EFFECTS ON PARTIAL FREQUENCY REDISTRIBUTION R_{II} ON THE LEVEL POPULATION RATIOS IN A RESONANCE LINE

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Abstract. Angle-averaged partial frequency redistribution R_{II} has been employed in obtaining a simultaneous solution of radiative transfer equation in the comoving frame and the statistical equilibrium equation for a non-LTE two level atom. We have obtained the ratios of population densities of the upper and lower levels of the resonance line of PV by utilizing the data given in Bernacca and Bianchi (1979). Line source functions are also obtained for different types of variations of density and velocity of the expanding gases. We have considered the atmosphere to be 11 times as thick as the stellar radius. The first iteration was started by putting the density of the upper level (N_2) equal to zero. However, the convergent solution shows a substantial increase in N_2 although it is still much less than the equilibrium value. The line source function and the ratio of the densities of the particles in the upper and lower levels fall sharply from a maximum at $\tau = \tau_{max}$ to minimum at $\tau = 0$. We have studied the scattering integral $\int_{-\infty}^{+\infty} J_x \phi_x \, dx$ and found that this quantity also varies quite similar to the ratio N_2/N_1 and the line source function S_L.

1. Introduction

It is well known that photon redistribution occurs due to scattering during the formation of spectral lines. Complete redistribution at a given frequency point in the line rarely occurs in a stellar atmosphere. Redistribution can occur not only in frequency but also in angle. The problems become highly complicated when radial motions of the gases are taken into account. It is the scattering integral which occurs in the source function that introduces major changes in the photon redistribution in frequency and angle. One must account for each photon that is scattered on emission and absorption. This is possible only when one takes account of all the partial redistributions of the photons scattered into other frequencies and angles. In a spherically-symmetric moving medium, the frequency redistribution strongly depends on the angle. This angle dependance of the frequency occurs irrespective of the fact whether we consider angle dependant or angle averaged redistribution functions in a moving medium. However, when we considers the formation of spectral lines in a comoving frame, we need not worry about this frequency redistribution dependency on the angle. It becomes important to consider this fact when the radiation field obtained in a co-moving frame is translated onto other frames of references, such as star's rest frame or observer's frame at infinity. The co-moving frame calculations have been done with partial frequency in Peraiah (1980a). The simultaneous solution of radiative transfer and the statistical equilibrium with complete redistribution has been obtained in Peraiah (1980b). In this paper, we present the results of partial frequency redistribution. We have employed the R_{II} function.

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2. Computational Procedure

The profile function $\phi(x)$ has been calculated from the relation

$$\phi(x) = \int R_{\text{II-A}}(x, x')\, \mathrm{d}x' , \tag{1}$$

where $R_{\text{II-A}}(x, x')$ is the angle averaged redistribution function given by (see Hummer, 1962)

$$R_{\text{II-A}}(x, x') = \frac{1}{\pi^{3/2}} \int_{1/2|\bar{x}-\underline{x}|}^{\infty} e^{-u^2} \left\{ \tan^{-1}\left(\frac{\underline{x} + u}{a}\right) + \tan^{-1}\left(\frac{\bar{x} - u}{a}\right) \right\} \mathrm{d}u , \tag{2}$$

where $\bar{x}$ and $\underline{x}$ are correspondingly the maximum and minimum values of x' and x (where $x = (\nu - \nu_0)/\Delta_s$, Δ_s being some standard frequency interval) the frequencies of the absorbed and emitted photons. As we are assuming that the emission profile is equal to the absorption profile x and x' can be interchanged. The symbol a represents the damping constant and is set equal to that of PV resonance line (1117.98 Å) whose damping constant is taken to be equal to 4×810^{-3}. From Equations (2) and (1), we obtain the line profile which is again normalized such that

$$\int_{-1}^{+1} \phi(x)\, \mathrm{d}x . \tag{3}$$

The absorption coefficient of this line is calculated by the formula

$$K(x, r) = \frac{h\nu_0}{4\pi\Delta\nu_{\text{D}}} [N_1(r)B_{12} - N_2(r)B_{21}]\, \phi(x) , \tag{4}$$

where ν_0 is the central frequency of the PV line, $\Delta\nu_{\text{D}}$ is the Doppler width; B_{12} and B_{21} are the Einstein coefficients $N_1(r)$ and $N_2(r)$ are the densities of the atoms of PV in the lower and upper levels respectively at the radius r. These are obtained from the statistical equilibrium equation

$$\frac{N_1(r)}{N_2(r)} = \frac{A_{21} + C_{21} + B_{21} \int \phi(x) J_x\, \mathrm{d}x}{C_{12} + B_{12} \int \phi_x J_x\, \mathrm{d}x} , \tag{5}$$

where A_{21} is the coefficient of stimulated emission and C_{12} and C_{21} are the rates of collisional excitation and de-excitation, respectively. They are given by Jefferies (1968) as

$$C_{12} \simeq 2.7 \times 10^{-10}\, \alpha_0^{-1.68} \exp(-\alpha_0)\, T^{-3/2} A_{21}\, \frac{g_2}{g_1} \left(\frac{I_H}{\chi_0}\right)^2 N_e \tag{6}$$

and

$$C_{21} = 2.7 \times 10^{-10}\, \alpha_0^{-1.68}\, T^{-3/2} A_{21} \left(\frac{I_H}{\chi_0}\right)^2 N_e\,, \tag{7}$$

where χ_0 is the excitation energy E_{12} and $\alpha_0 = \chi_0/kT$. I_H is the ionization potential of hydrogen; and J_x is the mean intensity given by

$$J_x = \frac{1}{2} \int_{-1}^{+1} I_x(\mu)\, d\mu\,; \tag{8}$$

$I_x(\mu)$ being the specific intensity of the ray making an angle $\cos^{-1}\mu$ with the radius vector. The specific intensity is obtained from the solution of the line transfer equation in the comoving frame

$$\mu \frac{\partial I(x, \mu, r)}{\partial r} + \frac{1 - \mu^2}{r} \frac{\partial I(x, \mu, r)}{\partial \mu} = $$
$$= K_L [\beta + \phi(x)][S(x, r) - I(x, \mu, r)] + $$
$$+ \left\{(1 - \mu^2) \frac{V(r)}{r} + \mu^2 \frac{dV(r)}{dr}\right\} \frac{\partial I(x, \mu, r)}{\partial x} \tag{9}$$

and

$$-\mu \frac{\partial I(x, -\mu, r)}{\partial r} - \frac{1 - \mu^2}{r} \frac{\partial I(x, -\mu, r)}{\partial \mu} = $$
$$= K_L [\beta + \phi(x)][S(x, r) - I(x, -\mu, r)] + $$
$$+ \left\{(1 - \mu^2) \frac{V(r)}{r} + \mu^2 \frac{dV(r)}{dr}\right\} \frac{\partial I(x, -\mu, r)}{\partial x}\,, \tag{10}$$

where β is the ratio of continuous absorption coefficient to line centre absorption coefficient. $V(r)$ is the radial velocity of the gas at the radius r. $S(x, \mu)$ is the source function given by

$$S(x, r) = \frac{\phi(x)}{\beta + \phi(x)} S_L(r) + \frac{\beta}{\beta + \phi(x)} S_C(r)\,. \tag{11}$$

We should note that $S(x, r)$ is frequency-dependent because of $\phi(x)$ and the line source function $S_L(r)$ is frequency-independent and is given by (when absorption and emission profiles are the same)

$$S_L(r) = \frac{A_{21}N_2(r)}{B_{12}N_1(r) - B_{21}N_2(r)} . \tag{12}$$

The quantity S_C is the continuum source function and is normally set equal to the Planck function. In the present investigation, we have put β equal to zero. The line source function with frequency dependency can also be written as

$$S_L(x, r) = \frac{1-\varepsilon}{\phi(x)} \int R_{\text{II-A}}(x, x')J(x')\,\mathrm{d}x' + \varepsilon B , \tag{13}$$

where ε is the probability of photon destruction by collisional de-excitation and is given by

$$\varepsilon = \frac{C}{C + A_{21}} , \tag{14}$$

where

$$C = C_{21}[1 - \exp(-h\nu_0/kT)] .$$

We have employed a straightforward iteration between the statistical equilibrium equation and radiative transfer equation in the comoving frame. We have assumed variation of the density of the atoms of PV together with the velocity so that they satisfy the equation of continuity. In the first iteration, we set $N_2(r) = 0$, i.e., $N(r) = N_1(r)$. This allows us to estimate the absorption coefficient from Equation (4). Then the equation of transfer given in (9) and (10) is solved and the mean intensities are calculated from this solution. The rates of collisional excitation and de-excitation are calculated from Equations (6) and (7). The mean intensities and the values of C_{12} and C_{21} enable us to estimate the ratio of population densities in the two levels, $N_2(r)/N_1(r)$. The quantities $N_1(r)$ and $N_2(r)$ are obtained from the equation $N(r) = N_1(r) + N_2(r)$. The second iteration is started with the new set of $N_1(r)$ and $N_2(r)$ and the whole process is repeated until we get convergence in the ratio $N_2(r)/N_2(r)$ at each radial point. The criterion for convergence is set equal to 1% in $N_2(r)/N_1(r)$ is two successive iterations. We have considered the PV resonance line 1117.979 Å in o^2 Ori (Bernacca and Bianchi, 1979). The atmospheric parameters from the above paper are employed and are given below for an isothermal medium:

$$A = \text{inner radius} = 5.2 \times 10^{11}\ \text{cm} ,$$

$$B = \text{outer radius} = 5.72 \times 10^{12}\ \text{cm} ,$$

$$B/A = 11 ,$$

$$T = 34674\ \text{K} ,$$

$$N_e(A) = 10^{12}\ \text{cm}^{-3} ,$$

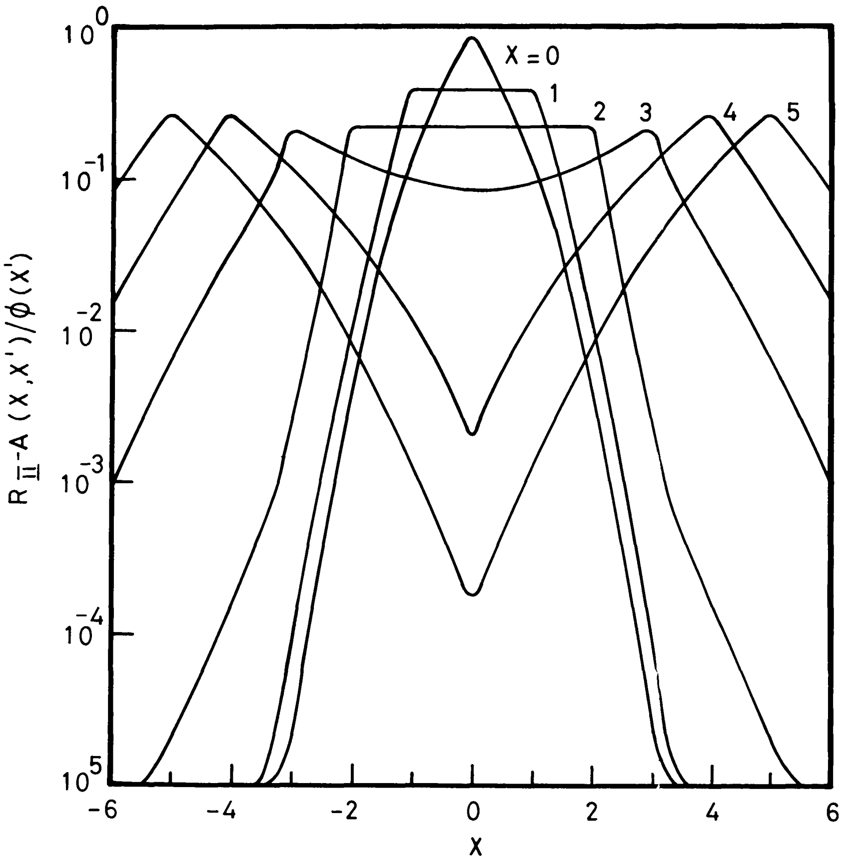

Fig. 1. The redistribution function $R_{II\text{-}A}$. The graph shows the probability of emission of a photon at frequency x' per absorption at frequency x. Ordinates: $R_{I\text{-}A}(x, x')/\phi(x)$ and abscissa: x'.

The results are presented in Figures 1, 2, 3, and 4. In Figure 1, we have given the probability of emission of a photon at frequency x' which is absorbed at frequency x. It is very interesting to note that the probability of emission of photon absorbed at frequency $x = 0$ is nearly equal to unity at the frequency $x' = 0$. This falls quite rapidly and becomes negligible beyond $x' > 3$.

If we consider the emission of a photon absorbed at $x > 3$, we notice that the probability of its emission at $x' = 0$ becomes minimum. At $x = 5$, this is reduced by nearly 4 orders of magnitude. This picture of absorption and emission has some interesting consequences in moving media of the stellar atmospheres. For example, when a photon is absorbed in the wings, it has a low probability of emission in the centre of the line. However, if the medium is in motion, then the frequency of the line photon change because of Doppler effect and may still have a high probability of emission at the centre of the line. The same process can occur in the case of a photon that is absorbed at the centre and may have high probability of emission in the wings when the gas is in motion. In Figures 2a, b, and c, we have described the scattering integral $\int J_x \phi_x \, dx$, N_2/N_1 and S_L with respect to the optical depth for $N_e(r) \sim 1/r$. In all cases three maximum velocities (V_B) are used: 10 km s^{-1}, 50 km s^{-1}, and 100 km s^{-1}. At the

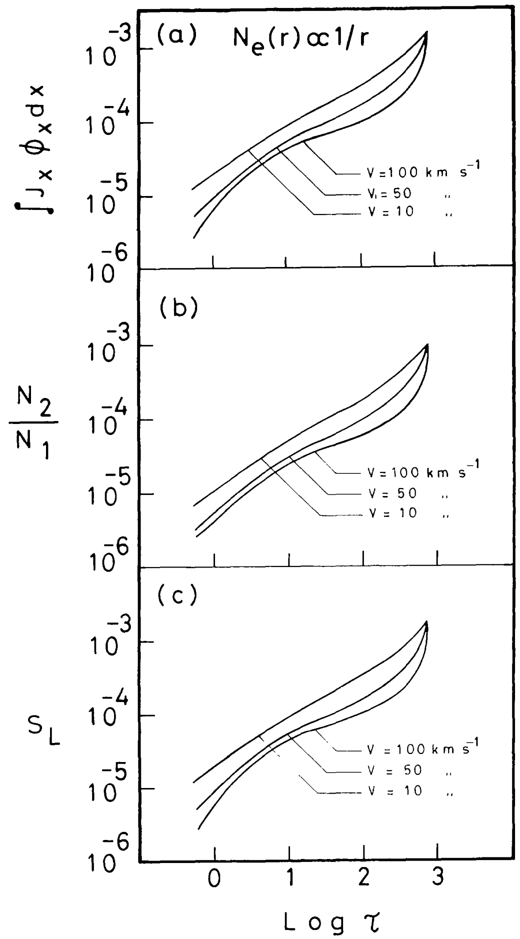

Fig. 2a–c. (a) The variation of scattering integral. (b) The level population ratio N_2/N_1. (c) The line source function S_L with optical depth for electron density. N_e proportional to $1/r$, r is the radial distance from the centre of the star.

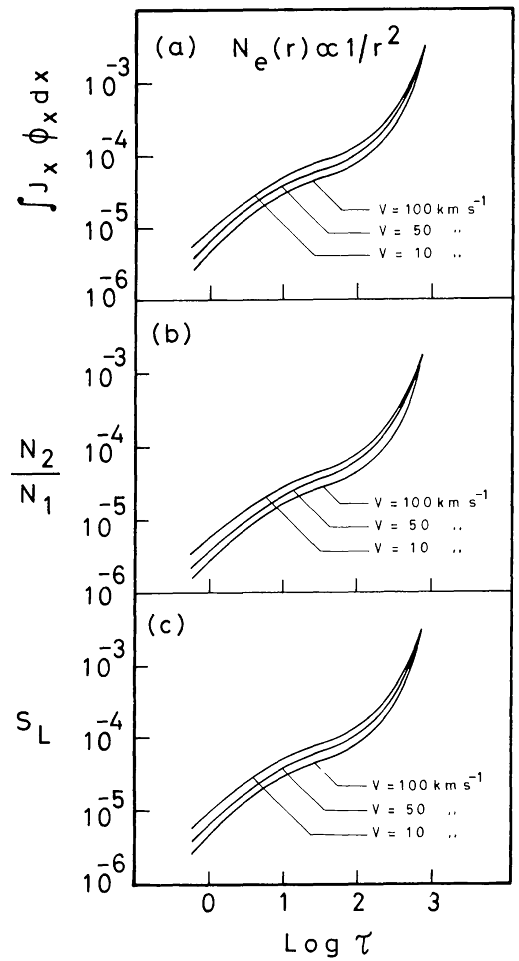

Fig. 3a–c. The same quantities as in Figure 2, but for electron density N_e proportional to $1/r^2$.

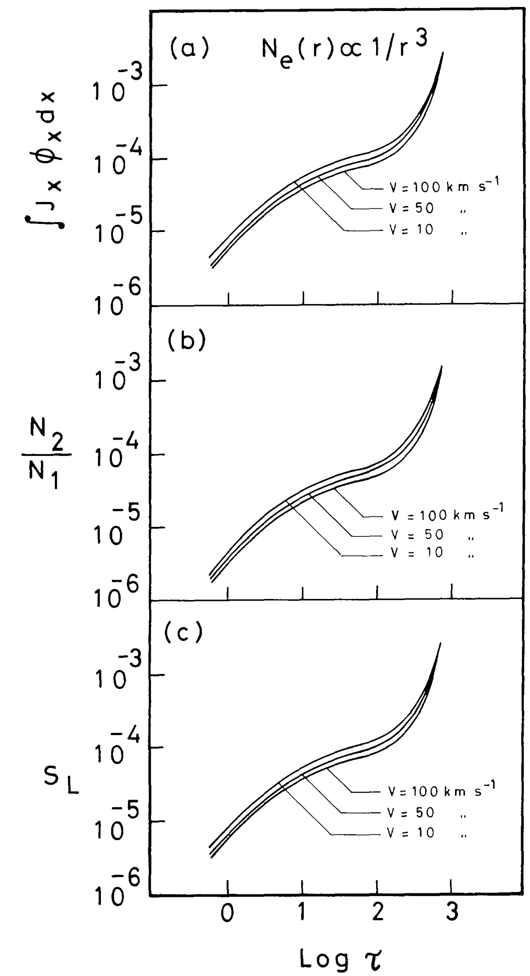

Fig. 4a–c. The same quantities as in Figures 2 and 3 for electron density N_e proportional to $1/r^3$.

maximum optical depth ($\tau = \tau_{max}$, $r = A$) all the three quantities have maximum value and at the minimum optical depth ($\tau = \tau_{min}$, $r = B$) they have been considerably reduced and gas motion accentuates this reduction. The ratio N_2/N_1 is maximum at $\tau = \tau_{max}$ and it becomes minimum at $\tau = \tau_{min}$. Although, we have set $N_2 = 0$ in the first iteration, it is interesting to note that, the upper level is considerably populated after the solution converges. The variation of S_L is quite similar to that of N_2/N_1.

In Figures 3a–c and 4a–c we have given the same quantities but with $N_e \sim 1/r^2$ and $1/r^3$. The variation of the quantities $\int J_x \phi_x \, dx$, N_2/N_1 and S_L are similar to those given in Figure 2a–c).

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