Bull. Astr. Soc. India (1996) 24, 125–128

The effect of p-modes on thin magnetic flux tubes

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Abstract. We examine the effect of p-modes impinging on thin magnetic flux tubes. The aim of the study is to see whether global p-modes can excite wave motions of significant amplitude in a magnetic flux tube. For simplicity we treat an isothermal flux tube with the same temperature as the external gas. We calculate the linear response of the tube when p-modes in the ambient are incident on it. This interaction leads to the excitation of a tube wave, which is allowed to leak out of the lower boundary. In general we find that the interaction is non-resonant, in contrast to the unstratified tube. The amplitude of the tube wave is largest for external p-modes of low degree and moderate order. We also calculate the absorption cross-section of p-modes by flux tubes.

Key words: MHD - Sun: oscillations - Sun: magnetic fields

1. Introduction

Intense magnetic flux tubes are ubiquitous in the solar photosphere and play an important role in the dynamics and energy transport of the atmosphere. Their field strengths are empirically known to be in the range 1-2 kG. Typically these tubes are *thin* with diameters of a few hundred kilometers, which is generally believed to be much less than the horizontal separation between adjacent tubes. The medium between the tubes is practically field-free and supports p-modes with periods in the 5-min range.

The aim of the present investigation is to examine the effect of p-modes on thin flux tubes. The interaction of p-modes with magnetic flux tubes has been extensively studied (e.g., Ryutov and Ryutova 1976; Thomas, Cram & Nye 1982; Bogdan & Zweibel, 1987; Hollweg 1988; Bogdan 1989; Spruit & Bogdan 1992; Ryutova & Priest 1993; Cally & Bogdan 1993; Cally, Bogdan & Zweibel 1994; Hasan 1995; Bogdan et al. 1995; Hasan

& Bogdan 1996; for upto date reviews see Bogdan 1995 and Spruit 1996). Most of the previous investigations have neglected the effect of stratification due to gravity. This effect is very important and qualitatively changes the nature of the interaction. In this paper we include gravity in the analysis and study the linear response of a tube when it is perturbed by a single p-mode in the ambient medium. Another significant difference between the present and previous work is that we determine an analytic solution of the forced wave equation.

2. Model and mathematical aspects

Let us consider a vertical magnetic flux tube surrounded by a field free atmosphere. For mathematical tractability we assume that the atmosphere in the flux tube as well as outside is isothermal with the same temperature. Since the radius of the tube is sufficiently small, we adopt the thin tube approximation (Defouw 1976, Roberts and Webb 1977). We now consider the linear interaction of the tube with a single p-mode in the external atmosphere with angular frequency ω and horizontal wave number k_x confined to a vertical cavity between $z = 0$ and $z = -d$, where z is the vertical co-ordinate (positive upwards), d is the depth of the lower turning point for the p-mode and x denotes the horizontal co-ordinate.

2.1 Vertical displacement in the tube

In the thin flux tube approximation, the linearized MHD equations for a sausage wave that is forced by a a external pressure perturbation Π_e with a variation of the form $e^{i(k_x x-\omega t)}$, can be reduced to a second order differential equation for the vertical displacement in the tube (Roberts 1983). Knowing Π_e, this equation can be solved analytically subject to specified boundary conditions for an isothermal atmosphere. We assume that the vertical displacement ξ_z vanishes at $z = 0$. For the lower boundary, we assume an outgoing wave solution, so that $\xi_z \sim e^{i(\omega t+kz)}$, where k denotes the vertical wave number of the tube mode. Thus it can be shown (for details see Hasan & Bogdan 1996) that the vertical displacement, in the limit $z \to -\infty$, has the form

$$\xi_z \sim e^{z/4H}(1+\beta)\frac{(\omega^2-N^2)}{c_S^2}\frac{k_e\alpha}{[(k_e^2-k^2+\alpha^2)^2+k^2\alpha^2]} \tag{1}$$

where k_e is the vertical wave numbers for the p-mode, β is the ratio of the gas to magnetic pressure in the unperturbed flux tube atmosphere, H is the pressure scale height, c_S is the sound speed, N is the Brunt-Väisälä frequency and $\alpha \equiv 1/4H$. It may be noted that in the present case β, H, N, k_e and k are constant with z.

From equation (1), we see that the displacement at a particular depth in the tube ξ_z is always finite for all values of k and ω, in contrast to the unstratified case, when a resonance exists for $k{=}k_e$. This demonstrates that the stratification qualitatively alters the nature of the interaction between a p-mode and thin flux tube. A careful examination of equation (1) (Hasan & Bogdan 1996) reveals that for small k_x, ξ_z is independent of k_x, whereas for large k_x, it drops off as $1/k_x^2$.

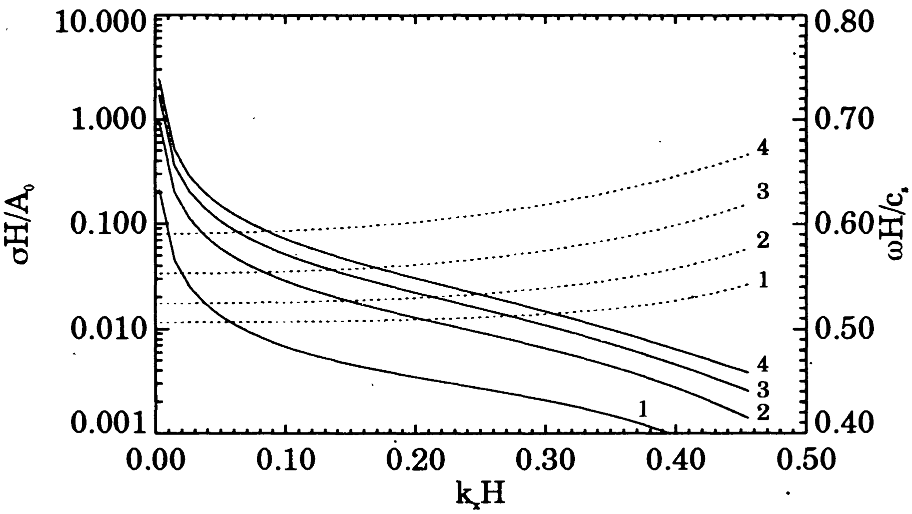

Figure 1. Variation of the absorption cross-section (solid lines) and frequency (dashed lines) as a function of horizontal wave number of the p-mode in the external atmosphere. For convenience we use non-dimensional units. The various numbers besides the curves denote the vertical order of the mode.

3. Results

In order to evaluate the interaction between a p-mode and flux tube, we calculate the absorption cross-section σ, which is the ratio of the time-averaged wave power going down the flux tube to the average incoming energy flux in the p-modes. Figure 1 shows the non-dimensional absorption cross-section ($\sigma H/A_0$) (solid lines) as a function of the wave number $k_x H$ in the external atmosphere for different orders (the number besides each curve) p-modes, assuming $d = 40H$ and $\beta = 1$. The dotted curves denote the p-mode frequencies (in units of c_S/H). The main feature of the results is that for a fixed order, corresponding to a specific value of the vertical wave number k_e, the absorption cross-section is largest at low k_x. However, as k_x increases, σ drops off fairly sharply. Also, for a fixed value of k_x, the response increases with mode order n (where $n = k_e d/\pi$) for low k_z, but when the latter becomes sufficiently large, it falls off. Table 1 shows the absorption cross-section for different mode orders, corresponding to $k_x H = 0.05$, assuming that other parameters are the same as before. We find that σ has a maximum for $n = 12$. The reason for this behaviour can be understood from an examination of equation (1).

4. Discussion and conclusions

We have examined the linear interaction of a flux tube that is forced by external p-modes. These calculations clearly indicate that in the presence of stratification, there is no resonance condition for an enhanced response, like for the unstratified case. Rather, we find that the interaction is significant only for external modes of small horizontal wave number or low degree and order. As the degree of the p-mode increases, the absorption cross-section drops off very sharply.

Table 1. Absorption cross-section (non-dimensional) for different values of the vertical mode order, corresponding to $d = 40H$, $\beta = 1$ and $k_x H = 0.05$.

n	$\sigma H/A_0$
1	1.327×10^{-2}
5	1.871×10^{-1}
9	2.778×10^{-1}
11	2.917×10^{-1}
13	2.928×10^{-1}
15	2.862×10^{-1}
19	2.635×10^{-1}
21	2.505×10^{-1}

We have considered a single mode. It is of interest to ask what would happen in the realistic situation when we have a wave packet in the external medium, consisting of a superposition of a large number of modes. Since we do not know the phases of the different modes, it is not possible to model this problem. However, on the basis of our calculations, it would appear that in a external pressure perturbation, the dominant contribution would from come the low degree components, which have a large response.

Finally, although the choice of an isothermal atmosphere is somewhat idealized, it nevertheless allows us to model the interaction analytically and thereby understand the conditions under which external p-modes can excite wave motions in the flux tube of significant amplitude. In future investigations, we hope to enlarge the scope of the study to treat more realistic cases.

One of us (SSH) is thankful to the High Altitude Observatory for support under the Visitor Program. The National Center for Atmospheric Research is sponsored by the National Science Foundation.

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