Tensor polarization of ω produced at threshold in $p{-}p$ collisions

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Abstract. It is shown that the dominant decay mode of $\vec{\omega} \rightarrow \pi^+\pi^-\pi^0$ can be employed to determine the Fano statistical tensor t^2_0 of $\vec{\omega}$ with respect to the quantization axis normal to the decay plane. In $pp \rightarrow pp\vec{\omega}$ one can choose decay planes with different orientations for a given ω direction, θ. By choosing three different experimentally convenient orientations of the decay planes for the same θ, one may determine empirically the t^2_0 and $t^2_{\pm 2}$ characterising the tensor polarization of ω in the Transverse Frame for $pp \rightarrow pp\vec{\omega}$.

Meson production in $N-N$ collisions has attracted considerable attention during the last decade of the past century as total cross section measurements for pion production in the early years of the decade were found to be more than a factor of 5 than the then available theoretical predictions. This catalyzed a variety of theoretical approaches and Hanhart *et al* [1] in 2000 have observed: " As far as microscopic model calculations of the reaction on $NN \rightarrow NN\pi$ are concerned one has to concede that theory is definitely lagging behind the development of the experimental sector". With measurements of spin observables in charged [2] as well as neutral [3] pion production employing a polarized beam on a polarized target, experimental studies have indeed reached a high degree of sophistication. Reference may be made to several excellent reviews on these developments [4]. The Jülich meson exchange model [1], which yielded theoretical predictions closer to data than most other models, has been more successful in the case of charged pion production [2] than with neutral pions [3]. A recent analysis [5] of $\vec{p}\vec{p} \rightarrow pp\pi^0$ measurements [3], following a model independent irreducible tensor approach [6], showed that the Jülich model deviates from the empirically extracted estimates quite significantly for the ${}^3P_1 \rightarrow {}^3P_0p$ and to a lesser extent for the ${}^3F_3 \rightarrow {}^3P_2p$ transitions; this analysis has also been carried out with and without taking into consideration the Δ contribution to emphasize its importance in the model calculation. As the c.m. energy is increased further, thresholds for heavier meson production are reached and one has to consider also contributions from excited nucleon states. Since theoretical models for the nucleon predict resonance states not seen in $\pi - N$ scatterng, experimental studies on meson production in $N-N$ collisions and on photo production of mesons [7] are useful to look for the 'missing resonances' [8] as well. It is also interesting to note that production of isoscalar mesons like ω and ϕ involve only excited nucleon states, in contrast to the case of isovector pion production. Moreover, since meson production in $N-N$ collisions probes short range hadron dynamics, it is particularly of interest to study heavy meson production. While the distance probed is ≈0.53 fm in the case of pion production, it goes down to 0.21 fm for ω and 0.18 fm for ϕ production [9]. In the absence of strange quarks in the initial state, the Okubo-Zweig-Iizuka (OZI) rule [10] supresses ϕ production relative to ω production. In view of the dramatic violation [11] of this rule observed in $\bar{p}p$ collisions, the ratio $R_{\phi/\omega}$ was measured [12] and was found to be an order of magnitude larger, after correction for the available phase space, than the theoretical estimate $R_{OZI} = 4.2 \times 10^{-3}$ [13]. The latest experimental estimate [14] is $R_{\phi/\omega} \approx 8 \times R_{OZI}$. Heavy meson production has also attracted attention in the context of di-lepton spectra and medium modifications [15]. There is a proposal [16] for the experimental study of heavy meson production in $\vec{N}\vec{N}$ collisions. Moreover the state of polarization of vector meson provides an interesting observable in addition to the differential cross section and analyzing powers. Concerning ourselves here with ω production, the total cross section has been measured at five different c.m. energies in the range 3.8 MeV to 30 MeV above threshold by Hibou *et al* [17] for $pp \rightarrow pp\omega$ and by Barsov *et al* [18] for $pn \rightarrow d\omega$. Measurements of the total cross section and angular distribution have also been reported on $pp \rightarrow pp\omega$ at excess energies of 92 MeV and 173 MeV [19] and at 60 MeV and 92 MeV [20] apart from data [21] yet to be published. The model independent irreducible tensor approach [6] has been extended to ω production [22] where it was pointed out that the state of polarization of ω can be studied experimentally using the decay mode $\omega \rightarrow \pi^0\gamma$. The importance of measuring the polarization of ω has also been highlighted to determine empirically the threshold partial wave amplitudes [23] for $pp \rightarrow pp\omega$. It is learnt [24] that it has been possible to fully reconstruct the decay of the ω into three pions and identify the orientation of the decay plane with respect to the ω direction as well as the beam direction. Therefore, the purpose of the present paper is to examine if the dominant decay mode of $\omega \rightarrow \pi^+\pi^-\pi^0$ could be utilized to study the state of polarization of ω.

Employing the irreducible tensor operators $\mathbf{S}^\lambda_\mu(s_f, s_i)$ of rank λ connecting initial and final channel spin states s_i and s_f of hadrons as defined in [25] and following the approach [26] of Dalitz, we may write [27], the decay matrix for $\omega \rightarrow \pi^+\pi^-\pi^0$ as

$$\mathbf{M} = f(\mathbf{S}^1(0,1).Q^1), \tag{1}$$

where f is a symmetric function of the energies of the 3 pions and Q^1_μ denote the spherical components of

$$\mathbf{Q} = [\mathbf{q_1} \times \mathbf{q_2} + \mathbf{q_2} \times \mathbf{q_3} + \mathbf{q_3} \times \mathbf{q_1}], \tag{2}$$

in terms of the momenta $\mathbf{q_1}, \mathbf{q_2}, \mathbf{q_3}$ of the three pions which add up to zero in the ω rest frame. We may define a right handed coordinate system with z-axis along $\mathbf{Q}$ and x-axis along say $\mathbf{q_1}$ in the decay plane. This may be referred to as the Decay Frame (DF).

Following the Madison convention [28], the spin density matrix $\boldsymbol{\rho}$ of a polarized ω may be written in the form

$$\boldsymbol{\rho} = \frac{\mathrm{Tr}\,\boldsymbol{\rho}}{3}[1 + \sum_{k=1}^{2}(\boldsymbol{\tau}^k \cdot t^k)], \tag{3}$$

in terms of the standard 3×3 matrices $\boldsymbol{\tau}^k_\mu, \mu = -k, .., k$ and the Fano statistical tensors t^k_μ of rank k, characterising the polarized ω. Noting that $\boldsymbol{\tau}^k_\mu = \mathbf{S}^k_\mu(1,1)$ and making use of the known [25] properties of the irreducible tensor operators along with Racah techniques, the angular distribution of the three pions in the decay plane is given by

$$\mathbf{M}\boldsymbol{\rho}\mathbf{M}^\dagger = |f|^2\frac{\mathrm{Tr}\,\boldsymbol{\rho}}{3}[(\mathbf{Q}\cdot\mathbf{Q}) - \sqrt{3}\sum_{k=1}^{2}(t^k \cdot (Q^1 \otimes Q^1)^k)], \tag{4}$$

where $\mathbf{M}^\dagger$ denotes the hermitian conjugate of $\mathbf{M}$. The first term $(\mathbf{Q}\cdot\mathbf{Q})$ leads to the well-known Dalitz plot for the 3π decay of unpolarized ω. Since $(Q^1 \otimes Q^1)^1_\mu = 0$, the $k = 1$ term drops out, so that (4) contains only the tensor analyzing power

$$A^2_0(DF) = -\sqrt{3}(Q^1 \otimes Q^1)^2_0, \tag{5}$$

since $(Q^1 \otimes Q^1)^2_\mu = \delta_{\mu 0}\sqrt{\frac{2}{3}}(\mathbf{Q}\cdot\mathbf{Q})$ in DF. We have

$$\mathbf{M}\boldsymbol{\rho}\mathbf{M}^\dagger = \frac{|f|^2\,\mathrm{Tr}\,\boldsymbol{\rho}}{3}(\mathbf{Q}\cdot\mathbf{Q})[1 - \sqrt{2}t^2_0(DF)], \tag{6}$$

where $t^2_0(DF)$ denotes the $\mu = 0$ component of the tensor polarization t^2_μ of the $\vec{\omega}$ in its decay frame, DF. Thus the distribution of points in the Dalitz plot retains the same shape, even when $t^2_0(DF) \neq 0$. However, since $-\sqrt{2} \leq t^2_0 \leq \frac{1}{\sqrt{2}}$, the total number of points N(DF) in the associated decay plane is sensitive to the tensor polarization $t^2_0(DF)$ of $\vec{\omega}$ as the multiplicative factor $(1 - \sqrt{2}t^2_0(DF))$ may vary between 0 and 3. In the case of $\vec{\omega}$ in $pp \to pp\vec{\omega}$, we may identify $\mathrm{Tr}\,\boldsymbol{\rho}$ with the unpolarized differential cross section $\frac{d\sigma_0}{d\Omega}$, which is given as a function of the angle θ between the momentum $\mathbf{q}$ of the ω and the momentum $\mathbf{p_i}$ of the proton beam in the c.m. frame for the reaction. A right handed Cartesian coordinate system with z-axis along $\mathbf{p_i} \times \mathbf{q}$ and x-axis along $\mathbf{p_i}$, was referred to [23] as the Transverse Frame(TF), where in the Fano statistical tensors t^2_μ characterising the tensor polarization of the $\vec{\omega}$ are such that $t^2_{\pm 1} = 0$ and

$$\begin{aligned}\mathrm{Tr}\,\boldsymbol{\rho}\,t^2_0 = \frac{1}{384\pi^2}\sqrt{\frac{1}{2}}\int dW\,([0.6|f'|^2 - |f_1|^2]\\ + 1.8\cos^2\theta[10|f_2|^2 + 2|f_3|^2 - |f'|^2]),\end{aligned} \tag{7}$$

$$\begin{aligned}\mathrm{Tr}\,\boldsymbol{\rho}\,t^2_{\pm 2} = \frac{1}{256\pi^2}\sqrt{\frac{1}{3}}\int dW\,([|f_1|^2 + 0.6|f'|^2] - 1.2\cos^2\theta[15|f_2|^2\\ + 3|f_3|^2 - 0.5|f'|^2] \mp 0.6i\sin 2\theta[15|f_2|^2 - 1.5|f_3|^2 - 0.5|f'|^2]),\end{aligned} \tag{8}$$

while

$$\begin{aligned}\mathrm{Tr}\,\boldsymbol{\rho} = \frac{d\sigma_0}{d\Omega} = \frac{1}{192\pi^2}\int dW\,([|f_1|^2 + 0.3|f'|^2]\\ + 0.9\cos^2\theta[10|f_2|^2 + 2|f_3|^2 - |f'|^2])\end{aligned} \tag{9}$$

in terms of the lowest three partial wave amplitudes f_1, f_2, f_3 for $pp \to pp\omega$ given in Table 1 of [23] and $f' = \sqrt{10}f_2 + f_3$. The integration with respect to the invariant mass W of the two protons in the final state may also be replaced by integration with respect to the c.m. energy E_ω of ω using eqn(1) of [23]. If (α, β, γ) denote the Euler angles of rotation for going from TF to DF, we have

$$t^2_0(DF) = \sum_{\mu=-2}^{2} D^2_{\mu o}(\alpha, \beta, \gamma) t^2_\mu. \tag{10}$$

Thus $t^2_0(DF)$ depends not only on the angle θ at which the $\vec{\omega}$ is produced, but also on the Euler angles (α, β, γ) which characterise the orientation of its decay plane with respect to TF. Counting N(DF) in a given decay plane provides, by virtue of (6), an estimate of the associated $t^2_0(DF)$. Noting that t^2_{-2} is the complex conjugate of t^2_2, it is clear that eqn.(10) is a linear equation in three real unknowns viz., $t^2_0, Re(t^2_2)$ and $Im(t^2_2)$ in the TF. One may conveniently choose three different sets of (α, β, γ) to measure experimentally the corresponding real numbers, $t^2_0(DF)$ on the left hand side of eqn.(10) for $\vec{\omega}$ produced in the same direction θ. Having set up an appropriate set of three linear equations, one may then solve for $t^2_0, Re(t^2_2)$ and $Im(t^2_2)$ in the TF which characterise completely the tensor polarization of ω produced in any chosen direction.

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