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Scattering Polarization and Hanle Effect: On the Importance of Angle-Dependent Frequency Redistribution

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Abstract. Some spectral lines formed by scattering of photons in stellar atmospheres or in circumstellar disks, show a linear polarization in the absence of magnetic fields. This so-called resonance scattering polarization is modified by the Hanle effect due to weak magnetic fields. This effect is used for the diagnostics of weak magnetic fields in the solar atmosphere, and it could be used in circumstellar disks too.

In the following we consider polarized spectral lines formed by scattering in dilute media. Partial frequency redistribution plays an important role in photon scattering processes. In the presence of Hanle effect, frequency redistribution is coupled with angular and polarization redistribution. However some approximate forms of the redistribution matrix, based on angle-averaging of the frequency redistribution function are often used.

The validity of these approximations has been verified in the non-magnetic case (Faurobert, 1987). We show here that they may lead to significant errors in the presence of a weak magnetic field. In particular the coupling between angular and frequency redistribution has an unexpected effect on the behaviour of the Stokes parameter U which becomes non-zero even in an axially symmetric medium.

1. Introduction

Resonance scattering and the Hanle effect in spectral lines can be described at the microscopic level by a redistribution matrix $\hat{\mathcal{R}}$ defined through the relation

$$\mathbf{S}_e(\nu, \mathbf{n}) = \int\int \hat{\mathcal{R}}(\nu, \mathbf{n}, \nu', \mathbf{n}'; \mathbf{B}) \mathbf{S}_\mathbf{i}(\nu', \mathbf{n}') \mathrm{d}\nu' \frac{\mathrm{d}\Omega'}{4\pi}. \quad (1)$$

Here $\mathbf{S_i}$ is the incident Stokes vector and $\mathbf{S_e}$ the scattered Stokes vector. The redistribution matrix $\hat{\mathcal{R}}$ is the differential scattering cross section for radiation with incoming frequency and direction $(\nu', \mathbf{n}')$, getting scattered into the outgoing frequency and direction $(\nu, \mathbf{n})$. Here we are concerned only with the Hanle effect on linear polarization. Hence $\hat{\mathcal{R}}$ is a 3x3 matrix and $\mathbf{S} = (I, Q, U)^\dagger$.

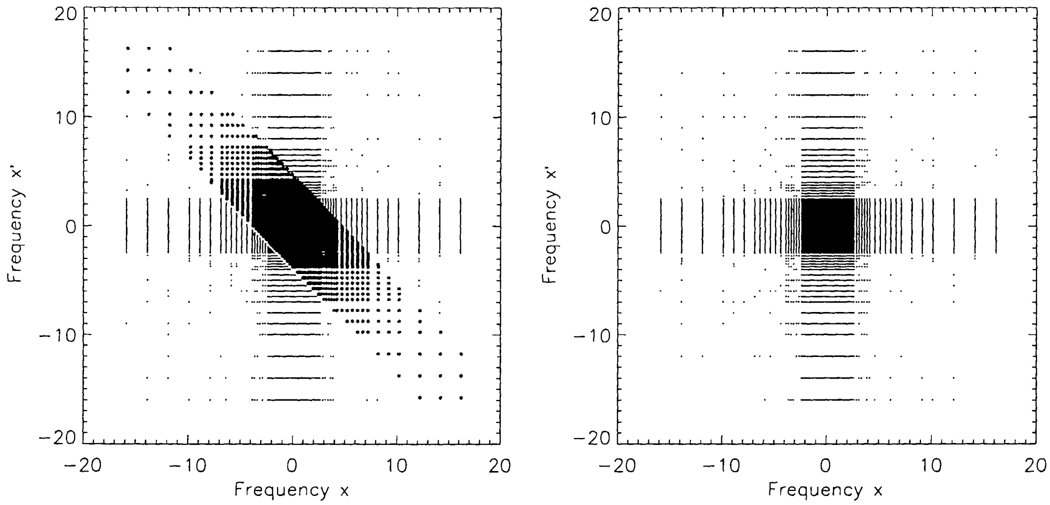

Figure 1. 2-D line core domains, left panel refers to the scattering at: right angle, $\Theta = 90°$, the right panel to the forward scattering $\Theta = 0°$.

The determination of $\hat{\mathcal{R}}$ is a difficult theoretical problem (see Omont, Smith, & Cooper, 1972, 1973; Domke & Hubeny, 1988; Bommier 1997a, b). In the following, two different levels of approximations are considered, corresponding to level II and III in Bommier 1997b, they lead to the derivation of angle-dependent redistribution functions and of their angle-averaged versions.

For magnetic field diagnostics with the Hanle effect, we need highly accurate solutions of the transfer problems. We are thus interested in finding out whether there are observable differences on the emergent polarization profiles depending on the level of approximation that is being used. We neglect depolarizing elastic collisions, the redistribution function is thus the so-called R_{II} redistribution function in standard notation (see Bommier 1997b). For frequencies in the line core, i.e.

$$|x + x'| < 2\nu_c(a/\cos(\Theta/2))\cos(\Theta/2),$$

the angle-dependent redistribution matrix is given by

$$\hat{\mathcal{R}}(x, x', \mathbf{n}, \mathbf{n}'; B) = R_{II}(x, x', \Theta)\hat{P}_H(\mathbf{n}, \mathbf{n}'; B). \qquad (2)$$

For the other frequencies

$$\hat{\mathcal{R}}(x, x', \mathbf{n}, \mathbf{n}'; B) = R_{II}(x, x', \Theta)\hat{P}_R(\mathbf{n}, \mathbf{n}'). \qquad (3)$$

As usual x denotes the frequency in Doppler width units, the scattering angle Θ is the polar angle between the propagation directions of the incident and emergent photons, $\hat{P}_R$ is the Rayleigh phase matrix and $\hat{P}_H(\mathbf{n}, \mathbf{n}'; B)$ is the Hanle phase matrix, ν_c is a cut-off frequency which depends on the value of the Voigt parameter of the line. It takes values of the order of a few Doppler widths.

Angular averaging with respect to the scattering angle Θ leads to similar expressions with R_{II} replaced by its angle average. The frontiers of the "line core" domain are modified in the averaging process (see Bommier 1997b).

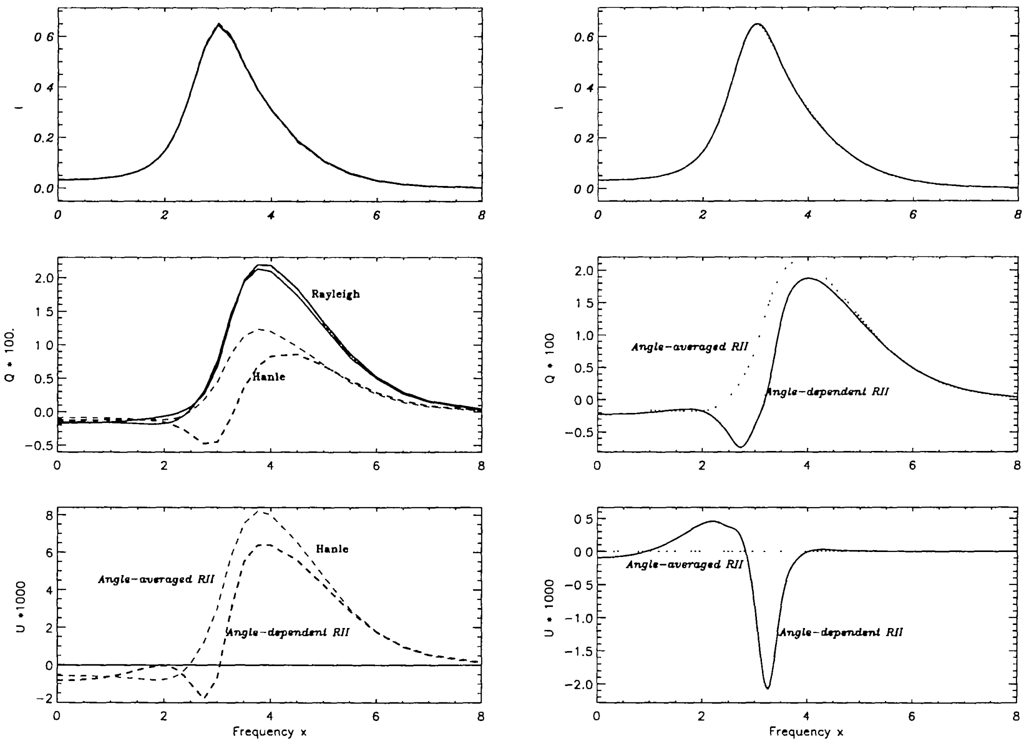

Figure 2. Emergent Stokes parameters I, Q, U as functions of frequency x, in Doppler width units. Profiles are symmetrical with respect to line center ($x = 0$). The inclination of the line of sight θ is such that $\cos\theta = 0.11$, its azimuth, $\phi = 0$. The line optical thickness is $T = 2X10^4$, the probability of collisional destruction of photons $\varepsilon = 10^{-3}$, the Voigt parameter of the line $a = 10^{-3}$, and $\gamma_B = 1$. Left panel: inclination of the magnetic field $\theta_B = 30°$, right panel: vertical magnetic field ($\theta_B = 0$). The magnetic field azimuth is $\phi_B = 0$.

2. Numerical tests

2.1. Numerical method

In a first step the intensity is computed, ignoring the polarization, through the CRD-CS iterative scheme of Paletou & Auer (1995). Then the polarization is computed by a Lambda-iteration. The short characteristic method is used to obtain at each iteration the formal solutions of the transfer equations for (I, Q, U). Less than 20 iterations are necessary to achieve a relative change of less than 0.1% in the polarization rate at the surface. The most demanding part, as far as CPU time and memory requirement are concerned, is the computation of the scattering integral over incident frequencies and directions.

2.2. Emergent polarization profiles

Figure 2 shows the profiles of the emergent Stokes parameters I, Q and U for isothermal slabs where the continuum absorption is ignored. The magnetic field is depth independent, and the Larmor frequency of the electron is equal to the radiative line width, so that the Hanle effect parameter $\gamma_B = 1$. The left panel corresponds to a magnetic field inclined with respect to the normal to the slab, with $\theta_B = 30°$. The right panel corresponds to a vertical magnetic field. The geometry of the problem, and the meaning of all the symbols can be found in Nagendra et al. (1998).

We notice that angle-averaging yields a good approximation in the case of Rayleigh scattering (no magnetic field, see also Faurobert 1987), but that it significantly affects the emergent polarization in the presence of Hanle effect.

An unexpected effect is obtained when the magnetic field is vertical (i.e. parallel to slab normal). When angle-averaged redistribution functions are used the parameter U of the emergent radiation vanishes because the normal to the slab is a symmetry axis. This is no longer true when the angle dependance of the redistribution function is taken into account, as shown in the right panel of Fig. 2.

2.3. Some explanations

Let us consider a single scattering event. We assume that the incident radiation is unpolarized but depends on frequency over the frequency domain of the line. This mimics the formation of the emergent polarization in an optically thick spectral line. Polarization arises from the surface layer, whereas the intensity is formed in deeper layers where the radiation field is almost isotropic.

In Eq. (1) we replace the redistribution matrix by the expressions given in Eqs. (2) and (3). As the angle-dependent frequency redistribution function R_{II} depends on the difference in azimuths of the incident and emergent directions, the product of R_{II} and of the incident intensity is not symmetrical with respect to the normal to the slab. This product plays the role of an effective incident radiation which is scattered according to the properties of the phase matrix.

One property of Hanle scattering is that an unpolarized but non-axially symmetric radiation field is scattered into a polarized radiation with non- zero values for both Stokes Q and U. The parameter U of the scattered radiation averages out to zero when we integrate over the incident frequency, if the atom is illuminated by a frequency-independent radiation field. But this cancellation effect does not take place if the incident frequency profile is not flat over the line domain. The explanations given here can be found in a more expanded form in Frisch, Nagendra, & Faurobert (2001).

References

Bommier, V. 1997a, A&A, 328, 706

Bommier, V. 1997b, A&A, 328, 726

Domke, H., & Hubeny, I, 1988, ApJ, 334, 527

Faurobert, M., 1987, A&A, 178, 269

Frisch, H., Nagendra, K.N., & Faurobert, M. 2001, in ASP Conf. Ser., Sacramento Peak Workshop on Solar Polarimetry, in press

Nagendra, K.N., Frisch, H., & Faurobert, M., 1998, A&A, 332, 610

Omont, A., Smith, E.W., & Cooper, J. 1972, ApJ, 175, 185

Omont, A., Smith, E.W., & Cooper, J. 1973 ApJ, 182, 283

Paletou, F., & Auer, L.H. 1995, A&A, 297, 771