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Non-linear planar oscillation of a satellite in elliptical orbit under the influence of solar radiation pressure—I

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Abstract. The non-linear oscillations of a satellite in an elliptic orbit around the earth under the influence of the solar radiation pressure have been studied in a series of papers I and II. In paper I, by using Melnikov's method the equations of motions have been shown to be non-integrable. The radiation pressure has been taken of the order of eccentricity e ($e << 1$) and therefore the solution is obtained by using BKM method in non-resonance case. It is observed that the amplitude of the oscillation remains constant up to second approximation. In paper II, we have studied the resonance case and the chaotic nature of the given dynamical system.

Key words : Celestial mechanics—solar system

1. Introduction

Planar oscillations of a satellite in an elliptic orbit have been studied by Beletskii (1965), Cheruousko, Zlatanstov *et al.* (1964) and by Singh (1973, 1983). In this problem we have studied the non-linear oscillation of a satellite in an elliptic orbit around the earth under the influence of the solar radiation pressure. We have determined-hyperbolic equilibrium solution and double asymptotic solutions corresponding to unperturbed Hamiltonian H_0. The non-integrability of the system has been shown through Melnikov's integral (1.2). Finally we have found out the solution in the non-resonance case by BKM method taking the solar radiation pressure parameter e of the order of the eccentricity e ($e << 1$).

Before proceeding further we would like to discuss the theorems on non-integrability and Melnikov's integral. Poincare (1972) found that transversal crossing of asymptotic surfaces of unstable periodic solutions leads to complex structure of phase curves. Melnikov (1963) paved the way of Bolotin (1986), Ziglin (1980, 1987) to construct theorems about the non-integrability of the non-linear dynamical systems with transversal homoclinic (heteroclinic) orbits. Kozlov (1983), Maciejewski (1992) have discussed the non-inegrability by using the method of separatrices splitting. Because of the facts given in Theorems 1.1 and 1.2 this method gives us more information about the chaotic nature of the dynamical system which we propose to study in paper II.

Theorem 1.1—If Σ is a phase space and σ is a shift map, then σ has (i) a countable infinity of periodic orbits of all possible periods, (ii) an uncountable infinity of non-periodic orbits, and (iii) a dense set.

Theorem 1.2—Suppose $f: \mathbb{R}^n \to \mathbb{R}^n$ is a c^r $(r \geq 2)$ diffeomorphism having a hyperbolic periodic point p. Furthermore, suppose that two asymptotic surfaces $W^s(p)$ and $W^u(p)$ have a point of transversal intersection. Then there exists some integer $n \geq 1$ such that f^n has an invariant cantor set Λ. Moreover, there exists a homeomorphism $\phi : \Lambda \to \Sigma$ such that

$$\phi \circ f^n \doteq \sigma \circ \phi$$

when the condition $\phi \circ f^n = \sigma \circ \phi$ holds, with $\phi : \Lambda \to \Sigma$, the dynamical systems $f^n : \Lambda \to \Lambda$ and $\sigma : \Sigma \to \Sigma$ are said to be topologically conjugate (see Smale (1963) or Wiggins (1988a, 1990a) for the proofs and details of the above theorem).

Our Hamiltonian function is 2π-periodic and is analytic with respect to its arguments and depends upon small parameter ε.

$$H = H(x, t, \varepsilon) = H_0(x) + \varepsilon H_1(x, t) + \ldots, \quad x = (q, p). \qquad \ldots (1.1)$$

As in Maciejewski (1993) the unperturbed Hamiltonian system ($\varepsilon = 0$) possesses hyperbolic equilibrium solution $x_0 = 0$. Suppose $\bar{x}(t)$ be double asymptotic solution to x_0, i.e. Lt $\bar{x}(t) = x_0$ as $t \to \pm\infty$. In the extended phase space (x, t) there are two asymptotic surfaces W_u^0 and W_s^0 formed by solutions tending asymptotically to x_0 as $t \to \pm\infty$ respectively. In the unperturbed system these are double (coincide). For the small perturbation ε there exists hyperbolic 2π-periodic solution $x_\varepsilon(t)$. The asymptotic surfaces of this solution do not coincide and cross transversely. The points common to both surfaces are called homoclinic points. In terms of Melnikov's integral the condition for the transversal crossing is given as, if a function

$$M(t_0) = \int_{-\infty}^{\infty} \{H_0, H_1\} (\bar{x}(t - t_0), t))dt \qquad \ldots (1.2)$$

has a simple zero then perturbed asymptotic surfaces cross transversely and the Hamiltonian function given by (1.1) is non-integrable.

2. Perturbed planar oscillations of a rigid satellite

2.1. *Equation of motion*

Let us consider a rigid satellite moving in an elliptic orbit (semi-major axis a, eccentricity e) around the earth under the influence of the solar radiation pressure $F_p = F_g(1 - q)$, F_g being solar gravitational attraction force. In general $q \simeq 1$ and so $0 < 1 - q << 1$. The satellite is assumed to be a triaxial ellipsoid with principal moments of inertia $A < B < C$, and C is the moment of inertia about the spin axis which is regarded as one of the principal axes. The torque caused by solar radiation pressure is assumed to be perpendicular to the orbital plane (centre of resultant radiation pressure lies on x'-axis as assumed by Maciejewski and is taken parallel to the major axis of the orbit) (figure 1).

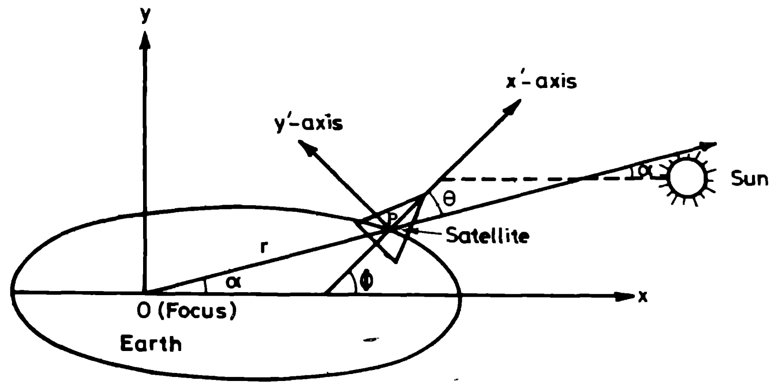

Figure 1. Motion of a satellite around the earth.

Let the instantaneous radius be r, the true anomaly α, the orientation of the satellite's long axis ϕ. Then $\phi - \alpha = \theta$, measures the orientation of the satellite's long axis relative to the satellite's radius vector. The equation of motion of satellite's planar oscillation is

$$(1 + e\cos\alpha)\frac{d^2\theta}{d\alpha^2} - 2e\sin\frac{d\theta}{d\alpha} - 2e\sin\alpha + \frac{n^2}{2}\sin 2\theta$$

$$+ \varepsilon(1 + e\cos\alpha)^{-3}\sin(\alpha + \theta) = 0 \qquad \dots (2.1.1)$$

where

$$n^2 = \frac{3(B - A)}{C}$$

and ε is proportional to solar radiation torque.

Taking $2\theta = q$, (2.1.1) can be written as

$$(1 + e\cos\alpha)\frac{d^2q}{d\alpha^2} - 2e\sin\alpha\frac{dq}{d\alpha} + n^2\sin q + 2\varepsilon(1 + e\cos\alpha)^{-3}$$

$$\times \sin\left(\frac{q}{2} + \alpha\right) - 4e\sin\alpha = 0. \qquad \dots (2.1.2)$$

Equation (2.1.2) is equivalent to the Hamilton's equations

$$\frac{dq}{d\alpha} = \frac{\partial H}{\partial p}, \quad \frac{dp}{d\alpha} = -\frac{\partial H}{\partial q} \qquad \dots (2.1.3)$$

where

$$H = H_0 + eH_1 + O(e^2)$$

$$H_0 = -2p + \frac{p^2}{2} - n^2\cos q$$

$$H_1 = p^2 \cos\alpha + n^2 \cos q \cos\alpha + 4\varepsilon_1 \cos\left(\frac{q}{2} + \alpha\right)$$

$$p = \frac{dq}{d\alpha} = \text{generalised momenta.}$$

Here, ε has been taken of the order ε, i.e.

$$\varepsilon = \varepsilon_1 e \ (0 < \varepsilon_1 << 1).$$

2.2. *Equilibrium and double asymptotic solution*

Equilibrium solution corresponding to H_0 is given by

$$\frac{dq}{d\alpha} = 0, \quad \frac{dp}{d\alpha} = 0.$$

So, by (2.1.3) the hyperbolic equilibrium solution is

$$q(\alpha) = \pi, \qquad p(\alpha) = 2.$$

Now, we determine the unperturbed double asymptotic solutions. Again from equations (2.1.3), we have

$$\frac{dp}{dq} = \frac{-n^2 \sin q}{-2 + p}.$$

On integration we obtain

$$\frac{p^2}{2} - 2p = n^2 \cos q + n^2 - 2.$$

But $$p = \frac{dq}{d\alpha} = \pm 2n \cos\frac{q}{2}.$$

Hence, the unperturbed double asymptotic solutions are given by

$$p^{\pm}(\alpha) = 2 \pm \frac{2n}{\cosh n\alpha}$$

$$\sin(q^{\pm}(\alpha)) = \pm\frac{2\sinh n\alpha}{\cosh^2 n\alpha}$$

$$\cos(q^{\pm}(\alpha)) = \frac{2}{\cosh^2 n\alpha} - 1.$$

2.3. (a) *Evaluation of Melnikov's integral*

Melnikov's integral is defined as :

$$M(\alpha_0) = \int_{-\infty}^{\infty} \{H_0, H_1\}(q^{\pm}(\alpha - \alpha_0), p^{\pm}(\alpha - \alpha_0), \alpha) d\alpha. \quad \ldots (2.3.1)$$

Its integrand is

$$\{H_0, H_1\}(q^{\pm}(\alpha - \alpha_0)), p^{\pm}(\alpha - \alpha_0), \alpha)$$

$$= \frac{\partial H_0}{\partial q^{\pm}(\alpha - \alpha_0)} \frac{\partial H_1}{\partial p^{\pm}(\alpha - \alpha_0)} - \frac{\partial H_0}{\partial p^{\pm}(\alpha - \alpha_0)} \frac{\partial H_1}{\partial q^{\pm}(\alpha - \alpha_0)},$$

$$= \mp 8n^2 \frac{\sinh n(\alpha - \alpha_0)}{\cosh^2 n(\alpha - \alpha_0)} \cos \alpha \mp 12n^3 \frac{\sinh n(\alpha - \alpha_0)}{\cosh^3 n(\alpha - \alpha_0)} \cos \alpha$$

$$\mp 4\varepsilon_1 n \frac{\sinh n(\alpha - \alpha_0)}{\cosh^2 n(\alpha - \alpha_0)} \cos \alpha \mp 4\varepsilon_1 n \frac{\sin n}{\cosh^2 n(\alpha - \alpha_0)}.$$

So, the Melnikov's integral becomes

$$M^{\pm}(\alpha_0) = \mp 8n^2 \int_{-\infty}^{\infty} \frac{\sinh n(\alpha - \alpha_0)}{\cosh^2 n(\alpha - \alpha_0)} \cos \alpha \, d\alpha$$

$$\mp 12n^3 \int_{-\infty}^{\infty} \frac{\sinh n(\alpha - \alpha_0)}{\cosh^3 n(\alpha - \alpha_0)} \cos \alpha \, d\alpha$$

$$\mp 4\varepsilon_1 n \int_{-\infty}^{\infty} \frac{\sinh n(\alpha - \alpha_0)}{\cosh^2 n(\alpha - \alpha_0)} \cos \alpha \, d\alpha$$

$$\mp 4\varepsilon_1 n \int_{-\infty}^{\infty} \frac{\sin \alpha}{\cosh^2 n(\alpha - \alpha_0)} d\alpha. \quad \ldots (2.3.2)$$

Integrals in (2.3.2) are evaluated by using the residual theorem. Thus

$$\mp 8n^2 \int_{-\infty}^{\infty} \frac{\sinh n(\alpha - \alpha_0)}{\cosh^2 n(\alpha - \alpha_0)} \cos \alpha \, d\alpha = \pm 8\pi \operatorname{sech} \frac{\pi}{2n} \sin \alpha_0,$$

$$\mp 12n^3 \int_{-\infty}^{\infty} \frac{\sinh n(\alpha - \alpha_0)}{\cosh^3 n(\alpha - \alpha_0)} \cos \alpha \, d\alpha = \pm 6\pi \operatorname{cosech} \frac{\pi}{2n} \sin \alpha_0,$$

$$\mp 4\varepsilon_1 n \int_{-\infty}^{\infty} \frac{\sinh n(\alpha - \alpha_0)}{\cosh^2 n(\alpha - \alpha_0)} \cos \alpha \, d\alpha = \pm \frac{4\varepsilon_1 \pi}{n} \operatorname{sech} \frac{\pi}{2n} \sin \alpha_0,$$

$$\mp 4\varepsilon_1 n \int_{-\infty}^{\infty} \frac{\sin \alpha}{\cosh^2 n(\alpha - \alpha_0)} d\alpha = \mp \frac{4\varepsilon_1 \pi}{n} \operatorname{cosech} \frac{\pi}{2n} \sin \alpha_0. \quad \ldots (2.3.3)$$

From (2.3.3) Melnikov's integral can be written as

$$M^{\pm}(\alpha_0) = \sin\alpha_0 \left[\pm 8\pi \operatorname{sech}\frac{\pi}{2n} \pm 6\pi \operatorname{cosech}\frac{\pi}{2n} \pm \frac{4\varepsilon_1\pi}{n}\operatorname{sech}\frac{\pi}{2n} \mp \frac{4\varepsilon_1\pi}{n}\operatorname{cosech}\frac{\pi}{2n}\right].$$

It is easy to observe that for any values of mass parameter $n > 0$ and solar radiation pressure parameter ε_1 $(0 < \varepsilon_1 < 1)$ the above function has a simple zero. Thus both pairs of asymptotic surfaces cross transversely and equations (2.1.3) are non-integrable.

2.3. (b) *Graphical representation of Melnikov's function*

(1) Figure 2 illustrates the graph of $M^{\pm}(\alpha_0, \varepsilon_1, n)$ for a fixed value $n = .8$, $\varepsilon_1 = .3$. These are simple sine curves, one corresponding to $M^{+}(\alpha_0)$ and the other $M^{-}(\alpha_0)$. It has a simple zero.

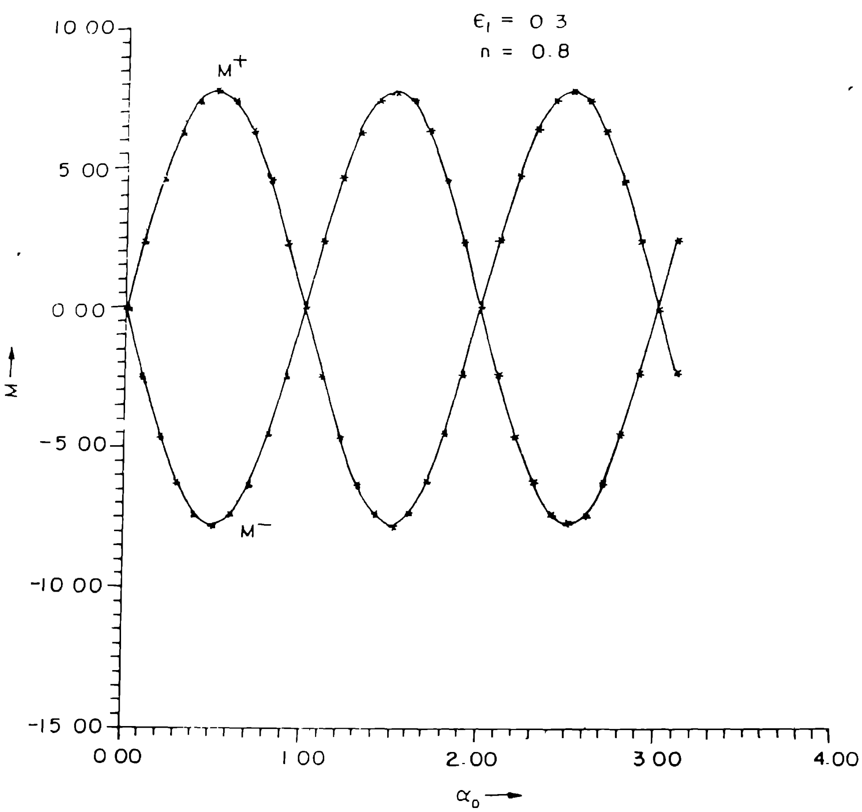

Figure 2. Melnikov's function $\varepsilon_1 = 0.3$, $n = 0.8$.

(2) Figure 3 illustrates the graph of $M^{+}(\alpha_0, \varepsilon_1, n)$ for a fixed value of $\varepsilon_1 = .2$ and $\alpha_0 = .1$. It has been observed that the Melnikov's function $M^{+}(\alpha_0, \varepsilon_1, n)$ is monotonically decreasing and $M^{-}(\alpha_0, \varepsilon_1, n)$ is monotonically increasing as n varies from .1 to .9.

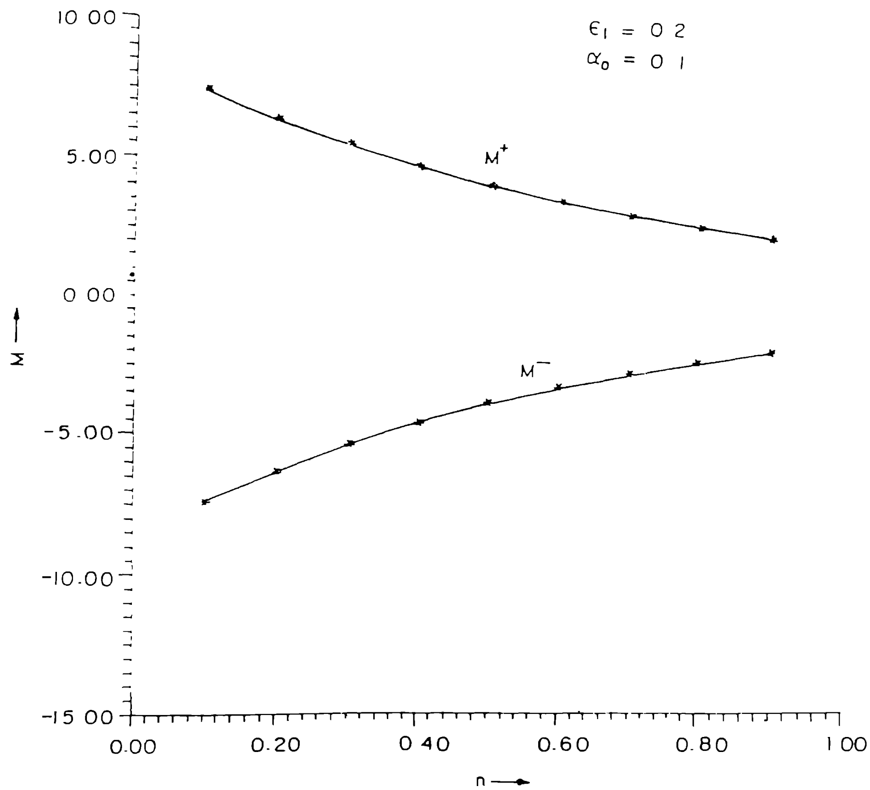

Figure 3. Melnikov's function $\varepsilon_1 = 0.2$, $\alpha_0 = 0.1$.

(3) Figure 4 illustrates the graph of $M^{\pm}(\alpha_0, \varepsilon_1, n)$ for a fixed value of $n = .2$, $\alpha_0 = .1$. It has been observed that the functions remain almost constant as ε varies.

3. Non-resonant planar oscillations of a satellite

Taking $\theta = n/2$, $\omega = n$, $\alpha = v$ and ε = order of e, i.e. $\varepsilon = \varepsilon_1 e$ $(0 < \varepsilon_1 << 1)$ in equation (2.1.2), we get

$$\frac{d^2\eta}{dv^2} + \omega^2\eta = 4e \sin v + 2e \sin v \frac{d\eta}{dv} - e \cos v \frac{d^2\eta}{dv^2}$$

$$- 2\varepsilon_1 e \sin\left(v + \frac{n}{2}\right) + \omega^2(\eta - \sin \eta). \qquad \ldots (3.1)$$

In equation (3.1) the non-linearity $(\eta - \sin \eta)$ is taken sufficiently weak and therefore it can also be taken of the order of e. So, by taking $\omega^2 = \alpha e$, equation (3.1) becomes

$$\frac{d^2\eta}{dv^2} + \omega^2\eta = e\Bigg[4 \sin v + 2 \sin \eta \frac{d\eta}{dv} - \cos v \frac{d^2\eta}{dv^2}$$

$$- 2\varepsilon_1 \sin\left(v + \frac{n}{2}\right) + \alpha(\eta - \sin \eta)\Bigg]. \qquad \ldots (3.2)$$

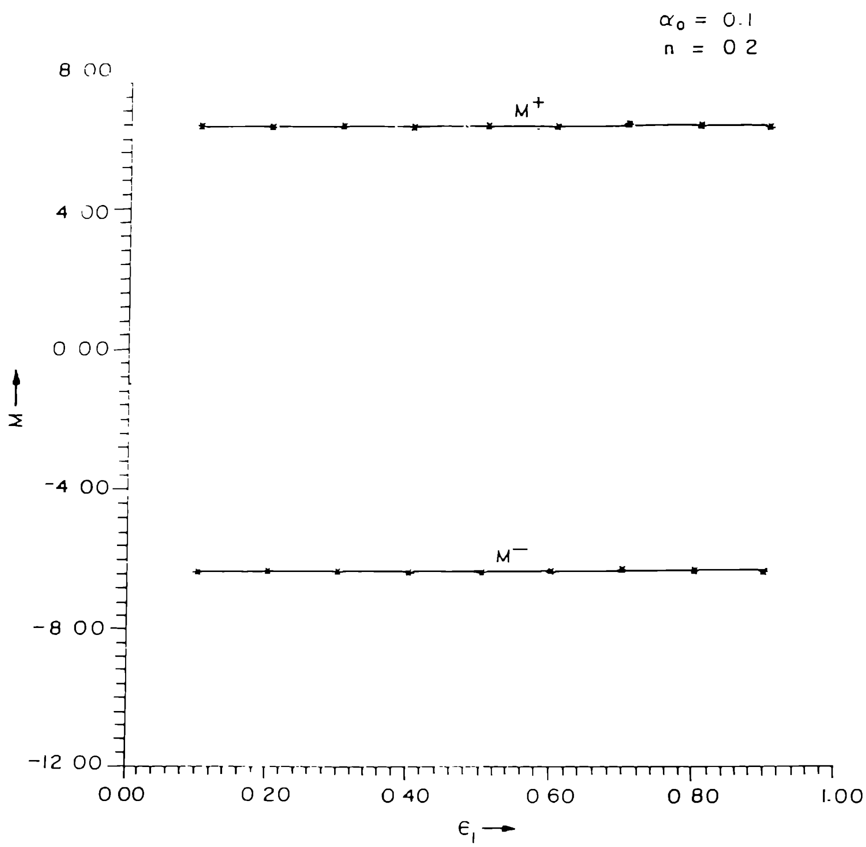

Figure 4. Melnikov's function $\alpha_0 = 0.1$, $M = 0.2$.

The dynamical system described by equation (3.2) moves under forced vibrations due to the presence of the periodic sine forces on the right-hand side of the equation. We are benefited by the smallness of the eccentricity 'e' in equation (3.2) and hence the solution may be obtained by exploiting the BKM method.

For $e = 0$, the generating solutions of the zeroeth order are

$$\eta = a \cos \psi, \qquad \psi = \omega v + \psi^*$$

where, amplitude a and phase ψ^* are constants which can be determined by the initial conditions. The solution of equation (3.2) is obtained in the form

$$\eta = a \cos \psi + e u_1(a, \psi, v) + e^2 u_2(a, \psi, v) + \dots \qquad \dots (3.3)$$

where the amplitude a and phase ψ are determined by the differential equations

$$\frac{da}{dv} = eA_1(a) + e^2 A_2(a) + \dots \qquad \dots (3.4)$$

$$\frac{d\psi}{dv} = \omega + eB_1(a) + e^2 B_2(a) + \dots \qquad \dots (3.5)$$

From (3.2) we find $\frac{d\eta}{dv}$ and $\frac{d^2\eta}{dv^2}$ and then substitute the values of η, $\frac{d\eta}{dv}$ and $\frac{d^2\eta}{dv^2}$ in equation (3.3). In the final equation equating the coefficients of like powers of e, we get

$$\omega^2 \frac{\partial^2 u_1}{\partial\psi^2} + 2\omega \frac{\partial^2 u_1}{\partial\psi\partial v} + \frac{\partial^2 u_1}{\partial v^2} - 2\omega A_1 \sin\psi - 2\omega a B_1 \cos\psi + \omega^2 u_1$$

$$= 4 \sin v - 2a\omega \sin v \sin\psi + \alpha\, a \cos\psi - \alpha \sin(a\cos\psi)$$

$$+ \omega^2 a \cos\psi \cos v - 2\varepsilon_1 \sin\left(v + \frac{a}{2}\cos\psi\right) \quad \ldots (3.6)$$

$$\omega^2 \frac{\partial^2 u_2}{\partial\psi^2} + 2\omega \frac{\partial^2 u_2}{\partial\psi\partial v} + \frac{\partial^2 u_2}{\partial v^2} - \left(aB_1 - A_1 \frac{\partial A_1}{\partial a}\right)\cos\psi$$

$$- \left(A_1 a \frac{\partial B_1}{\partial a} + 2A_1 B_1\right)\sin\psi + 2\omega B_1 \frac{\partial^2 u_1}{\partial\psi^2} + 2A_1 \frac{\partial^2 u_1}{\partial a\partial v}$$

$$- 2B_1 \frac{\partial^2 u_1}{\partial\psi\partial v} + 2\omega A_1 \frac{\partial^2 u_1}{\partial a\partial\psi} + 2a\omega B_2 \cos\psi$$

$$- 2\omega A_2 \sin\psi + \omega^2 u_2$$

$$= 2\sin v \left[\frac{\partial u_1}{\partial v} + A_1 \cos\psi - B_1 a \sin\psi + \omega \frac{\partial u_1}{\partial\psi}\right]$$

$$+ \alpha\,[1 - \cos(a\cos\psi)]\, u_1$$

$$- \cos v \left[\omega^2 \frac{\partial^2 u_1}{\partial\psi^2} + 2\omega \frac{\partial^2 u_1}{\partial\psi\partial v} + \frac{\partial^2 u_1}{\partial v^2} - 2\omega A_1 \sin\psi\right.$$

$$\left.- 2\omega B_1 a \cos\psi\right] - \varepsilon_1 \cos\left(v + \frac{a}{2}\cos\psi\right) u_1 . \quad \ldots (3.7)$$

Using Fourier expansions given by

$$\sin(a\cos\psi) = 2\sum_{k=0}^{\infty} (-1)^k J_{2k+1}(a) \cos(2k+1)\psi$$

$$\cos(a\cos\psi) = J_0(a) + 2\sum_{k=1}^{\infty} (-1)^k J_{2k}(a) \cos 2k\psi$$

where J_k, $k = 0, 1, \ldots$ stands for Bessel's function; in equation (3.6) and then comparing the coefficients of $\cos\psi$ on the both sides of the resultant equations in such a way that $u_1(a, \psi, v)$ should not contain the resonance terms, we get

$$A_1(a) = 0$$

$$B_1(a) = \alpha\,[2J_1(a) - a]/2a\omega.$$

Substituting the values of $A_1(a)$ and $B_1(a)$ in equation (3.6), and then on solving, we get

$$\begin{aligned}
u_1 = {} & \frac{4\sin\nu}{\omega^2 - 1} - \frac{a\omega}{2\omega + 1}\cos(\nu + \psi) - \frac{a\omega}{2\omega - 1}\cos(\nu - \psi) \\
& - \frac{a\omega^2}{2\,(2\omega + 1)}\cos(\nu + \psi) + \frac{a\omega^2}{2\,(2\omega - 1)}\cos(\nu - \psi) \\
& + \frac{\alpha}{2\omega^2}\sum_{k=1}^{\infty}\frac{(-1)^k J_{2k+1}(a)\cos(2k+1)\psi}{k(k+1)} \\
& - \frac{2\varepsilon_1 J_0(a/2)}{\omega^2 - 1}\sin\nu + 2\varepsilon_1\sum_{k=1}^{\infty}(-1)^k J_{2k}\left(\frac{a}{2}\right) \\
& \times\left[\frac{1}{(\omega(2k+1)+1)\,(\omega(2k-1)+1)}\sin(\nu + 2k\psi)\right. \\
& \left. + \frac{1}{(\omega(2k+1)-1)\,(\omega(2k-1)-1)}\sin(\nu - 2k\psi)\right] \\
& + 2\varepsilon_1\sum_{k=0}^{\infty}(-1)^k J_{2k+1}\left(\frac{a}{2}\right)\left[\frac{1}{(2k\omega + 1)\,(2(k+1)\omega + 1)}\cos(\nu + \overline{2k+1}\,\psi)\right. \\
& \left. + \frac{1}{(2k\omega - 1)\,(2(k+1)\omega - 1)}\cos(\nu - \overline{2k+1}\,\psi)\right].
\end{aligned}$$

Substituting the values of A_1, B_1, $\frac{\partial u_1}{\partial \psi}$ and $\frac{\partial u_1}{\partial \nu}$ in equation (3.7) and then equating the coefficients of $\cos\psi$ and $\sin\psi$ to zero, to avoid resonant terms, we may have

$$A_2 = 0$$

$$\begin{aligned}
B_2 = {} & \frac{\alpha^2}{2a\omega^3}\sum_{k=1}^{\infty}(-1)^k\frac{J_{2k+1}(a)\,J'_{2k+1}(a)}{k(k+1)} - \frac{\alpha^2}{8a^2\omega^2}(2J_1(a) - a)^2 \\
& + \frac{3\omega(\omega^2 - 1)}{4\,(4\omega^2 - 1)} + \frac{\varepsilon_1 J_1(a/2)\,(\omega^2 - 1)}{a\omega(4\omega^2 - 1)}.
\end{aligned}$$

Therefore, in the second approximation the solution is

$$\eta = a\cos\psi + e\left[4 - 2\varepsilon_1 J_0\left(\frac{a}{2}\right)\right]\frac{\sin\nu}{\omega^2-1}$$

$$+\left[\frac{a\omega(\omega-2)}{2} - 2\varepsilon_1 J_1\left(\frac{a}{2}\right)\right]\frac{\cos(\nu-\psi)}{2\omega-1}$$

$$-\left[\frac{a\omega(\omega+2)}{2} - 2\varepsilon_1 J_1\left(\frac{a}{2}\right)\right]\frac{\cos(\nu+\psi)}{2\omega+1}$$

$$+\frac{\alpha}{2\omega^2}\sum_{k=1}^{\infty}(-1)^k\frac{J_{2k+1}(a)\cos(2k+1)\psi}{k(k+1)}$$

$$+2\varepsilon_1\sum_{k=1}^{\infty}(-1)^k\left[\frac{J_{2k}(a/2)\sin(\nu+2k\psi)}{(\omega(2k+1)+1)(\omega(2k-1)+1)}\right.$$

$$+\left[\frac{J_{2k}(a/2)\sin(\nu-2k\psi)}{(\omega(2k+1)-1)(\omega(2k-1)-1)}\right.$$

$$+\left[\frac{J_{2k+1}(a/2)\cos(\nu+\overline{2k+1}\,\psi)}{(2k\omega-1)(2\omega(k+1)-1)}\right.$$

$$\left.+\frac{J_{2k+1}(a/2)\cos(\nu+\overline{2k+1}\,\psi)}{(2k\omega+1)(2\omega(k+1)+1)}\right]. \quad \ldots (3.8)$$

In the first approximation the solution is

$$\eta = a\cos\psi \quad \ldots (3.9)$$

$$\frac{da}{d\nu} = 0$$

$$\frac{d\psi}{d\nu} = \omega + \frac{\omega}{2a}[2J_1(a) - a]. \quad \ldots (3.10)$$

In the second approximation η is given in (3.8) and

$$\frac{da}{d\nu} = 0 \quad \ldots (3.11)$$

$$\frac{d\psi}{d\nu} = \omega + \frac{\omega}{2a}(2J_1(a) - a) + \frac{\omega}{2a}\left[\sum_{k=1}^{\infty} J_{2k+1}(a)J'_{2k+1}(a) - \frac{1}{4a}(2J_1(a)-a)^2\right]$$

$$+\frac{e^2(\omega^2-1)}{4\omega^2-1}\left[\frac{3\omega}{4} + \frac{\varepsilon_1 J_1(a/2)}{a\omega}\right]. \quad \ldots (3.12)$$

Thus from equations (3.11) we observe that the amplitude of the oscillation remains constant even up to the second approximation and equation (3.8) gives us the main resonance at $\omega = 1$, $\omega = \pm 1/k$ ($k \in I - \{0\}$) and parametric resonance at $\omega = \pm 1/2$.

4. Conclusions

We conclude that the non-linear rotational equations of motion of the planar oscillation of a satellite in an elliptic orbit around the earth are non-integrable. By assuming the solar radiation pressure parameter ε of the order of the eccentricity e ($e << 1$), we have found out the solutions by BK4 method and have observed that amplitude remains constant even up to the second order of approximation. The main resonance occurs at $\omega = \pm 1$, $\omega = \pm 1/k$ ($k \in I - \{0\}$) and the parametric at $\omega = \pm 1/2$.

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