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A note on the rotational motion in quiescent prominences

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Abstract. Following an earlier paper (Pande & Bondal 1991) we have, on the basis of Kippenhahn and Schlüter's magnetohydrostatic model of a quiescent prominence, attempted to study the effect of a rotational velocity field in the vertical plane. However, it was not possible to establish whether the rotational velocity field in the vertical plane could be accomodated in the model. We now find that the rotational motion is possible both in the vertical and horizontal planes, without violating the basic assumptions in that model; this is a modification over our earlier conclusion.

Key words : quiescent prominences — rotational motion

1. Introduction

Mass motions have been known to exist in quiescent prominences for a long time, and rotational motions were reported in them by Liggett & Zirin (1984). Following their findings, we investigated the effect of rotational velocity fields in Kippenhahn and Schlüter's magnetohydrostatic model (KS model) of a quiescent prominence (Pande & Bondal 1991). We found that a rotational velocity field is possible in the *x-y* plane, i.e. in the horizontal plane. Under certain assumptions, we derived iso-velocity contours. However, we stated that the solution for a coplanar rotational velocity field in the *x-z* plane, i.e. in the vertical plane did not seem possible, without violating the basic assumptions in the KS model, including div $\vec{B}$=0, or should be accommodated in some alternate way not known then. We now show that the rotational motion is physically possible in the *x-z* plane also.

2. Formulation of the Problem. Equations.

To show that the rotational motion is possible in the *x-z* plane, the assumptions made and the notations used are the same as reported earlier (Pande & Bondal 1991). We take equation (9) as the starting point, i.e. :

$$B_x \frac{\partial^2 B_z}{\partial x^2} + \left(\frac{Sz-4\pi g}{N} \right) B_z \frac{\partial B_z}{\partial x} = -Sx\frac{\partial \rho}{\partial z} - \left(Sz-4\pi g \right) \frac{Sx\rho}{N} \tag{1}$$

where B_x and B_z are the magnetic field components, ρ the mass density, g the gravitational acceleration on the Sun. Further $N = 4\pi RT/m$, where R is the gas constant, T the temperature and m the mass of the hydrogen atom. Also $S = 4\pi (a_{11}^2 - a_{12}^2)$ where a_{11} and a_{12} are the coefficients related to the velocity components in a rotational velocity field in the x - z plane defined as follows (Smirnov 1962):

$$V_x = a_{11}x - a_{12}z$$

and

$$V_z = a_{12}x - a_{11}z\,. \tag{2}$$

These coefficients have been determined earlier (Pande & Bondal 1991). The first term on the RHS of equation (1) has a negative sign which was erroneously omitted in our earlier investigation (Pande & Bondal 1991).

We now make use of the initial assumptions inherent in the KS model. To avoid the dependence of B_z or z, *i.e.* to preserve the condition div $\vec{B}=0$, we thought it prudent to compare the first term on the LHS of the equation (1) with the corresponding term on the RHS and so also the second term on the LHS with the corresponding term on the right, to see whether the KS field geometry is satisfied. This gives us :

$$B_x \frac{\partial^2 B_z}{\partial x^2} = - Sx\frac{\partial \rho}{\partial z}\,. \tag{3}$$

Dividing by ρ, we get

$$\frac{B_x}{\rho} \frac{\partial^2 B_z}{\partial x^2} = - \left(\frac{1}{\rho} \frac{\partial \rho}{\partial \rho}\right) Sx\,. \tag{4}$$

Further,

$$\left\{\left(\frac{Sz-4\pi g}{N} \right)\right\} B_z \frac{\partial B_z}{\partial x} = -\left\{\left(\frac{Sz-4\pi g}{N} \right)\right\} Sx\rho \tag{5}$$

or,

$$B_z \frac{\partial B_z}{\partial x} = -Sx\rho,$$

or,

$$\frac{1}{2\rho} \frac{\partial B_z^2}{\partial x} = -Sx\,. \tag{6}$$

Dividing (4) by (6), we get

$$2B_x \frac{\partial^2 B_z}{\partial x^2} \bigg/ \frac{\partial B_z^2}{\partial x} = \left(\frac{1}{\rho} \quad \frac{\partial \rho}{\partial z}\right),$$

or,

$$2B_x \frac{\partial^2 B_z}{\partial x^2} \bigg/ \frac{\partial B_z^2}{\partial x} = -\frac{1}{h},$$

or,

$$\frac{\partial^2 B_z}{\partial x^2} \bigg/ \frac{\partial B_z^2}{\partial x} = -\frac{1}{2hB_x},$$

$$= -\frac{\alpha}{B_z(\infty)}, \tag{7}$$

where

$$\alpha = \frac{B_z(\infty)}{2hB_x},$$

$$B_z = B_z(\infty) \tanh\left(\frac{B_z(\infty)x}{B_x 2h}\right)$$

$$B_z(\infty) \tanh(\alpha x). \tag{8}$$

From the field geometry of KS model we consequently have to prove that

$$\frac{\partial^2 B_z}{\partial x^2} \bigg/ \frac{\partial B_z^2}{\partial x} = -\alpha/B_z(\infty),$$

where,

$$B_z = B_z(\infty) \tanh(\alpha x). \tag{9}$$

From 9 we have

$$\frac{\partial^2 B_z}{\partial x^2} = -2B_z(\infty)\alpha^2 \operatorname{sech}^2(\alpha x) \tanh(\alpha x). \tag{10}$$

Also,

$$B_z^2 = B_z^2(\infty) \tanh^2(\alpha x),$$

giving

$$\frac{\partial B_z}{\partial x} = \left\{\frac{\partial}{\partial(\alpha x)} \; B_z^2(\infty) \tanh^2(\alpha x)\right\}\alpha$$

$$= 2\, B_z^2(\infty)\, \alpha \tanh(\alpha x) \operatorname{sech}^2(\alpha x). \tag{11}$$

Dividing (10) by (11) we get, after simplification,

$$\frac{\partial^2 B_z}{\partial x^2} \Big/ \frac{\partial^2 B_z^2}{\partial x} = -\frac{\alpha}{B_z(\infty)} \tag{12}$$

which proves the condition stated above, and thus this makes the rotational velocity field possible in the x - z plane also.

Results and Discussions

In the earlier paper (Pande & Bondal 1991) we found that although iso-velocity contours could be obtained in the x-z plane also, the solution for $\vec{B}$ as a function of z contained a dependence of B_z on z thus violating the basic assumption in the KS model and also the physical condition div $\vec{B}$=0.

Here we have shown that the rotational motion can be sustained in the x-z plane also, without violating the condition div $\vec{B}$=0, and without making B_z dependent on z - consistent with the original KS model. The iso-velocity contours in the x - z plane would be similar to those in the x-y plane given earlier, and can be obtained by an appropriate replacement of the coordinates in fig. 1 of the paper (Pande & Bondal 1991).

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