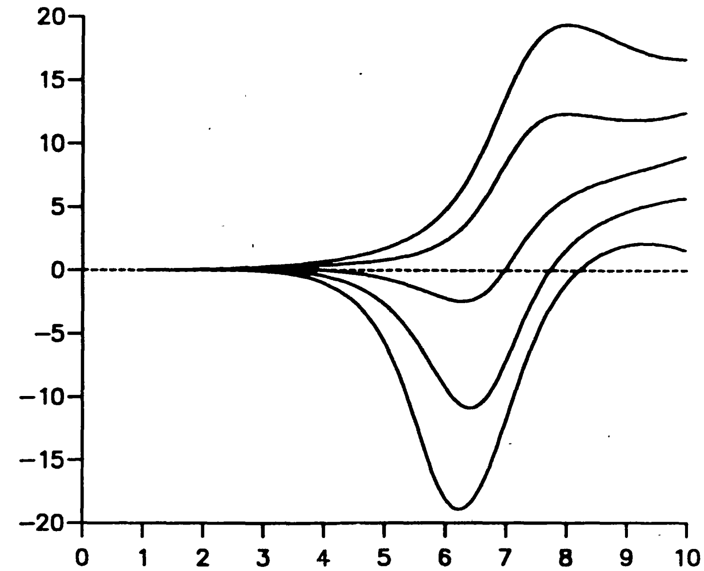

Figure 2. Calculated frequency shifts $\Delta\nu$ (in μHz) as a function of frequency ν (in mHz), for p-modes of degree $l = 50$. The shifts arise for an increase in chromospheric field strength from $B_c = 10$ G to $B'_c = 30$ G, with no change in chromospheric temperature (top curve), a rise from $T_c = 4170$ K to $T'_c = 5000$ K, 6000 K, 7000 K, and to 8000 K (bottom curve). (From Johnston et al. 1995)

Finally, we consider the f-mode. The classic f-mode in a planar atmosphere has dispersion relation $\omega^2 = gk$, $\nu = (gk)^{1/2}/2\pi \approx 100l^{1/2}\mu$Hz. Observations (Bachmann et al. 1995, and references therein; see also the overview by Kosovichev 1995) have revealed that in fact the f-mode departs from this parabola: for $l < 800$ frequencies are up to 5 μHz above the basic parabola, whereas for $l > 1000$ frequencies lie significantly below the parabola; fractional frequency shifts are perhaps as much as +1% at low l, falling to −2% or −3% for $l > 2000$.

The frequency difference between the classic f-mode parabola and the measured frequencies, with the indication of a cross-over, raises the question of what physical processes are responsible for that behaviour. One the one hand, it is known from theoretical studies that the magnetism of the chromosphere *raises* the frequency above the standard parabola (Campbell & Roberts 1989; Evans & Roberts 1990; Miles & Roberts 1992; Miles et al. 1992; Jain & Roberts 1994b), a consequence of the increased elasticity of the atmosphere. Specifically, for a uniform chromospheric field the difference $\Delta\nu (\equiv \nu(B'_c) - \nu(B_c))$ between the frequency in the presence of a magnetic atmosphere of strength B'_c and one

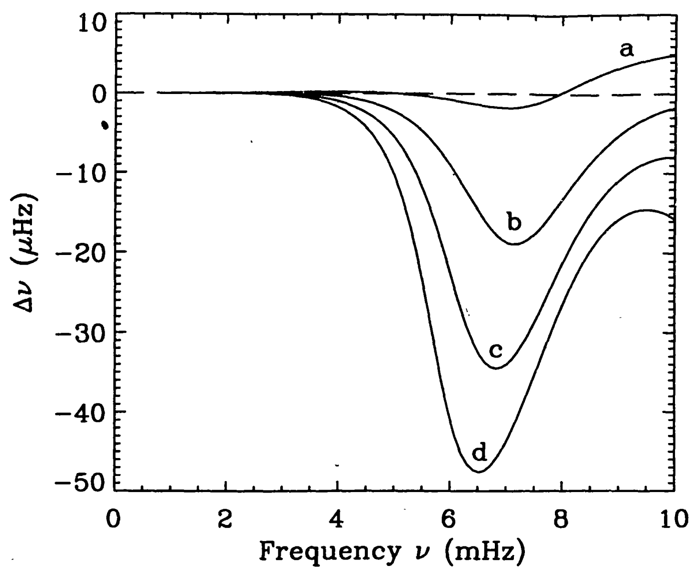

Figure 3. Calculated frequency shifts $\Delta\nu$ (in μHz) as a function of frequency ν (in mHz), for p-modes of degree $l = 100$. The shifts arise for an increase in chromospheric field strength from $B_c = 20$ G to $B_c' = 30$ G, combined with a chromospheric temperature rise from $T_c = 4170$ K to $T_c' = 5000$ K (case a), to 6000 K (b), to 7000 K (c), and to 8000 K (case d). (From Jain & Roberts 1996)

with strength B_c follows from equation (9), valid for $kz_0 \ll 1$, by taking the limit $n \to 0$. The result is (Evans & Roberts 1990)

$$\frac{1}{\nu}\Delta\nu \approx E_m \frac{1}{p_p}\Delta(B_c^2/2\mu)(\frac{\nu^2}{\nu_0^2})^{m+1}, \qquad E_m = \frac{2^m}{\Gamma(m+2)m^{m+1}}. \tag{13}$$

An illustration is of interest. Take $m = 3/2$, $p_p = 80$ N m^{-2}, and consider a chromospheric field strength of $B_c' = 25$ G and set $B_c = 0$; then (13) gives $\Delta\nu/\nu \approx 3 \times 10^{-5}(l/100)^{5/2}$. For modes of degree $l \leq 100$ (frequencies $\nu \leq 1$ mHz) the influence of the field is negligible; however, by $l = 400$, $\nu = 2$ mHz the fractional shift has risen to 0.1%, and by $l = 600$, $\nu = 2.45$ mHz we have $\Delta\nu/\nu \approx 0.26\%$, giving a shift of $\Delta\nu \approx 6\,\mu$Hz. Thus magnetic effects can explain the observed positive departure of the f-mode frequency above the classical parabola for modes of moderate degree.

On the other hand, theoretical studies have indicated that the presence of convective flows, such as the granulation or supergranulation, has an effect on the oscillations (Gough & Toomre 1983; Murawski & Roberts 1993a, b; Rosenthal et al. 1995). Specifically, for a simple incompressible fluid, Murawski & Roberts (1993a, b) have demonstrated that f-mode frequencies are *reduced* in the presence of a stochastic flow. A negative frequency shift, broadly consistent with that observed, is found for very small (10^2 km) or very large (4×10^3km) convective cells with flows of about 1 km s^{-1}, suggesting that granules may have a significant influence on the f-mode (and presumably p-modes too). Thus we

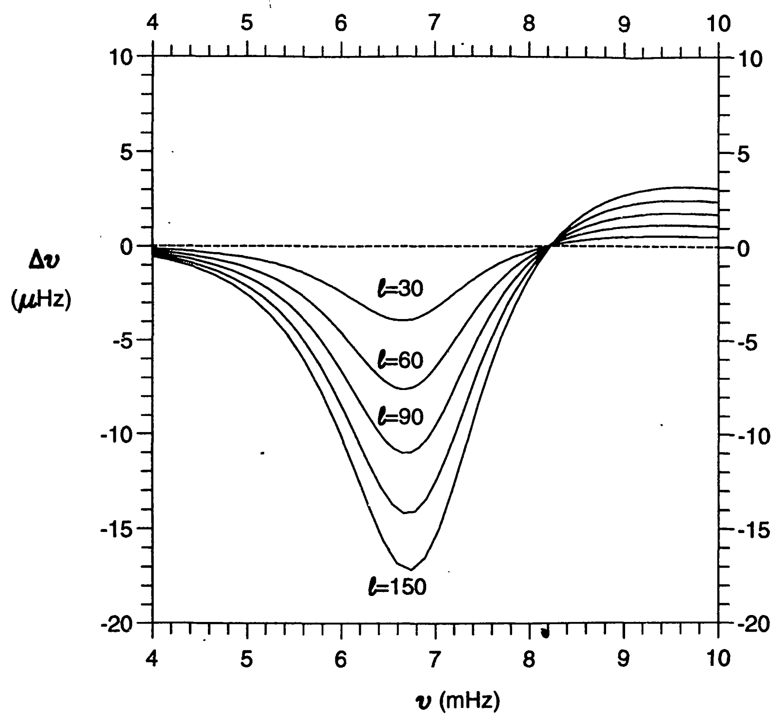

Figure 4. Calculated frequency shifts $\Delta\nu$ (in μHz) as a function of frequency ν (in mHz), for p-modes of degrees $l = 30$ to 150. The shifts arise for an increase in chromospheric field strength from $B_c = 10$ G to $B_c' = 15$ G, combined with a chromospheric temperature rise from $T_c = 4170$ K to $T_c' = 5000$ K. (From Jain & Roberts 1996)

have two effects, flows and magnetism, that influence the f-mode, and these effects may act in opposition to one another. This suggests a cross-over effect, much as is observed. An assessment of the combined influences of flow and magnetism has been made for a simple model of surface waves (Murawski & Goossens 1993), and it would be especially interesting to extend that analysis to more realistic circumstances. Of course, other factors may be at work too. Ghosh et al. (1995; see also Ghosh 1996) considers the role of various flow profiles, and finds that in addition to a negative shift at high wavenumber there is a positive shift at low wavenumber. In other words, a flow on its own is able to produce frequency cross-over. The effect warrants further study.

3. Conclusions

We have outlined the manner in which chromospheric magnetism influences the frequencies of p- and f-modes in a distinctive fashion. For the p-modes we have argued that a simultaneous rise in chromospheric magnetism and temperature leads to frequency shifts that are similar to those observed in the rise phase of the present solar cycle. Similarly, we have indicated that the f-mode may suffer a positive frequency shift from the standard $\omega^2 = gk$ when it propagates in the presence of a magnetic chromosphere, and a negative shift when it propagates in the presence of granules. These influences are expected to combine to produce a positive shift at low degree and a negative shift at high degree, though the effect has yet to be demonstrated for realistic solar conditions. Other, non-magnetic, models also seem capable of producing positive and negative shifts, and warrant further study. We suggest that the f-mode should vary over the solar cycle, though this has yet to be demonstrated observationally.

The precision of current observations will go a long way to resolving the many questions concerning the importance of all the various effects on p- and f-modes we have outlined. Currently, there is no consensus of opinion as to precisely what is the f-mode, let alone what produces frequency shifts. We take the view that the f-mode is basically the exact solution $\omega^2 = gk$ of the gas dynamic equations, modified slightly by the presence of a magnetic chromosphere and granules. In the absence of a magnetic field, the exact solution produces a motion that grows with height, though its kinetic energy density declines provided $2H_ck < 1$ (Roberts 1991). In the presence of a uniform magnetic atmosphere, though, the motion itself will ultimately decline with height. Thus, the presence of a uniform field in the atmosphere significantly changes the nature of motions within the atmosphere, but not the motion within the field-free zone below (where the motion is essentially one of exponential decline). Other views of the f-mode have been explored. Rosenthal & Gough (1994), Hindman & Zweibel (1994) and Rosenthal & Christensen-Dalsgaard (1995) have considered the f-mode as an interface wave between two non-magnetic atmospheres, representing the chromosphere and corona. Further studies in all these areas are warranted if we are to understand fully the actual f-mode.

The advent of routine high precision measurements of the oscillation frequencies, combined with magnetogram and velocity measurements at the solar surface, expected from GONG and SOHO, will shed considerable light on the theoretical issues addressed here. We await such developments with much interest.

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