Bull. Astr. Soc. India (1995) **23**, 77–83

The Hulse-Taylor binary pulsar PSR 1913+16

C. Sivaram
Indian Institute of Astrophysics, Bangalore 560 034

Received 22 March 1994; accepted 23 September 1994.

Abstract. The significance of the binary pulsar 1913 + 16 in establishing evidence for gravitational radiation emission from binary systems is described. It is pointed out that the system has enabled verification of general relativity predictions to a high level of precision. This article was written on the occasion of the 1993 Physics Nobel Prize awarded to Hulse and Taylor.

Key Words : binary pulsar — PSR 1913-16

The 1993 Nobel Prize in Physics was given jointly to R.A. Hulse and J.H. Taylor for their 1974 discovery of the first binary pulsar PSR 1913 + 16 (Hulse & Taylor 1974) and the subsequent painstaking use of its precise orbital parameters to obtain strong indirect evidence for gravitational radiation emission from the system, verifying general relativity predictions to about a per cent. Thus the physics prize has been given to astronomers after a gap of ten years, the 1983 prize was shared by S. Chandrasekhar and W. Fowler. This is the second time it is being given for work involving pulsars, the earlier one was the 1974 prize to Martin Ryle and A. Hewish, latter getting it for his seminal discovery of the first pulsar in 1968. Prior to this, Hans Bethe got the 1967 prize for his pioneering work on the nuclear reaction cycle powering the energy generation in the sun and other stars. In 1978, Penzias and Wilson shared half the prize (the other half going to P. Kapitza for his work on superfluid helium and low temperature phenomena) for their 1965 serendipitous discovery of the cosmic microwave background radiation. Why is this particular pulsar so important?

This particular binary pulsar is about 5 kpc away. It was quickly realized that apparent changes in pulsar frequency could be explained by the Doppler effect due to orbital motion about an unseen compact companion (later on also shown to have a mass around 1.4 $M_{\odot}$, with a period ~ 8 hrs.

The presence of a natural celestial high precision clock (pulsar) moving with a velocity ~ 300 km/sec through the gravitational field of an orbiting companion led to a flurry of hectic scientific activity in the community of general relativity (GR) specialists. It was very soon realised that it was a natural testing laboratory for various general relativistic effects with much larger values than in the solar system. The three or four solar system based observational tests had been hitherto the main sources of empirical support for GR. But this system of the binary pulsar soon provided the possibility of testing the theory to higher levels of precision including indirect support for the existence of gravitational waves. As an example, compared to the well known 43" (seconds) *per century* of additional i.e., perihelion advance for the planet Mercury, GR predicts a much larger periastron advance of $\omega' \cong 4.2°$ per year for this system. This is actually seen and is some 30,000 times greater than in the case of Mercury. It implies that the perihelion shift due to GR, in this system for one day is about as much as that of Mercury in a century! Moreover as we shall see the effects of gravitational radiation in this system are significant and measurable. (Scaling as $(V/C)^5$, i.e. > 10^6 times more important than in the solar system).

The general relativistic perihelion (or periastron in this case!) precession is given by the formula for the two-body problem in GR as (first given by Robertson (1938):

$$\omega' = \frac{3G^{2/3}(M_1+M_2)^{2/3}}{(1-e^2)c^2}\left(\frac{2\pi}{P_{orb}}\right)^{5/3}. \tag{1}$$

Here the orbital period P_{orb} is known to a high precision as P_{orb} = 27906.982151 sec or 7.751939337 hr, the masses M_1 and M_2= 2.82787 $M_\odot$.

For solar system applications $M_2 << M_1$. It is seen that a higher orbital eccentricity e enhances ω'. For this binary pulsar system e is quite high, being e = 0.6171395. Again the orbital period is only ~ 8 hr, compared to about 3 months for Mercury, the effect going as $P_{orb}^{-5/3}$.

The mass function is defined in the usual way as:

$$f = \frac{(M^2 \sin i)^3}{(M_1+M_2)^2} = \frac{(a_1 \sin i)^3}{G}\left(\frac{2\pi}{P_{orb}}\right)^2.$$

Variations in arrival times of pulses at earth provide information about the orbit. Because of the high precision of pulsar timing ($P = 8.626 \times 10^{-18}$) transverse Doppler shift and gravitational red shift can be measured.

$$\frac{(\delta t)\ \text{received}}{(\delta t)\ \text{emitted}} = 1 + \frac{V_1}{c} n + \frac{1}{2}\frac{V_1^2}{c^2} + \frac{GM_2}{r_1 c^2} \tag{2}$$

$$V_1^2 = r_1^2 + r_1^2\phi^2 = (2\pi P)^2 \frac{a_1^2}{1-e^2}(1+2e\cos\phi + e^2).$$

Here

$$r_1 = \frac{a_1(1-e^2)}{1+e\cos\phi}$$

is the "true anomaly" i.e. polar Co-ord measured from periastron; $X = r_1 \cos\varphi$, $y = r_1 \sin\varphi$(x axis along line of nodes) and $\varphi = \omega + \phi, \omega$ is the angular distance of periastron from the node, measured in the orbital plane.

The energy equation is

$$\frac{1}{2} V_1^2 + \frac{GM_2}{r_1} = \beta \cos\phi + \text{constant}, \tag{3}$$

$$(2\pi/P)^2 = \frac{GM_2^2}{(M_1+M_2)\,a_1^3}$$

$$GM_2/r_1 = GM_2^2/(M_1+M_2)r_1$$

$$\beta = \frac{GM_2^2\ (M_1+2M_2)e}{(M_1+M_2)^2 a_1 (1-e^2)}\ . \tag{4}$$

The second-order Doppler shift and gravitational redshift gives rise to one new measurable quantity β.

As usual two combinations of the parameters, i.e. a_1 sin i and mass function f are measured by first order ~ v/c Newtonian effects. Four key parameters characterize the system: i.e. masses M_1 and M_2; a_1 = semi major axis of pulsar orbit; i = inclination of orbit plane to line-of-sight. Thus general relativity enables three more combinations of parameters to be measured: one from periastron advance, one from combined second-order Doppler and gravitational redshift and one from decay of orbital period of system due to gravitational radiation emission. As we shall see the actually observed orbital period change of about 76 microseconds per year is completely consistent with that expected from orbital decay from emission of gravitational radiation as given by the standard quadrupole formula in Einstein's GR theory. This constitutes the strongest evidence as yet (there being no direct detection so far) for the reality of the existence of gravitational waves. By monitoring the orbital parameters for about 20 years, Taylor and his colleagues (Taylor et al. 1992) have verified the agreement between theory and observation for this system to be 1.0023, a very impressive support for GR.

The mass estimate has been derived from independent fit-of-four parameters: (Taylor & Weisberg 1984, 1989) inclination of orbital plane to line-of-sight, advance of periastron, energy loss and light deflection parameter. The post-Newtonian orbital effects are described by the parameter.

$$\gamma = \frac{\beta P(1-e^2)}{2\pi c^2} \tag{5}$$

where β is given as in equation (3). The observed γ_{obs} = (0.00438 ± 0.00024).

In the solar system post-Newtonian periodic deviations from elliptic motion have not yet been verified. A few remarks about gravitational radiation would be appropriate at this juncture. The lowest order gravitational radiation is quadrupolar. Monopole radiation vanishes as in electromagnetics, because of conservation of charge $\sum e$ = constant which in this case is total mass $\sum M$ = constant. The lowest order radiation in electromagnetism is dipolar, i.e. given by the time varying dipole moment ex i.e. the radiation emitted is proportional to $e|\ddot{x}|^2$. (the second time derivative squared). In gravity, this vanishes because of the conservation of momentum. The gravitational dipole moment (mass being counter-part of charge) can be written:

$$\dot{d}_j = \sum m_A x_j^A ,$$

(for system of j particles).

Then $\ddot{d}_j = \sum m_A \ddot{x}_j^A = \sum \dot{P}_j^A = 0.$

($\dot{P}_A$ is the momentum of the Ath particle). Thus conservation of momentum, makes the rate of change of momentum $\dot{P}_j$ vanish, thus giving, $\ddot{d}_j = 0$, and therefore no gravitational dipole radiation.

The lowest order is thus quadrupolar radiation given by:

$$\dot{E}_G = P_G \sim \frac{G}{45} \dddot{Q}^{0}/^2 \frac{1}{c^5} , \tag{6}$$

(where Q° is the mass quadrupole moment of the system).

Now G/C^5 is a very small quantity, i.e. ~ 10^{-59} erg^{-1} sec, which makes gravitational radiation a very weak effect to detect. For two bodies of masses M_1 and M_2 in circular orbit the gravitational radiation loss is:

$$\dot{E}_G = \frac{dE}{dt} \sim \frac{32\,G}{5c^5} \mu^2 \omega^6 R^4 , \tag{7}$$

(the triple time derivative equation (6) squared gives ω^6, ω being the frequency of the system, the mass quadrupole moment (μR^2) squared gives $\mu^2 R^4$, $\mu = M_1 M_2 / M_1 + M_2$ is the reduced mass, (For Sun-Jupiter equation (7) predicts only a few watts!).

For rate of change of the separation R between the two components due to emission of gravitational radiation we have

$$dE\,/\,dt \;=\; \frac{1}{2}\,\frac{GM\mu}{R^2}\,\frac{dR}{dt}$$

and equating this to $\dot{E}_G$ in (6)

we get,

$$R^3 dR/dt \;=\; -\,\frac{64}{5}\,\frac{G^3}{c^5}\,M^2\mu \qquad (8)$$

and integrating

$$R^4 = -\,(256/5)\,(G^3/c^5)M^2\mu t + \text{constant}.$$

System will collapse when $R \to 0$ at $t = t_o$. This gives the time scale for collapse of orbit from R_{now} *as:*

$$t_0 = \frac{5}{256}\,\frac{c^5}{G^3}\,\frac{R^4{}_{now}}{M^2\mu}\,G^3. \qquad (9)$$

For the earth-sun system, $t_0 \sim 10^{20}$ *yr* and for the binary pulsar on the other hand $t_o \sim 10^8$ *yr.*

As one consequence, for example, this would rule out binary neutron stars as models for pulsars as $t_o/P \sim 10^5\ s \sim 1$ day for $P \leq 1$ second.

The above relations (i.e. equations (7) and (9) can be generalised to eccentric orbits $e \neq 0$. In this case, one can integrate dE/dt over orbital period as $1/T \int_0^{\tau} dE/dt.\ dt$, giving (a being the semi-major axis):

$$\dot{E}_G = \frac{-32\,G}{5c^5}\,\frac{G^3 M_1^2 M_2^2\ (M_1+M_2)}{a^5(1-e^2)^{7/2}}\left(1+\frac{73}{24}e^2+\frac{37}{96}e^4\right).$$

This gives in turn the rate of change of the semi-major length a as:

$$da/dt \;=\frac{-64}{5c^5}\,G^3\,\frac{(M_1\ \ M_2)\ \ (M_1+M_2)}{a^3(1-e^2)^{7/2}}\left(1+\frac{73}{24}e^2+\frac{37}{96}e^4\right), \qquad (11)$$

These formulae applied to the binary pulsar implies a change in orbital period due to shrinking of the orbit of ~ 76 micro seconds per year and this has indeed been verified to about 1% accuracy, provides impressive evidence for gravitational radiation. Combined with the large observed periastron precession (4.2° per year as expected) along with the measured second-order Doppler and gravitational redshift it enables a completely self consistent determination of the five orbital parameters to an unprecedented accuracy of seven significant figures!

From the values of P, e and ω periastron, we find

$$M_1 + M_2 = 2.82787\, M_O \tag{12}$$

Care must be taken not to separately use values of G and M in Cgs units. The gravitational constant G is only known to less than five significant digits. The quantity $GM_\odot/c^3 = 4.925495 \times 10^{-6}$ S is known much more accurately, so $M_1 + M_2$ can be given much more accurately in solar mass units than in grams.

The above equations for *da/dt* etc. enable the determination of the rate of change in orbital period P_{orb} due to emission of gravitation radiation as

$$\dot{P}_{orb} = \frac{-192\pi}{5} G^{5/3} \frac{M_1 M_2 f(e)}{c^5 (M_1+M_2)^{1/3}} \left(\frac{2\pi}{P_{orb}}\right)^{5/3}$$

$$= -1.202333 \times 10^{-12} M_2 (2.82787 - M_2)\,. \tag{13}$$

γ, the Post-Newtonian parameter defined earlier can be written:

$$\gamma = \frac{G^{2/3} M_2 (M_1 + 2M_2) e}{(M_1+M_2)^{4/3} c^2} \left(\frac{P_{orb}}{2\pi}\right)^{1/3}$$

$$= (0.00073445) M_2 (2.82787 + M_2) \tag{14}$$

From the observed $\gamma = (0.00438 \pm 0.00025)$, we find $M_2 = (1.44 \pm 0.006)$ M_O and from eq. (12), M_1 has almost the same value. Then equation (13) predicts $P_{orb} = -2.39 \times 10^{-12}$, in execellent agreement with the observed or measured value of $P_{orb} = (-2.31 \pm 0.22) \times 10^{-12}$.

The latest measurements based on observations by Taylor and his collaborators over a period of nearly two decades (Taylor et al. 1992) implies confirmation of the quadrupole formula of GR for gravitational wave emission to within measurement errors ~ 1%. This demonstration of the existence of gravitational waves has the significance of Hertz, experiment to confirm Maxwell's prediction of electromagnetic waves. The cosmos has fortunately supplied us with the right laboratory source for this experiment. (The factor G/C^5 suggests that laboratory experiments are unlikely to generate gravitational waves of sufficient intensity for detection: a 1 metre rod spun to its breaking stress limit would generate less than 10^{-30} erg/s!). Even gravitational waves from the binary pulsar are not intense enough to be directly detected in gravitational wave detectors in terrestrial laboratories (The source being 5 kpc away). The amplitude of the gravitational waves from this system corresponds to $h \sim 10^{-23}$ at frequency $\sim 2/P_{orb} \sim 10^{-4}$ Hz and may be directly detectable by laser interferometry in space, next century!

It is the remarkable precision of the timing of the radio signals from the pulsar that enables the change in predicted orbital period due to emission of gravitational radiation to be confirmed, thanks to the painstaking, horological book keeping by Taylor and his collaborators over a period of nearly two decades. The orbit is slowly shrinking however and the two neutron stars would merge in a time period of ~ 10^9 yr. The merger would be accompanied by the release of ~ 5 x 10^{52} erg of gravitational radiation at ~ KHz frequency in an intense pulse lasting about a second. Such a pulse should be detectable in existing bar detectors even if it occurs ~ 100 Mpc away, i.e. such a merger of a binary pulsar or neutron star system can be detected by its gravitational wave emission even if it occurs in one in about ten million galaxies. It is one of the models advocated for the formation of millisecond pulsars (Sivaram & Kochhar 1984).

Estimates of the corresponding amounts of gravitational radiation emitted in the merger of galaxies as well as (Sivarman & Alladin 1986) in the collapse of cosmic strings (Sivaram 1987). The binary pulsar has also enabled elimination of some alternative theories of gravity (like Rosen's bimetric theory, etc.) which were consistent with solar system tests and stringent limits on some others. Again tight constraints can be put on the parameters of any additional long range or intermediate range forces which can contribute to the radiation damping of binary systems although such forces may give null results in laboratory experiments (Bertotti & Sivaram 1991).

At least two more binary neutron stars (pulsars) (there are also several pulsars forming binaries with main-sequence stars, white dwarfs etc.) are known in our galalxy.

PSR 1534 + 12 and PSR 2127 + IIC. These are also expected to coalese in about 3 x 10^9 yr and 2 x 10^8 yr respectively due to emission of gravitational radiation. In the final phases of coalescence the binary pulsar would be a strong emitter of gravitational waves with amplitude $h \sim 10^{-17}$, very easily detectable (but we will have to wait > 10^8 years for that!). Statistically it is expected that within 500 Mpc, about 3 such events per year may occur.

The reader should not confuse *binary models* of individual pulsars with binary pulsars. Some early models of *individual pulsars* visualised them as binary neutron stars in close orbit with periods of a second or less, if this were so, equation (9) shows that such systems would last *less than a day*, whereas pulsars have been known to pulse for decades. So such binary models of pulsars are ruled out, not binary pulsars!.

References

Bertotti B., Sivaram, C., 1991, Nuovo Cimento, 106 b, 1299

Hulse R A., Taylor J.H., 1975, ApJ, 195, L51-55.

Peters P.C., 1964, PR, 136, 1224.

Robertson H P., 1938, Ann. Math., 39, 101.

Sivaram C , 1987 Nature, 327, 108.

Sivaram C., Alladin S M , 1986, BASI, 14, 229-32.

Sivaram C., Kochhar R K., 1984, Astrophys. Space Sci., 98, 201, 203

Taylor J.H. et al., 1992, Nature, 355, 132.

Taylor J.H., Stinebring D.R., 1986, ARA&A, 24, 285.

Taylor J.H., Weisberg J.M , 1982, ApJ, 253, 908.

Taylor J.H., Weisberg J M., 1984, PRL, 52, 1348.

Taylor J.H. et al. Weisberg J.M ,1989, ApJ, 345, 434