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Equilibrium structures of differentially rotating white dwarf stars

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Abstract. In one of our earlier papers (Mohan *et al.* 1992) we proposed a method for computing the equilibrium structures of polytropic models of stars rotating differentially according to the law $\omega^2 = b_0 + b_1 s^2 + b_2 s^4$ (ω being the angular velocity of rotation of a fluid element distant s from the axis of rotation and b_0, b_1, b_2 suitably chosen numerical constants). The method uses Kippenhahn and Thomas approach and Kopal's results on Roche equipotentials to obtain explicit form of stellar structure equations which incorporate rotational effects up to second order terms. In the present paper we use this method to compute the equilibrium structures of certain differentially rotating white dwarf stars.

Key words : white dwarf—stellar structure—differential rotation—Roche equipotential

1. Introduction

White dwarf models have been frequently used in literature to depict the inner structures of realistic low mass stars in their last stages of evolution. A white dwarf star is largely supported against gravity by the pressure provided by the kinetic energy of the degenerate electrons. In contrast, its luminosity is almost entirely derived from the thermal energy of the non-degenerate ions, when nuclear process no longer comes into play and gravitational contraction has almost ceased. A completely degenerate white dwarf very much resembles polytropic configurations, ranging from the polytropes of index $N = 1.5$ (in the limit $M \to 0$) to the polytrope of index $N = 3.0$ (in the limit $M \to M_3$, where M_3 is the mass of polytropic star of index 3.0).

Some of the white dwarf stars are known to be rotating stars. The white dwarf stars of class DC, those which have no observable lines, are possible candidates for having differential rotation. By virtue of the Poincare-Wavré theorem, a barotropic configuration in a state of permanent rotation must necessarily comply with the condition $\Omega = \Omega(s)$, where Ω is the angular velocity of rotation and s is the distance from the axis of rotation. The particular case of constant angular velocity of rotation has been considered by several authors such as

James (1964), Anand & Dubas (1968), Roxburgh (1965), Ostriker & Hartwic (1968) etc. Their results show that solid body rotation does not induce any substantial change in the global structure of degenerate dwarfs. However, the intense study carried out by Hoyle (1947) and Roxburgh (1965) on some problems of differentially rotating white dwarf stars assuming an angular momentum distribution law of the type $J = J(m_s)$, where m_s is the mass fraction interior to the cylinder of radius s with axis as axis of rotation, pointed out completely different picture. Detailed models of massive white dwarfs in fast non-uniform rotation have been also constructed by Ostriker *et al.* (1966).

In the present paper we consider the feasibility of using the approach developed in our earlier paper (cf. Mohan *et al.* 1992) to determine the effects of differential rotation on the equilibrium structures of white dwarf models rotating differentially according to the law $\omega^2 = b_0 + b_1 s^2 + b_2 s^4$ (ω being the angular velocity of rotation of a fluid element distant s from the axis of rotation and b_0, b_1, b_2, suitably chosen numerical constants). The approximation of exact equipotential surfaces of rotating white dwarfs by corresponding Roche equipotential used in this method may not be, perhaps, very much justified. However still, it will be of interest to see how the equilibrium structures of white dwarf stars are affected by the differential rotation with the present approach vis-a-vis other approaches being used in literature. The boundary value problem determining the equilibrium structures of differentially rotating white dwarf models of stars has been set up in section 2. Expressions determining the volume, surface area and other physical parameters of differentially rotating white dwarf models are next obtained in section 3. Numerical results for the equilibrium structures of certain differentially rotating white dwarf models have been obtained in section 4. The results are analysed in section 5 and certain conclusions of practical significance drawn in section 6.

2. Equilibrium structures of differentially rotating white dwarf models

White dwarf models have been extensively studied in literature as representative models of low mass stars in their last stages of evolution. (see for instance Chandrasekhar 1939, Chapter XI). In the present section we investigate the problem of determining the equilibrium structures of differentially rotating white dwarf models rotating differentially according to the law

$$\omega^2 = b_0 + b_1 s^2 + b_2 s^4 \qquad \dots (1)$$

where ω is the angular velocity of rotation of a fluid element distant s from the axis of rotation and b_0, b_1, b_2 are suitably chosen numerical constants. The equilibrium structures of such differentially rotating white dwarfs can be computed using the technique earlier used by us (Mohan *et al.* 1992) for obtaining the equilibrium structures of differentially rotating polytropic stars. This approach uses the averaging technique of Kippenhahn & Thomas (1970) to account for the distortional effects caused by rotation. For computing the distortional effects caused by rotation, the actual equipotential surfaces of the star are approximated by Roche equipotentials and Kopal's (1972) results on Roche equipotentials used to express the problem in an explicit form convenient for numerical work.

In a self-graviting equilibrium configuration equipotential surfaces are also surfaces of equipressure and equidensity. Let P_ψ and ρ_ψ denote respectively the pressure and the density on the equipotential surface ψ = constant of a differentially rotating white dwarf model. Assuming the distorted model to be a completely degenerate white dwarf P_ψ and ρ_ψ of such a configuration will be connected through the relations of the type

$$P_\psi = Af(x) \quad \text{and} \quad \rho_\psi = Bx^3 \qquad \dots (2)$$

where following Chandrasekhar (1939), we have

$$A = 6.01 \times 10^{22}, \qquad B = 9.82 \times 10^5 \mu_e, \qquad \dots (3)$$

$$f(x) = x(2x^2 - 3)\,(x^2 + 1)^{1/2} + 3 \sinh^{-1} x, \qquad \dots (4)$$

$x = P_0/mc$ a relativistic constant. Also P_0 denotes momentum, m the mass of the particle and c the velocity of light and μ_e the mean molecular weight per electron.

Following Chandrasekhar (1939) and adopting the approach used by us in our earlier paper (Mohan *et al.* 1992), the differential equation governing the equilibrium structure of a white dwarf star rotating differentially according to the law (1) can be written explicitly in the non-dimentional form as

$$\frac{d}{d r_0}\left[A(b_0, b_1, b_2, r_0)\,\frac{d\phi_\psi}{d r_0}\right] = -\eta_u^2 r_0^2 B(b_0, b_1, b_2, r_0)\left(\phi_\psi^2 - \frac{1}{\phi_0^2}\right)^{3/2} \qquad \dots (5)$$

where
$$A(b_0, b_1, b_2, r_0) = r_0^2\left[1 - \frac{1}{15}b_0^2 r_0^6 - \frac{8}{105}b_0 b_1 r_0^8 - \left(\frac{16}{315}b_0 b_2 + \frac{41}{1575}b_1^2\right)r_0^{10} - \frac{128}{3465}b_1 b_2 r_0^{12} + \dots\right].$$

$$B(b_0, b_1, b_2, r_0) = 1 + 2b_0 r_0^3 + \frac{24}{5}b_0^2 r_0^6 + \frac{16}{21}b_2 r_0^7 + \frac{44}{7}b_0 b_1 r_0^8 + \left(\frac{1664}{315}b_0 b_2 + \frac{208}{105}b_1^2\right)r_0^{10} + \frac{2240}{693}b_1 b_2 r_0^{12} + \dots$$

and $r_0 = 1/\psi$ is a non-dimensional measure of the distance of the fluid element from the centre. In the above expression terms up to second order of smallness in b_0, b_1, b_2 and up to r_0^{12} in r_0 have been retained. The differential equation (5) is to be solved to satisfy the boundary conditions :

$$\phi_\psi = 1, \ \frac{d\phi_\psi}{dr_0} = 0, \text{ at the centre : } r_0 = 0,$$

$$\phi_\psi = \frac{1}{\phi_0} \text{ at the surface : } r_0 = r_{0s}, \qquad \dots (6)$$

r_{0s} being the value of r_0 at the outermost surface of the star.

Equation (5) subject to the boundary conditions (6) determines the equilibrium structure of a differentially rotating white dwarf model. On setting $b_0 = b_1 = b_2 = 0$, (5) reduces to the usual equation of a white dwarf model. On setting $b_0 = 2n$ and $b_1 = b_2 = 0$, equation (5) can be used to determine the equilibrium structure of a white dwarf model distorted by solid body rotation alone.

In order to determine the equilibrium structure of a differentially rotating white dwarf model second-order nonlinear differential equation (5) has to be solved numerically subject to the boundary conditions (6). Techniques normally used for numerically solving white dwarf structure equation can also be used here. One may start numerical integration of (5) (for specified values of $1/\phi_0^2$, η_u b_0, b_1 and b_2) from the centre using $\phi_\psi = 1$ and $d\phi_\psi/dr_0 = 0$ as the initial conditions. The integration is to be continued till ϕ_ψ equals to $1/\phi_0$. The value r_{0s} of r_0 for which ϕ_ψ becomes $1/\phi_0$, determines the outermost free surface of the model.

Once the numerical solution of (5) is obtained, we know the values of ϕ_ψ for values of the non-dimensional independent variable r_0 varying from zero to r_{0s}. The values of pressure P_ψ and the density ρ_ψ on the various equipotential surfaces of the distorted model may now be obtained using (2) in the same manner as is done for the undistorted white dwarf models (see for instance Chandrasekhar 1939, Chapter XI).

3. Computation of various physical parameters

Once the stellar structure equation (5) of a differentially rotating white dwarf model has been numerically integrated, values of various physical parameters may be computed as follows.

Following the approach adopted in our earlier paper (Mohan *et al.* 1992), the shape of the distorted configuration may be computed using

$$r = (\alpha\eta_u) r_{0s} \left[1 + \frac{1}{2} b_0 x r_{0s}^3 + \frac{1}{4} b_1 x^2 r_{0s}^5 + \frac{3}{4} b_0^2 x^2 r_{0s}^6 + \frac{1}{6} b_2 x^3 r_{0s}^7 + b_0 b_1 x^3 r_{0s}^8 + \left(\frac{5}{6} b_0 b_2 + \frac{5}{16} b_1^2 \right) x^4 r_{0s}^{10} + \frac{1}{2} b_1 b_2 x^5 r_{0s}^{12} + \ldots \right] \quad \ldots (7)$$

where η_u is the value of η when ϕ equals $1/\phi_0$ for the undistorted model and α is a measure of the distance given by $\alpha^2 = 2A/\pi G B \phi_0^2$. Also $x = 1 - \nu^2$, where $\nu = \cos\theta$.

The volume V_ψ and the surface area S_ψ of such differentially rotating white dwarf model are given respectively by

$$V_\psi = \frac{4\pi}{3} (\alpha\eta_u)^3 r_{0s}^3 \left[1 + b_0 r_{0s}^3 + \frac{2}{5} b_1 r_{0s}^5 + \frac{8}{5} b_0^2 r_{0s}^6 + \frac{8}{35} b_2 r_{0s}^7 + \frac{8}{35} b_0 b_1 r_{0s}^8 + \left(\frac{128}{105} b_0 b_2 + \frac{16}{35} b_1^2 \right) r_{0s}^{10} + \frac{64}{99} b_1 b_2 r_{0s}^{12} + \ldots \right] \quad \ldots (8)$$

and

$$S_\psi = 4\pi (\alpha\eta_u)^2 r_{0s}^2 \left[1 + \frac{2}{3} b_0 r_{0s}^3 + \frac{4}{15} b_1 r_{0s}^5 + \frac{14}{15} b_0^{12} r_{0s}^6 + \frac{16}{105} b_2 r_{0s}^7 + \frac{36}{35} b_0 b_1 + \left(\frac{704}{945} b_0 b_2 + \frac{88}{315} b_1^2 \right) r_{0s}^{12} + \frac{832}{2079} b_1 b_2 r_{0s}^{12} + \ldots \right] \quad \ldots (9)$$

Following Chandrasekhar (1939), the mass of the whole configuration of differentially rotating white dwarf is given by

$$M = -4\pi\left(\frac{2A}{\pi G}\right)^{3/2}\frac{1}{B^2}\left(-r_0^2\eta_u\frac{d\phi_\psi}{dr_0}\right)_{r_{0s}}$$

so that

$$\frac{M}{M_\odot} = \frac{2.8521}{\mu_e^2}\left(-r_0^2\eta_u\frac{d\phi_\psi}{dr_0}\right)_{r_{0s}} \qquad \ldots (10)$$

where $M_\odot$ is the mass of the sun. The ratio of the central density to the mean density $(\rho_c/\bar{\rho})$ can be computed using

$$\frac{\rho_c}{\bar{\rho}} = -\frac{1}{3}\left(1-\frac{1}{\phi_0^2}\right)^{3/2}\Bigg/\left[\frac{1}{r_0\eta_u^2}\frac{d\phi_\psi}{dr_0}\right]_{r_{0s}} \qquad \ldots (11)$$

The values of r_e/r_p, the ratio of the equitorial radius to the polar radius, and ω_e/ω_p, the ratio of angular velocity at the equator to the angular velocity at the pole are given by

$$\frac{r_e}{r_p} = 1 + \frac{1}{2}b_0r_{0s}^3 + \frac{1}{4}b_1r_{0s}^5 + \frac{3}{4}b_0^2r_{0s}^6 + \frac{1}{6}r_{0s}^7 + b_0b_1r_{0s}^8$$
$$+\left(\frac{5}{6}b_0b_2 + \frac{5}{16}b_1^2\right)r_{0s}^{10} + \frac{1}{2}b_1b_2r_0^{12} + \ldots, \qquad \ldots (12)$$

$$\frac{\omega_e}{\omega_p} = \frac{(b_0 + b_1r_e^2 + b_2r_e^4)^{1/2}}{b_0} \qquad \ldots (13)$$

where
$$r_e = r_{0s}\left[1 + \frac{1}{2}b_0r_{0s}^3 + \frac{1}{4}b_1r_{0s}^5 + \frac{3}{4}b_0^2r_{0s}^6 + \frac{1}{6}b_2r_{0s}^7 + \right.$$
$$\left. + b_0b_1r_{0s}^8 + \left(\frac{5}{6}b_0b_2 + \frac{5}{16}b_1^2\right)r_{0s}^{10} + \frac{1}{2}b_1b_2r_{0s}^{12} + \ldots\right].$$

Following Kopal (1972), values of gravity at any point in the differentially rotating white dwarf model is given by

$$g = \frac{GM}{r_e^2}\left[1 - b_0xr_0^3 - b_1x^2r_0^5 - \left(2b_0^2x^2 - \frac{1}{2}b_0^2x\right)r_0^6 - b_2x^3r_0^7\right.$$
$$-\left(\frac{17}{4}b_0b_1c^3 - b_0b_1x^2\right)r_0^8 - \left(5b_0b_2x^4 - b_0b_2x^3 + \frac{7}{4}b_1^2x^4 - \frac{1}{2}b_1^2x^3\right)r_0^{10}$$
$$\left.-\left(\frac{43}{12}b_1b_2 - b_1b_2x^4\right)r_0^{12} + \ldots\right] \qquad \ldots (14)$$

So that the ratio of the value of gravity at the pole (g_p) and the value of gravity at the equator (g_e) of the surface is given by

$$\frac{g_e}{g_p} = \left(\frac{r_p}{r_e}\right)^2 \left[1 - b_0 r_{0s}^3 - b_1 r_{0s}^5 - \frac{3}{2} b_0^2 r_{0s}^6 - b_2 r_{0s}^7 - \frac{13}{4} b_0 b_1 r_{0s}^8 - \left(4 b_0 b_2 + \frac{3}{2} b_1^2\right) r_{0s}^{10} - \frac{31}{12} b_1 b_2 r_{0s}^{12} - \ldots\right]. \quad \ldots (15)$$

Assuming non-degerate state conditions on the outermost surface of the star the value of T_e/T_p and L_e/L_p may now be computed using

$$\frac{T_e}{T_p} = \left(\frac{g_e}{g_p}\right)^{1/4} \quad \ldots (16)$$

and

$$\frac{L_e}{L_p} = \left(\frac{r_p}{r_e}\right)^3 \left[1 - b_0 r_{0s}^3 - b_1 r_{0s}^5 - \frac{3}{2} b_0^2 r_{0s}^6 - b_2 r_{0s}^7 - \frac{13}{4} b_0 b_1 r_{0s}^8 - \left(4 b_0 b_2 + \frac{3}{2} b_1^2\right) r_{0s}^{10} - \frac{31}{12} b_1 b_2 r_{0s}^{12} - \ldots\right]. \quad \ldots (17)$$

4. Numerical results

To obtain the inner structure, the shape, the volume, the surface area and the values of various other physical parameters of a differentially rotating white dwarf model, equation (5) has to be integrated numerically subject to the boundary conditions (6) for the specified values of the parameter $1/\phi_0^2$, the radius of the undistorted white dwarf η_u, and the value of constants b_0, b_1 and b_2 appearing on the right-hand side of the law of differential rotation (1). Numerical integration of this equation may be performed by using fourth-order Runge-Kutta method. Since the centre and the surface of the star are singularities of (5), for starting numerical integration, a series solution should preferably be developed near the centre. Such a solution for the present case is given by

$$\phi_\psi = 1 - \frac{\eta_u^2}{6} q^3 r_0^2 + \frac{\eta_u^4}{40} q^4 r_0^4 - \frac{b_0 \eta_u^2}{15} q^3 r_0^5 - \frac{q^5 (5 q^2 + 14) \eta_u^6}{5040} r_0^6 + \frac{3 b_0 \eta_u^4}{140} q^4 r_0^7 + \left\{\frac{\eta_u^8 (339 q^2 + 280) q^6}{1088640} - \frac{5 \eta_u^2}{72} q^3 b_0^2\right\} r_0^8 + \ldots \quad \ldots (18)$$

where

$$q^2 = (1 - 1/\phi_0^2).$$

Numerical integrations have been performed to obtain the inner structures of certain differentially rotating white dwarf models for values of b_0, b_1 and b_2 as listed in table 1. The variation of angular velocity with s in the case of these models is depicted through graphs in figure 1. Whereas Model 1 is a non-rotating spherical model, Model 2 has a solid body rotation. Models 3 to 10 are rotating differentially. Also whereas Models 1 to 9 are stable according to Stoeckly (1970) criteria, Model 10 is unstable.

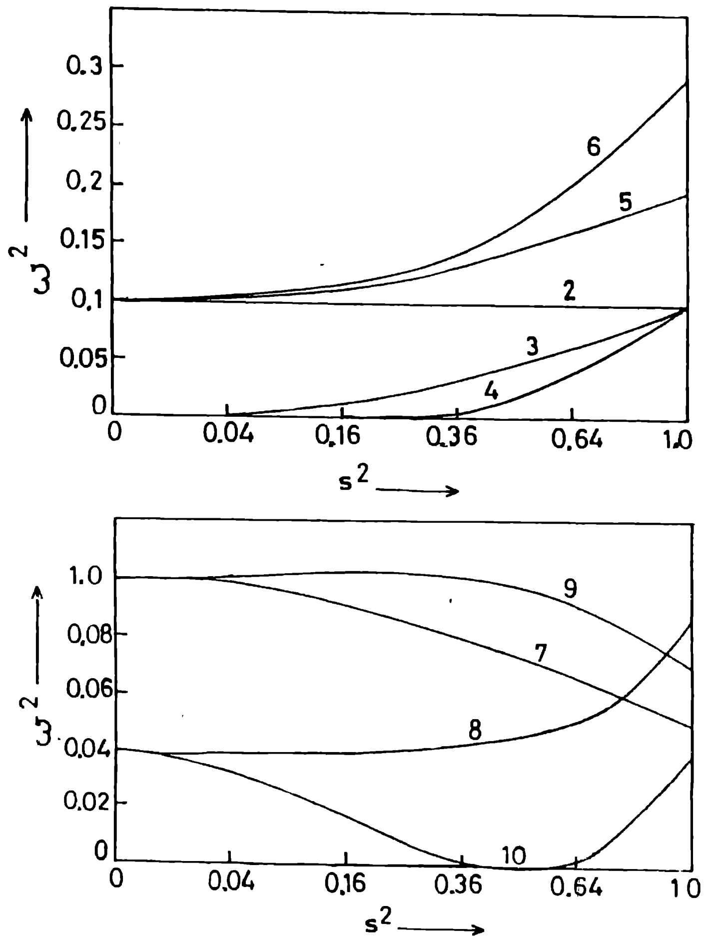

Figure 1. Graph of ω^2 versus s^2 in certain differentially rotating steller models (Model numbers indicated on graphs).

Computations have been performed for values of $1/\phi_0^2$ as 0.01, 0.05, 0.2, 0.4, 0.6 and 0.80. After obtaining the starting value of ϕ_ψ from the series solution (10) at $r_0 = 0.005$, numerical integration of equation (5) is carried forward using Runge-Kutta method of fourth order for a step length 0.005 and is continued till ϕ_ψ equals $1/\phi_0^2$. The results of numerical integration are presented in table 1. Volumes, surface areas, shapes and other physical parameters of these differentially rotating white dwarf models are presented in tables 2(a)-2(f).

5. Discussion on the results

The results for the volumes and surface areas given in tables 2(a) to 2(f) show that except for model 10 (which is unstable according to Stoeckly criteria), the volumes and the surface areas of the distorted models are larger compared to their corresponding values for the undistorted models. However, the actual increase in the volume and surface area differs from model to model. The maximum increase is noticed in the case of Models 5 and 6 for all the values of parameter $1/\phi_0^2$ considered in our present study. In the case of model 10 whereas for $1/\phi_0^2 = 0.2$, 0.4, 0.6 and 0.8, both volume as well as surface area are less than the original non-rotating model 1 for $1/\phi_0^2 = 0.05$, only volume is less.

The values of r_e/r_p given in these tables give a reasonable idea of the distortion in the shapes of these models. The values of T_e/T_p and L_e/L_p shown in these tables give an insight into the magnitude of the effects produced by differential rotations on the values of surface temperature and luminosity of such differentially rotating white dwarf stars assuming that the surfaces of such stars are in non-degenerate state. The results indicate that because of differential rotation, the values of luminosity as well as temperature are less on equator as compared to their values at the poles. This is probably due to the fact that because of equitorial bulges, the points on the equator are farther from the centre as compared to the poles. It is also observed that whereas the value of $\rho_c/\bar{\rho}$ decreases the value of $M/M_\odot$ increases with the introduction of differential rotation in the model.

Table 1. Values of r_0 and $(-d\phi_\psi/dr_0)$ at the surface for various types of different rotating white dwarf models (for different values of $1/\phi_0^2$)

Model No.	b_0	b_1	b_2	0.01	0.05	0 2	0 4	0.6	0 8
1	0 0	0.0	0 0	1.00000	1 00000	1.00000	1.00000	1 00000	1.00000
				0.36067	0.38333	0.33354	0 24399	0 15763	0.07645
2.	0.1	0.0	0 0	0.98888	0 98727	0 98584	0.98520	0 98487	0 98465
				0.37285	0.39883	0.34907	0.25604	0.16565	0.08041
3.	0.0	0.1	0.0	0.99827	0.99775	0 99722	0.99697	0.99683	0.99674
				0.36303	0.38677	0.33738	0.24710	0 15975	0 07751
4.	0.0	0.0	0.1	0.99956	0.99937	0.99916	0.99905	0.99898	0.99894
				0 36136	0 38445	0.33490	0.24512	0.15842	0.07685
5.	0.1	0 1	0 0	0.98699	0.98484	0 98289	0 98200	0.98154	0.98123
				0 37578	0 40302	0.35368	0.25975	0.16818	0.08168
6	0.1	0.1	0.1	0 98642	0 98406	0.98188	0 98089	0.98036	0 98001
				0.37697	0.40479	0.35571	0.26142	0 16932	0 08226
7.	0.1	−0.05	0.0	0.98979	0 98845	0.98728	0.98677	0.98650	0.98633
				0.37149	0.39687	0 34691	0.25429	0.16446	0 07982
8.	0 04	−0.01	0.0625	0.99546	0 99475	0.99409	0.99378	0.99362	0.99352
				0.36543	0 38976	0 34009	0.25610	0 16106	0.07815
9.	0.1	0.02	−0.05	0.98877	0 98717	0.98575	0 98511	0 98478	0.98457
				0.37288	0.39883	0.34903	0.25599	0.16561	0.08039
10*.	0.04	−0.16	0.16	0 99765	0.99754	0.99745	0.99742	0.99740	0 99739
				0.36267	0.38557	0 33554	0 24546	0 15858	0.07691

Note : For each model entries in the first row are the values of r_0 at the surface and the entries in the second row are the values of $(-d\phi_\psi/dr_0)$ at the surface.

Table 2(a). Values of certain structure parameters and related quantities for differentially rotating white dwarf models of stars for $1/\phi_0^2$ = 0.01 and η_u = 5.3571

Model No.	$V_\psi \times 10^{-25}$ (cm^3)	$S_\psi \times 10^{-17}$ (cm^2)	r_e/r_p	ω_p/ω_e	g_e/g_p	$M/M_\odot$	T_e/T_p	L_e/L_p	$\rho_c/\bar{\rho}$
1	3.6908	5.3615	1.0000	—	1 000	1.3776	1.0000	1 0000	26 1265
2	3.9675	5.6267	1 0554	1 0000	0.7984	1.3927	0.9453	0.7565	24 9921
3	3.8337	5 4989	1.0279	0.0000	0.8411	1.3819	0.9577	0.8183	25.9118
4	3.7699	5.4382	1.0166	0.0000	0.8711	1.3791	0.9661	0.8569	26.0653
5	4 1442	5.7944	1.0902	0.6808	0.6362	1 3983	0.8931	0.5836	24 7497
6	4.2663	5 9102	1 1166	0.5209	0.4885	1.4011	0 4375	0.8360	24.6572
7	3.8872	5.5502	1.0398	1.4580	0.8774	1.3902	0.9678	0.8439	25.1066
8	3.8366	5.5019	1.0299	0.6371	0 8490	1.3840	0.9599	0.8244	25 6550
9	3.9457	5.6060	1.0501	1.2555	0.8436	1.3925	0.9584	0.8033	24 9870
10	3.6910	5.3629	1.0020	1.0013	0.9831	1 3788	0.9957	0.9811	25 9212

Table 2(b). Values of certain structure parameters and related quantities for differentially rotating white dwarf models of stars for $1/\phi_0^2$ = 0.05 and η_u = 4.4601

Model No.	$V_\psi \times 10^{-26}$ (cm^3)	$S_\psi \times 10^{-18}$ (cm^2)	r_e/r_p	ω_p/ω_e	g_e/g_p	$M/M_\odot$	T_e/T_p	L_e/L_p	$\rho_c/\bar{\rho}$
1	2.3813	1 8582	1.0000	—	1.0000	1.2190	1.0000	1 0000	16.0169
2	2.5459	1.9430	1.0551	1.0000	0 7994	1.2362	0 9456	0.7577	15.1983
3	2.4693	1 9036	1.0278	0 0000	0 8415	1 2244	0 9578	0.8188	15.8386
4	2.4309	1.8840	1.0166	0.0000	0.8713	1.2211	0.9661	0 8571	15.9601
5	2.6528	1.9976	1 0893	0 6819	0.6394	1.2431	0.8942	0 5870	15.0033
6	2.7272	2.0356	1.1151	0.5232	0.4945	1.2465	0.8386	0.4435	14.9258
7	2.4971	1 9179	1.0396	1 4555	0.9770	1 2331	0.9679	0.8442	15.2917
8	2.4697	1.9039	1.0298	0.6378	0.8496	1.2265	0.9601	0.8250	15.6699
9	2 5322	1.9360	1 0499	1.2521	0.8439	1.2360	0.9585	0.8038	15.1967
10	2 3806	1.8582	1.0020	1.0018	0.9831	1.2201	0.9957	0.9811	15.8845

Table 2(c). Values of certain structure parameters and related quantities for differentially rotating white dwarf models of stars for $1/\phi_0^2$ = 0.20 and η_u = 3.7271

Model No.	$V_\psi \times 10^{-27}$ (cm^3)	$S_\psi \times 10^{-18}$ (cm^2)	r_e/r_p	ω_p/ω_e	g_e/g_p	$M/M_\odot$	T_e/T_p	L_e/L_p	$\rho_c/\bar{\rho}$
1	1.1116	5.1902	1.0000	—	1.0000	0 8863	1.0000	1.0000	9.9336
2	1 1828	5.4096	1.0548	1.0000	0 8003	0.9015	0.9458	0 7588	9.3570
3	1 1508	5.3111	1 0277	0 0000	0.8420	0 8910	0.9579	0.8193	9.7930
4	1.1340	5 2599	1.0166	0 0000	0.8715	0.8884	0.9662	0.8573	9 8848
5	1.2295	5.5530	1.0885	0 6829	0.6423	0.9080	0 8952	0 5901	9 2075
6	1.2623	5.6535	1 1137	0.5253	0.5000	0.9113	0.8409	0 4489	9.1455
7	1 1613	5.3436	1.0395	1.4536	0.8780	0.8986	0.9680	0.8446	9.4291
8	1.1504	5 3102	1.0297	0 6385	0.8501	0.8931	0.9602	0.8256	9 6845
9	1.1765	5.3905	1.0497	1.2491	0.8442	0.9013	0.9586	0.8042	9.3572
10	1.1110	5.1895	1 0020	1 0022	0.9831	0.8871	0.9957	0.9811	9 8490

Table 2(d). Values of certain structure parameters and related quantities for differentially rotating white dwarf models of stars for $1/\phi_0^2$ = 0.40 and η_u = 3 5245

Model No.	$V_\psi \times 10^{-27}$ (cm^3)	$S_\psi \times 10^{-18}$ (cm^2)	r_e/r_p	ω_p/ω_e	g_e/g_p	$M/M_\odot$	T_c/T_p	L_c/L_p	$\rho_c/\bar{\rho}$
1	2.6587	9.2822	1 0000	—	1.0000	0.6132	1.0000	1.0000	7.8873
2	2.8227	9.6609	1 0547	1 0000	0.8007	0.6245	0 9460	0.7592	7.4046
3	2.7500	9.4934	1.0276	0 0000	0 8422	0 6172	0 9580	0.8195	7.7641
4	2.7113	9.4048	1.0166	0.0000	0.8716	0.6148	0.9662	0 8574	7 8432
5	2 9312	9.9097	1.0881	0.6833	0.6436	0 6294	0 8957	0.5914	7.2752
6	3.0073	10.0845	1.1131	0.5261	0.5025	0.6320	0 8419	0 4514	7 2205
7	2.7728	9.5461	1.0395	1 4525	0.8781	0.6222	0.9680	0.8446	7.4674
8	2.2870	9.3803	1 0289	0.6438	0 8545	0 6287	0.9614	0.8305	7 4266
9	2.8080	9.6271	1.0497	1.2478	0 8444	0.6243	0.9586	0.8044	7.4054
10	2.2627	9.2805	1.0020	1.0023	0.9831	0.6136	0.9957	0.9811	7.8195

Table 2(e). Values of certain structure parameters and related quantities for differentially rotating white dwarf models of stars for $1/\phi_0^2$ = 0.60 and η_u = 3.6038

Model No.	$V_\psi \times 10^{-27}$ (cm^3)	$S_\psi \times 10^{-19}$ (cm^2)	r_e/r_p	ω_p/ω_e	g_e/g_p	$M/M_\odot$	T_c/T_p	L_e/L_p	$\rho_c/\bar{\rho}$
1	5 2213	1 4557	1.0000	—	1 0000	0.4050	1 0000	1.0000	6 9479
2	5.5372	1.5139	1.0546	1.0000	0 8009	0.4128	0.9460	0.7595	6.5112
3	5 3965	1.4880	1.0276	0.0000	0.8508	0.4078	0.9580	0.8198	6 8329
4	5 3236	1.4747	1.0165	0.0000	0.8717	0 4062	0.9662	0.8575	6.9059
5	5.7466	1.5523	1.0880	0.6835	0 6443	0 4163	0.8959	0.5922	6.3915
6	5 8938	1.5793	1.1128	0.5266	0.5038	0.4181	0.8425	0.4527	6 3409
7	5.4407	1.4962	1.0394	1.4520	0.8781	0 4112	0.9680	0 8448	6 5691
8	5.3954	1 4879	1.0296	0.6389	0.8505	0.4086	0.9603	0.8260	6 7562
9	5.5086	1 5087	1.0496	1.2471	0.8444	0.4127	0 9586	0 8045	6 5122
10	5.2179	1.4554	1.0020	1.0024	0 9831	0.4053	0.9957	0.9811	6.8880

Table 2(f). Values of certain structure parameters and related quantities for differentially rotating white dwarf models of stars for $1/\phi_0^2$ = 0.80 and η_u = 4.0446

Model No	$V_\psi \times 10^{-28}$ (cm^3)	$S_\psi \times 10^{-19}$ (cm^2)	r_e/r_p	ω_p/ω_e	g_e/g_p	$M/M_\odot$	T_c/T_p	L_e/L_p	$\rho_c/\bar{\rho}$
1	1.1364	2.4447	1.0000	—	1.0000	0 2204	1 0000	1.0000	6 3797
2	1 2043	2 5413	1.0546	1.0000	0.8011	0.2248	0.9461	0.7596	5 9721
3	1.1746	2 4991	1.0276	0 0000	0.8424	0.2221	0.9580	0.8197	6.2716
4	1.1585	2.4765	1.0165	0.0000	0.8717	0.2211	0.9663	0 8575	6.3395
5	1.2493	2.6050	1.0878	0.6837	0.6447	0 2268	0.8961	0.5927	5.8589
6	1.2810	2.6500	1.1125	0 5269	0 5046	0.2278	0.8428	0.4536	5.8103
7	1 1835	2.5118	1.0394	1.4517	0.8782	0.2239	0 9680	0.8449	6 0265
8	1.1739	2 4982	1.0296	0.6390	0.8506	0.2224	0.9603	0.8261	6.2002
9	1 1981	2 5325	1.0496	1.2467	0.8445	0.2247	0.9586	0.8046	5 9731
10	1 1357	2.4442	1.0020	1 0024	0.9831	0.2206	0.9957	0.9811	6 3247

6. Concluding remarks

In our present study we have presented a method based on the averaging concept of Kippenhahn and Thomas for computing the effects of differential rotation on the equilibrium structures of white dwarf stars. The method is convenient to use in practice. It approximates the actual equipotential surfaces of the star by Roche equipotential surfaces, and incorporates in the stellar structure equation the effects of rotational terms up to second order of smallness. In view of this limitation the method should be used with caution for rapidly rotating white dwarf stars.

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