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The Hanle Effect with Angle Dependent Redistribution Functions

H. Frisch, M. Faurobert

Laboratoire G.D. Cassini, OCA/CNRS, BP 4229, 06304 Nice Cedex 4, France

K.N. Nagendra

Indian Institute of Astrophysics, Sarjapur Road, Koramangala Layout, Bangalore 560034, India

Laboratoire G.D. Cassini, OCA/CNRS, BP 4229, 06304 Nice Cedex 4, France

Abstract. Numerical work with the Hanle effect, taking into account Partial Frequency Redistribution (PRD) effects has so far been studied with the approximation that frequency redistribution can be described by an angle-averaged frequency redistribution function. This assumption is questionable when resonance polarization or the Hanle effect are considered since it is exactly the anisotropy of the radiation field which is the driving source of the polarization. Redistribution matrices for the Hanle effect, containing an angle-dependent frequency redistribution, have been proposed in the literature (Stenflo 1994; Bommier 1997b). As shown here, they have an unexpected behavior on the Stokes parameter U which becomes non-zero even in an axially symmetric medium.

1. Introduction

Resonance scattering and the Hanle effect in spectral lines can be described at the microscopic level by a redistribution matrix $\hat{\mathcal{R}}$ which we define at each point in space by the relation

$$\mathbf{S}_{e}(\nu, \mathbf{n}) = \int \int \hat{\mathcal{R}}(\nu, \mathbf{n}, \nu', \mathbf{n}'; \mathbf{B}) \, \mathbf{S}_{i}(\nu', \mathbf{n}') \, d\nu' \, \frac{d\Omega'}{4\pi}. \tag{1}$$

Here \mathbf{S}_i is the incident Stokes vector and \mathbf{S}_e the scattered Stokes vector. The redistribution matrix $\hat{\mathcal{R}}$ is the differential scattering cross section for radiation with incoming frequency and direction (ν', \mathbf{n}') , getting scattered into the outgoing frequency and direction (ν, \mathbf{n}) . The integration is over the frequency ν' and direction \mathbf{n}' of the incident photons. For the Hanle effect the redistribution matrix depends on the magnetic field \mathbf{B} . Here we are concerned only with the Hanle effect on linear polarization. Hence $\hat{\mathcal{R}}$ is a 3×3 matrix and $\mathbf{S} = (I, Q, U)$.

The determination of \mathcal{R} is a difficult theoretical problem. The first quantum mechanical attempt was by Omont et al. (1972, 1973). The first paper deals

with resonance scattering. The effect of a magnetic field is considered in the second one. Partial frequency redistribution (PRD) is accounted for in these seminal papers which are based on the theory of redistribution in frequency of radiation during scattering. The redistribution matrices were not explicitly written. For resonance polarization (zero magnetic field) Domke & Hubeny (1988) derived a tractable analytical expression of the redistribution matrix. It contains frequency coherent and frequency incoherent terms with branching ratios depending on the atomic model and various collision rates. In this paper 'coherent' refers always to frequency redistribution in the atomic frame.

The density matrix technique, combined with the formalism of the irreducible tensor components, offers a different and powerful approach (Landi degl' Innocenti 1983, 1984, 1985). The standard theory can handle multi-level atoms but is restricted to complete frequency redistribution. Landi degl'Innocenti et al. (1996) suggested the use of a continuous distribution of sub-levels to incorporate PRD mechanism into the density matrix approach. The problem of PRD was reanalyzed by Bommier (1997a,b) who could show that PRD appears automatically in the density matrix formalism when the expansion in the atomradiation interaction is continued to all orders in a quantum electro-dynamical (QED) theory. Complete redistribution corresponds to the dominant term in the expansion.

The classical oscillator model described in Stenflo (1994) offers a more intuitive picture of PRD. Bommier & Stenflo (1999) have shown that the stationary solutions of the harmonic oscillator correspond to the frequency coherent terms and the transitory solutions to the frequency incoherent ones.

In all these theories, the redistribution matrix is first determined in the atomic reference frame. Doppler effects due to atomic velocities are then incorporated by means of a convolution with a velocity distribution to obtain the laboratory (observer) frame redistribution matrix. In general the velocity distribution function used is a Maxwellian. The assumption underlying this procedure is discussed in Landi degl'Innocenti (1996).

For resonance polarization, the redistribution matrix in the atomic frame can be written as

$$\hat{\mathcal{R}} = \sum_{\alpha} r_{\alpha}(\xi, \xi') \hat{P}_{\alpha}(\mathbf{n}, \mathbf{n}'), \tag{2}$$

where ξ and ξ' are rest frame frequencies, r_{α} a frequency redistribution function and \hat{P}_{α} a phase matrix. Only the Rayleigh and the isotropic matrix contribute to $\hat{\mathcal{R}}$. In the laboratory frame, the above expression for the redistribution matrix becomes

$$\hat{\mathcal{R}} = \sum_{\alpha} R_{\alpha}(\nu, \mathbf{n}, \nu', \mathbf{n}') \hat{P}_{\alpha}(\mathbf{n}, \mathbf{n}'), \tag{3}$$

where the R_{α} are 'laboratory frame redistribution functions' which take Doppler effects into account.

In numerical solutions of the transfer equations, the $R_{\alpha}(\nu, \mathbf{n}, \nu', \mathbf{n}')$ are in general replaced by their angle-average $\bar{R}_{\alpha}(\nu, \nu')$, or by the expression for complete redistribution $R(\nu, \nu') = \phi(\nu)\phi(\nu')$, with ϕ being the normalized line absorption profile (e.g. Rees & Saliba 1982; Paletou & Faurobert-Scholl 1997; Faurobert-Scholl et al. 1997; Faurobert-Scholl et al. 1999). For non-polarized transfer with an isotropic phase function, this simplification is justified on the

grounds that the radiation field is almost isotropic. For resonance polarization this approximation is more questionable since the anisotropy of the radiation field is actually the source of the polarization. It has been verified numerically (Faurobert 1987, 1988) that this approximation introduces errors of a few percent compared to the angle dependent case (see also section 2.2).

For the Hanle effect, the situation is much more complicated. Even in the atomic rest frame, the redistribution matrix cannot be written as a sum of factorized terms as in equation (2) (Stenflo 1994, p. 81; Bommier 1997b, Bommier & Stenflo 1999). Numerical work has been carried out with ad hoc redistribution matrices of the form

$$\hat{\mathcal{R}} = \sum_{\alpha} \bar{R}_{\alpha}(\nu, \nu') \hat{P}_{\alpha}(\mathbf{n}, \mathbf{n}'), \tag{4}$$

the Hanle phase matrix appearing now in some of the terms (e.g. Faurobert-Scholl 1991; Nagendra, Paletou, Frisch, & Faurobert-Scholl 1999).

Bommier (1997b) has determined the redistribution matrix in the atomic frame, and also proposed well controlled approximations for the laboratory frame redistribution matrix that can be incorporated into radiative transfer calculations. The 2-D frequency space (ν, ν') is divided into several regions. In each region the redistribution matrix is a sum of frequency coherent and frequency incoherent terms, each term being the product of a redistribution function and a phase matrix. Two different levels of approximation are proposed: level I incorporates angle-dependent redistribution functions and level II their angle-averaged version as in equation (4).

For magnetic field diagnostics with the Hanle effect, we need very accurate solutions of the transfer problems. We were thus interested in finding out whether there could be observable differences on the surface polarization depending on the level of approximation that was being used. A numerical code for radiative transfer in a plane parallel slab was written for that purpose. To check it we considered the case of a magnetic field normal to the surface of the slab. For a thermal creation of photons and no radiation incident on the slab, the model is axially symmetric. Hence it was expected that Stokes U would be zero. We found that U was zero in the wings where we have only Rayleigh scattering but not in the line core where the Hanle effect is acting. This somewhat surprising result is explained now. It is sufficient to consider the effect of a single scattering.

2. Analysis of Stokes U

For the purpose of the analysis, we consider a redistribution matrix of the form

$$\hat{\mathcal{R}}(\nu, \mathbf{n}, \nu', \mathbf{n}') = R(\nu, \mathbf{n}, \nu', \mathbf{n}') \hat{P}(\mathbf{n}, \mathbf{n}'). \tag{5}$$

 $R(\nu, \mathbf{n}, \nu', \mathbf{n}')$ is an angle dependent redistribution function and $\hat{P}(\mathbf{n}, \mathbf{n}')$ a polarization phase matrix which is here the Hanle or Rayleigh phase matrix. We assume that equation (5) holds for all frequencies and directions. We rewrite the scattering integral as

$$\mathbf{S}_{e}(x,\mu) = \int_{-\infty}^{+\infty} \frac{1}{2} \int_{-1}^{+1} \frac{1}{2\pi} \int_{0}^{2\pi}$$

$$R(x,\mu,\varphi,x',\mu',\varphi')\hat{P}(\mu,\varphi,\mu',\varphi')\mathbf{S}_{i}(x',\mu')\,d\varphi'\,d\mu'\,dx'. \tag{6}$$

We can think of R, as being one of the standard frequency redistribution function $(R_{\rm I}, R_{\rm II}, R_{\rm III}, \cdots)$ (Hummer 1962; Mihalas 1978). The frequency variable is $x = (\nu - \nu_o)/\Delta\nu_{\rm D}$, the standard non-dimensional frequency variable. The direction $\bf n$ (respectively $\bf n'$) is defined by its polar angle θ with respect to the normal to the surface and an azimuthal angle φ measured with respect to an arbitrarily oriented axis on the surface of the slab (respectively θ' and φ'). As usual $\mu = \cos \theta$ and $\mu' = \cos \theta'$. Since we are considering a problem which is axially symmetric, we can assume that $\bf S_i$ is independent of the azimuth.

We concentrate on the integration over φ' which can be performed as an independent calculation since S_i does not depend on φ' . Let

$$\hat{T}(x,\mu,x',\mu') = \frac{1}{2\pi} \int_0^{2\pi} R(x,\mu,\varphi,x',\mu',\varphi') \hat{P}(\mu,\varphi,\mu',\varphi') \, d\varphi'. \tag{7}$$

Following Domke & Hubeny (1988), we represent the angle-dependent redistribution function with a Fourier azimuthal expansion. This allows us to write

$$R(x, \mu, \varphi, x', \mu', \varphi') = R_0(x, \mu, x', \mu') + R_1(x, \mu, x', \mu') \cos(\varphi - \varphi') + R_2(x, \mu, x', \mu') \cos 2(\varphi - \varphi') + \cdots$$
 (8)

This expansion has terms to all orders but involves only cosines since the redistribution function is an even function of $(\varphi - \varphi')$. The first coefficients $R_i(x, \mu, x', \mu')$ are plotted in Domke & Hubeny (1988) as function of x for some choices of μ , μ' and x'. It can be observed that they are decreasing with increasing values of the index i, but not in a monotonic way for a given x.

2.1. The Hanle phase matrix

An elegant expression for the Hanle phase matrix $\hat{P}_{\rm H}$ corresponding to the geometry of the problem is given in Stenflo (1994, p. 88-89). Ignoring circular polarization and assuming a normal Zeeman triplet, it can be written as

$$\hat{P}_{\rm H} = \hat{E}_{11} + \frac{3}{4}\hat{P}_{\rm H}^2. \tag{9}$$

Here \hat{E}_{11} is the isotropic matrix (all its elements are zero except the element (1,1) which is equal to 1). The matrix $\hat{P}_{\rm H}^2$, which describes linear polarization, can be written as

$$\hat{P}_{H}^{2} = \hat{P}_{0}^{2}(\mu, \mu')
+ 2\cos\alpha_{1}\sin\theta\sin\theta'[\hat{P}_{1}^{2}\cos(\varphi - \varphi' - \alpha_{1}) + \hat{P}_{-1}^{2}\sin(\varphi - \varphi' - \alpha_{1})]
+ \cos\alpha_{2}[\hat{P}_{2}^{2}\cos(2(\varphi - \varphi') - \alpha_{2}) + \hat{P}_{-2}^{2}\sin(2(\varphi - \varphi') - \alpha_{2})],$$
(10)

where

$$\hat{P}_0^2 = \frac{1}{2} \begin{pmatrix} \frac{1}{3} (1 - 3\mu^2)(1 - 3{\mu'}^2) & (1 - 3\mu^2)(1 - {\mu'}^2) & 0\\ (1 - {\mu}^2)(1 - 3{\mu'}^2) & 3(1 - {\mu}^2)(1 - {\mu'}^2) & 0\\ 0 & 0 & 0 \end{pmatrix}, \tag{11}$$

$$\hat{P}_1^2 = \mu \mu' \begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} + \hat{E}_{33}, \tag{12}$$

$$\hat{P}_{-1}^2 = \begin{pmatrix} 0 & 0 & \mu \\ 0 & 0 & \mu \\ -\mu' & -\mu' & 0 \end{pmatrix},\tag{13}$$

$$\hat{P}_{2}^{2} = \frac{1}{2} \begin{pmatrix} (1 - \mu^{2})(1 - {\mu'}^{2}) & -(1 - \mu^{2})(1 + {\mu'}^{2}) & 0\\ -(1 + \mu^{2})(1 - {\mu'}^{2}) & (1 + \mu^{2})(1 + {\mu'}^{2}) & 0\\ 0 & 0 & 4\mu\mu' \end{pmatrix}, \tag{14}$$

$$\hat{P}_{-2}^{2} = \begin{pmatrix} 0 & 0 & -(1-\mu^{2})\mu' \\ 0 & 0 & (1+\mu^{2})\mu' \\ \mu(1-\mu'^{2}) & -\mu(1+\mu'^{2}) & 0 \end{pmatrix}.$$
 (15)

All the elements of \hat{E}_{33} are zero except the element (3,3) which is equal to one. Equation (10) is actually a Fourier azimuthal expansion of $\hat{P}_{\rm H}^2$. Note that it is limited to second order terms. The Hanle angles α_1 and α_2 depend on the strength of the magnetic field only. They are zero when the magnetic field is zero. In this case the Hanle phase matrix reduces to the Rayleigh phase matrix. The analytical expressions of α_1 and α_2 can be found in several papers (e.g. Landi degl'Innocenti & Landi degl'Innocenti 1988; Faurobert-Scholl 1991; Stenflo 1994) but are irrelevant here.

We now multiply the redistribution function by the phase matrix and calculate the contribution from each of the Fourier coefficient in equation (8). Coefficients with $i \geq 3$ will not contribute to the scattering integral (6), because the Fourier expansion of $\hat{P}_{\rm H}^2$ is limited to terms of second order. Straight-forward algebra leads to

$$\hat{T}(x,\mu,x',\mu') = R_0(x,\mu,x',\mu')[\hat{E}_{11} + \frac{3}{4}\hat{P}_0^2(\mu,\mu')]$$

$$+ \frac{3}{4}R_1(x,\mu,x',\mu')\cos\alpha_1\sin\theta\sin\theta'[\cos\alpha_1\,\hat{P}_1^2(\mu,\mu') - \sin\alpha_1\,\hat{P}_{-1}^2(\mu,\mu')]$$

$$+ \frac{3}{8}R_2(x,\mu,x',\mu')\cos\alpha_2\,[\cos\alpha_2\,\hat{P}_2^2(\mu,\mu') - \sin\alpha_2\,\hat{P}_{-2}^2(\mu,\mu')].$$
(16)

We assume that the incident field is unpolarized $(Q_i = 0 \text{ and } U_i = 0)$ and look for terms which can contribute to U_e , the U component of \mathbf{S}_e . Equations (11) to (15) show that there are two such terms, namely the terms proportional to $R_1(x,\mu,x',\mu')\sin\alpha_1$ and $R_2(x,\mu,x',\mu')\sin\alpha_2$. Two conditions are necessary for these terms to be present: a non-zero magnetic field and a frequency redistribution function depending on the scattering angle. From equation (13) we see that \hat{P}_{-1}^2 couples Stokes U_e to Stokes I_i (and Stokes Q_i), and thus contributes to creating a non-zero U_e component even when the incident Stokes vector is unpolarized. The same property also holds for \hat{P}_{-2}^2 . To calculate U_e we still have to integrate over x' and μ' . Since \mathbf{S}_i is in general anisotropic and frequency dependent, there will be no cancellations that could lead to a destruction of U_e .

Thus for a redistribution matrix of the form given in equation (5), U_e is not zero, even when the magnetic field is normal to the medium. However, as shown in section 3, the integral of U_e over frequency is zero. We also show in section 3 that U_e is zero when the incident field is independent of frequency.

2.2. The Rayleigh phase matrix

The Rayleigh phase matrix is given by equations (9) and (10) with $\alpha_1 = \alpha_2 = 0$. Equation (16) reduces then to

$$\hat{T}(x,\mu,x',\mu') = R_0(x,\mu,x',\mu')[\hat{E}_{11} + \frac{3}{4}\hat{P}_0^2(\mu,\mu')] + R_1(x,\mu,x',\mu')\sin\theta\sin\theta'\frac{3}{4}\hat{P}_1^2(\mu,\mu') + R_2(x,\mu,x',\mu')\frac{3}{8}\hat{P}_2^2(\mu,\mu').$$
(17)

The matrices \hat{P}_{-1}^2 and \hat{P}_{-2}^2 contributing to $U_{\rm e}$ do not appear anymore. Hence, for Rayleigh scattering, $U_{\rm e}(x,\mu)=0$ when the magnetic field is normal to the medium. Angle-averaged and angle-dependent redistribution functions yield somewhat different values of $Q_{\rm e}$ (Faurobert 1987, 1988). The changes are well correlated with the order of magnitude of the functions $\bar{R}_i(x,\mu.x',\mu')$.

3. Integrated quantities

The standard redistribution functions (Hummer 1962; Mihalas 1978) satisfy the conservation relations

$$\int_{-\infty}^{+\infty} R(x, \mathbf{n}, x', \mathbf{n}') dx' = \phi(x); \quad \int_{-\infty}^{+\infty} R(x, \mathbf{n}, x', \mathbf{n}') dx = \phi(x'), \tag{18}$$

where ϕ is the absorption profile. These relations are easy to establish when the redistribution function is represented by a double Fourier transform with respect to x and x' (Rybicki 1976; Frisch 1988). An equivalent method is to start from the expression of $R(x, \mathbf{n}, x', \mathbf{n}')$ still written as an integral over the velocity distribution function. Equation (18) implies that the integrals over x or x' of the expansion coefficients $R_i(x, \mu, \varphi, x', \mu', \varphi')$, $i \geq 1$, are zero.

When the incident field is independent of frequency, i.e. $\mathbf{S}_{i}(x,\mu) = \mathbf{S}_{i}(\mu)$, we can integrate equation (6) over x' first. Taking equation (18) into account and integrating over the azimuthal angles φ' , we obtain

$$\mathbf{S}_{e}(x,\mu) = \phi(x)\frac{1}{2} \int_{-1}^{+1} \left[\hat{E}_{11} + \frac{3}{4}\hat{P}_{0}^{2}(\mu,\mu')\right] \mathbf{S}_{i}(\mu') d\mu'. \tag{19}$$

Looking at the structure of equation (11), we see immediately that $U_e(x, \mu) = 0$ for all values of x and μ .

When the incident field depends on frequency, but is a known function, it may be possible to integrate explicitly over x'. An example can be found in Sahal-Bréchot et al. (1998), where the incident field is non-polarized and Gaussian and the redistribution function is $R_{\rm I}$. For this specific problem, it can be shown hat $U_{\rm e}(x,\mu)=0$ when the geometry of the problem as an axial symmetry (e.g. solar wind perpendicular to the surface of the sun).

We now consider the integral of $S_e(x,\mu)$ over x. Using, as above, equations (6), (9), (10) and (18), we find

$$\int_{-\infty}^{+\infty} \mathbf{S}_{\mathbf{e}}(x,\mu) \, dx = \frac{1}{2} \int_{-\infty}^{+\infty} \int_{-1}^{+1} \phi(x') [\hat{E}_{11} + \frac{3}{4} \hat{P}_{0}^{2}(\mu,\mu')] \mathbf{S}_{i}(x',\mu') \, d\mu' \, dx'. \quad (20)$$

The r.h.s. involving only the isotropic matrix and $\hat{P}_0^2(\mu,\mu')$, the integral of $U_{\rm e}(x,\mu)$ over frequency is clearly zero. There is an interesting remark to be made. When there is no systematic velocity, as is implicitly assumed here, the incident and scattered Stokes vectors are symmetric with respect to the line center at x=0. The condition that the integral of $U_{\rm e}$ over the line is zero implies

$$\int_0^\infty U_{\rm e}(x,\mu) \, dx = 0,\tag{21}$$

for all values of μ and all points in space. This result can be used to check numerical codes.

With a redistribution matrix of the type shown in equation (4) which has no angle-frequency coupling, the Hanle effect disappears when the magnetic field is normal to the slab. This is easily verified with the equations given here. Replacing in equation (6), $R(x, \mu, \varphi, x', \mu', \varphi')$ by its angle-average $\bar{R}(x, x')$ and integrating over φ' , one finds that $S_{\rm e}(x, \mu)$ is given by the r.h.s. of equation (20) with $\phi(x')$ replaced by $\bar{R}(x, x')$. This is the situation which prevails in numerical work done up to now with PRD.

4. Concluding remarks

For the Hanle effect, the redistribution matrix is far from having the simple form written in equation (5). As explained in Section 1, Bommier (1997b) has shown that the (x,x') frequency space can be divided in several domains where the redistribution matrix is of the form shown in equation (3). The expressions proposed by Bommier (1997b) are consistent with the property that the Hanle effect acts only in the line core (Omont et al. 1972; Stenflo 1994, p. 81). With these expressions, as demonstrated in this paper, Stokes U will not be zero when the magnetic field is normal to the surface of the medium. The question arises whether the integral of U over a half-profile, say for x > 0, (see equation (21)) is zero. Preliminary numerical calculations performed with expressions proposed by Bommier show that this condition could be at least approximately satisfied. In the line wings, U=0 because there is no Hanle effect. In the line core, say for $x < x_{\rm c}$, with $x_{\rm c}$ some cut-off frequency, U has both positive and negative values that could canceled out by integration. More systematic numerical work has to be performed to see how close to zero the integral over a half-profile is and how sensitive it is to cut-off parameters in the redistribution matrix. We can however already point out that observing a non zero Stokes U does not necessarily mean that the magnetic field is not normal to the surface. The integral of U over the profile has also to be considered before any conclusion can be drawn.

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References

Bommier, V. 1997a, A&A, 328, 706 Bommier, V. 1997b, A&A, 328, 726 Bommier, V., Stenflo, J.O. 1999, A&A, 350, 327

Domke, H., Hubeny, I. 1988, ApJ, 334, 527

Faurobert, M. 1987, A&A, 178, 269

Faurobert, M. 1988, A&A, 194, 268

Faurobert-Scholl, M. 1991, A&A, 246, 469

Faurobert-Scholl, M., Frisch, H., Nagendra, K.N. 1997, A&A, 322, 896

Faurobert-Scholl, M., Paletou, F., Bommier, V. 1999, in Solar Polarization (2nd SPW), eds. K.N. Nagendra, J.O. Stenflo (Kluwer Academic Publishers, Dordrecht), 115

Frisch, H. 1988, in *Radiation in Moving Gaseous Media*, eds. Y. Chmielewski, T. Lanz, (Geneva Observatory, Switzerland), 337

Hummer, D. 1962, MNRAS, 125, 21

Landi degl'Innocenti, E., 1983, Solar Phys., 85, 3

Landi degl'Innocenti, E., 1984, Solar Phys., 91, 1

Landi degl'Innocenti, E., 1985, Solar Phys., 102, 1

Landi degl'Innocenti, E., 1996, Solar Phys., 164, 29; also in *Solar Polarization* (1st SPW), eds. J.O. Stenflo, K.N. Nagendra & (Kluwer Academic Publishers, Dordrecht),

Landi degl'Innocenti, M., Landi degl'Innocenti, E., 1988, A&A, 192, 374

Landi degl'Innocenti, E., Landi degl'Innocenti, M., Landolfi, M. 1996, in Forum Thémis, Science with Thémis, eds. N. Mein, S. Sahal-Bréchot (Observatoire de Paris), 59

Mihalas, D. 1978, Stellar Atmospheres (2nd edition), Freeman, San Francisco

Nagendra, K.N., Frisch, H., Paletou, F., Faurobert-Scholl, M. 1999, in *Solar Polarization* (2nd SPW), eds. K.N. Nagendra, J.O. Stenflo (Kluwer Academic Publishers, Dordrecht), 127

Omont, A., Smith, E.W., Cooper, J. 1972, ApJ, 175, 185

Omont, A., Smith, E.W., Cooper, J. 1973, ApJ, 182, 283

Paletou, F., Faurobert-Scholl, M. 1997, A&A, 328, 343

Rees, D.E., Saliba, G. 1982, A&A, 115, 1

Rybicki, G. 1976, personnal communication

Sahal-Bréchot, S., Bommier, V., Feautrier, N. 1998, A&A, 340, 579

Stenflo, J.O. 1994, Solar Magnetic Fields, Kluwer Academic Publishers, Dordrecht