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External perturbations and rate of cometary capture

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Abstract. Capture of comets by Sun is attributed to some disturbing force field (Hills 1981; Yabushita 1971, 1972, 1979; Biermann *et al.* 1983 etc.). In the present exposition an attempt has been made to study the mechanism of the capture process in terms of the perturbation $\vec{U}'$ imposed on cometary velocity $\vec{U}$. For this purpose Hasegawa's (1976) capture hypothesis has been modified. A concise closed form for capture rate has been obtained which gives an insight into the mechanism of this process; consequently a captivity parameter (ε) has been defined here which gives a measure for the Sun's total capture efficiency. Under the influence of the external disturbance the magnitude of the Sun's relative velocity $\vec{V}_0$ is found to be changed to $V_0 \pm U'$ for the apex and antepex directions, respectively. Accordingly the total captivity parameter has been split into partial captivity parameters ε_1 (for apexial direction) and ε_2 (for antepexial direction) in order to explain the observed anisotropy in capture from the two directions. Further, parameter hyperspace has been scanned for σ, V_0, U', n so that the computed capture rate and anisotropy are in agreement with their observed values. It has been found that the best-fit values σ = 1.25 km/s, V_0 = 1 km/s, U' = 0.2 km/s and $n = 1.5 \times 10^{14}/\text{pc}^3$ not only conform to physical requirements of the process but are also in agreement with the values obtained by other researchers (Hasegawa 1976; Valtonen & Innanen 1982; Valtonen 1983 etc.).

Key words : comets—rate of capture—external perturbations

1. Introduction

The observed characteristics of long-period comets with regard to their influx rate and the statistics of their orbital elements have led to many speculations about their origin. But none of the theories seems capable of explaining successfully all the observed facts. Hence it is speculated that the observed statistical features of cometary orbits may be even due to some process taking place after their formation. This process may be a continuation of their

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formation process as discussed by Lyttleton (1948). The other possibility is that after their formation the comets suffer some perturbations and get stored in a reservoir at large heliocentric distance (Oort 1950; Fernández 1982). This reservoir becomes the ultimate source of comets visiting the solar system for the first time as 'new' comets. Once the reservoir has been formed the development of cometary orbits takes place irrespective of their mode of formation. In the context of these theoretical studies another important concept was introduced by Hasegawa (1976). According to the capture hypothesis proposed by him the comets are captured by the Sun during its transit through interstellar space. In this process the external perturbations play an important role since these direct the comets towards the Sun. Hence it is the triggering agencies and the resulting mechanism which decide the statistics of orbits of 'new' comets. These agencies are external to the solar system. The external perturbations have been assigned to either the stars or the galaxy by different researchers (Hoek 1865, Oort 1950, Hills 1981, Kuiper 1951, Cameron 1962, Whipple 1964, Kresák 1982, Yabushita 1971, 1972, 1979, Tyror 1957, Hurnik 1959, Oja 1975, Khanna & Sharma 1983 and Biermann *et al.* 1983). Hasegawa (1976) has shown that the stars passing through cometary cloud are capable of perturbing and directing the comets towards the Sun. According to Fernández (1982) the stellar perturbations are effective in such a way that the comets with $a \leq 5 \times 10^4$ AU are sent towards the solar system while the others are sent away from it. From this discussion it follows that there does exist some external force field (galactic or stellar) which plays a significant role in sending the comets towards Sun thereby assisting the capture process. In this perspective, capture hypothesis is formulated and discussed below with regard to the total capture rate and anisotropy in the capture of comets by the Sun.

2. Capture process and its mathematical formulation

As mentioned above the external perturbations direct the comets towards the Sun. Their relative velocities ($\vec{V}$) with respect to the Sun have all possible magnitudes ranging from zero to infinity. Hence only those comets will have bound motion about the Sun whose speed is less than their escape speed (V_e) at R—the heliocentric distance at which they come within Sun's gravitational field. These comets become and remain permanent members of the solar system until the planetary perturbations change their orbits to hyperbolic form. Thus according to capture hypothesis, during the Sun's transit through the cometary medium, the comets with $V < V_e$ are captured by it. The external perturbations assist the Sun in this process since the cometary speed V is changed under the influence of the perturbations by external force fields. A mathematical formulation based on the assumption of Maxwellian distribution for cometary speeds at heliocentric distance $> 5 \times 10^4$ AU was given by Shimizu as reported by Hasegawa (1976). Although Hasegawa (*op. cit.*) discussed separately in quantitative manner the importance of the external (stellar) perturbations in directing the cometary velocities towards the Sun, yet he has not used these perturbations explicitly in his formulation to explain the observed rate of 'new' comets per year. In the present work due consideration has been given to the effect of external force fields in terms of the changes produced by them in cometary velocities. Accordingly Shimizu's formulation has been modified here.

Like Hasegawa's (1976) treatment, the present formulation is also based upon the assumption of Maxwellian distribution for cometary speeds beyond a heliocentric distance $R = 5 \times 10^4$ AU. As discussed by him the justification for taking $R = 5 \times 10^4$ AU is that it is most frequently observed original aphelion distance of long-period comets and it also covers the Oort's cometary cloud. Let u, v, w and $\dot{x}$, $\dot{y}$, $\dot{z}$ be the components of velocity in the coordinate systems, respectively, in the body fixed frame of the comet (at the origin P) and the space fixed frame with the Sun at the origin O. The w-axis makes an angle χ with the z-axis (figure 1)*. Maxwellian distribution of cometary speeds U (where $U^2 = u^2 + v^2 + w^2$) is given by

$$f(u, v, w)\, du\, du\, dw = \frac{1}{(2\pi\sigma^2)^{3/2}} \exp\left[-\frac{1}{2\sigma^2}(u^2 + v^2 + w^2\right] du\, du\, dw. \quad \ldots (1)$$

Let $\vec{U}'$ be the perturbation imposed on $\vec{U}$ by the external force fields**. $\vec{U}'$ has all possible orientations w.r.t. $\vec{U}$. In order to take into account the net effect of these varying orientations on $\vec{U}'$, $\bar{l}, \bar{m}, \bar{n}$ may be taken as the average direction cosines of $\vec{U}'$ w.r.t. body fixed frame of the comet. Then the components of $\vec{U}'$ in the directions of u, v, w axes are respectively given by

$$u' = \bar{l}U', \quad v' = \bar{m}U', \quad w' = \bar{n}U'. \quad \ldots (2)$$

Hence the cometary velocities change from $\vec{U}$ to $\vec{U} + \vec{U}'$ under the influence of external perturbations. In other words the Sun's relative velocity (w.r.t. cometary medium) changes

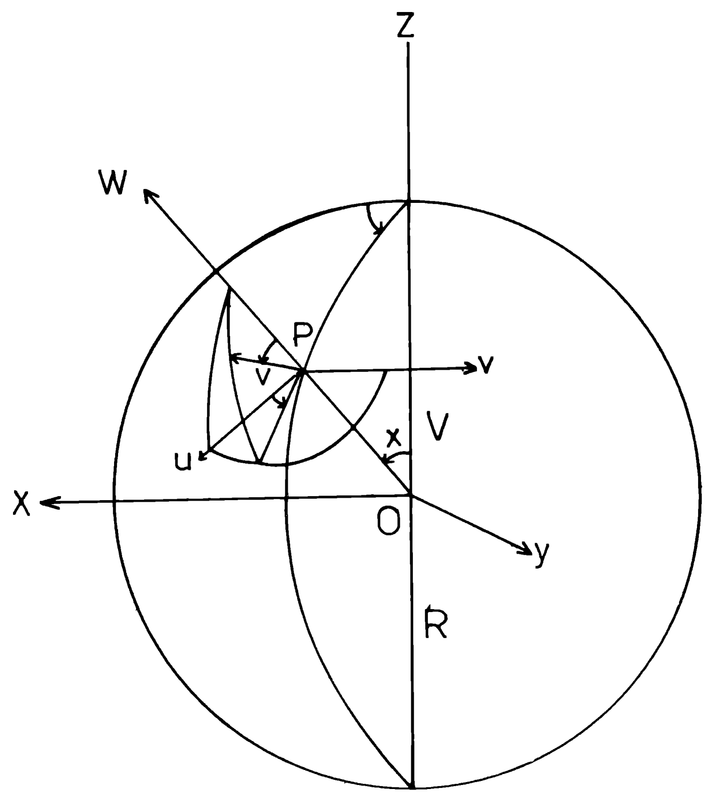

Figure 1. The relative velocity of a comet at P with respect to the sun.

*These coordinate systems are in accordance with Shimizu's formulation as reported by Hasegawa (1976).
**Since the external perturbations have not been assigned here to any specific agent so the perturbative effects are being studied only in terms of the velocity imposed on the comets.

from $\vec{V}_0$ to $\vec{V}_0 - \vec{U}'$. It may be added that the perturbations will not change Sun's velocity to any appreciable extent but will change its relative velocity w.r.t. the comets†. Thus the relative velocity of comets w.r.t. the Sun is given by

$$\vec{V} = \vec{U} - (\vec{V}_0 - \vec{U}'). \qquad \dots (3)$$

If (V, θ, ϕ) are the polar coordinates of relative velocity $\vec{V}$ in the cometocentric coordinate system, then

$$u - u_0 + u' = V \sin\theta \cos\phi,\ u_0 = -V_0 \sin\chi;$$

$$v - v_0 + v' = V \sin\theta \sin\phi,\ v_0 = 0;$$

$$w - w_0 + w' = V \cos\theta,\ w_0 = V_0 \cos\chi \qquad \dots (4)$$

where $u_0^2 + v_0^2 + w_0^2 = V_0^2$

and $u'^2 + v'^2 + w'^2 = U'^2. \qquad \dots (5)$

Substituting equations (4) in equation (1) the number of comets entering the sphere per unit time through an area element dS at a point $P(R, \chi, \lambda)$ with the relative velocities of V to $V + dV$, θ to $\theta + d\theta$ and ϕ to $\phi + d\phi$ is given by

$$dN(V, \theta, \phi; \chi, \lambda) = \frac{nR^2}{(2\pi\sigma^2)^{3/2}} \exp\left[-\frac{1}{2\sigma^2}\{(V_0^2 + u'^2 + v'^2 + w'^2 + V^2)\right.$$
$$-2VV_0(\sin\theta \sin\chi \cos\phi - \cos\theta \cos\chi)$$
$$-2V(u' \sin\theta \cos\phi + v' \sin\theta \sin\phi + w' \cos\theta)$$
$$\left. + 2V_0(u' \sin\chi - w' \cos\chi)\}\right] V^3\, dV \sin\theta \cos\theta$$
$$d\theta\, d\phi \sin\chi\, d\chi\, d\lambda \qquad \dots (6)$$

where n is the number density per pc^3 of comets entering the sphere of radius R. This equation reduces to the equation (A4) of Hasegawa (1976) when $u' = v' = w' = 0$.

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†As shown by Hasegawa (1976), the change in relative cometary velocity w.r.t. the Sun is given by

$$\Delta V = \Delta V_c - \Delta V_s = \frac{2GM_*}{V_*}\left(\frac{1}{D_c} - \frac{1}{D_s}\right) = 89\left(\frac{1}{D_c} - \frac{1}{D_s}\right)$$

where M_* = star's mass = $M_\odot$
V_* = 20 km/sec
D_c = Comet-star distance
D_s = Sun-star distance

and Δv is in units of km/sec while D_c, D_s are in units of AU. Hence for $D_c \sim 500$ AU and $D_s \sim 5 \times 10^4$ AU, $\Delta V = 89/D_c \simeq 0.18$ km/s.

Integration of this expression over $0 \le \lambda \le 2\pi$, $0 \le \phi \le 2\pi$, $0 \le \chi \le \pi$, $\pi/2 \le \theta \le \pi$ and $0 \le V \le \infty$ yields the total number of comets entering the sphere of radius R per unit time. Out of these comets only those will be captured by the Sun for which $0 \le V \le V_e$ where V_e is the escape velocity of comets at distance R from the Sun. Since we are interested here only in finding the number of captured comets so the above equation can be expanded up to first term only under the approximation

$$(V + V_0 - U')^2 < 2\sigma^2. \qquad \ldots (7)$$

So

$$dN\,(V, \theta, \phi; \chi, \lambda) = \frac{\pi n R^2}{(2\pi\sigma^2)^{5/2}} [(2\sigma^2 - u'^2 - v'^2 - w'^2 - V_0^2)$$
$$-V^2 + 2VV_0\,(\sin\theta \sin\chi \cos\phi - \cos\theta \cos\chi)$$
$$+ 2V\,(u' \sin\theta \cos\phi + v' \sin\theta \sin\phi + w' \cos\theta)$$
$$- 2V_0\,(u' \sin\chi - w' \cos\chi)]\; V^3\, dV \sin\theta \cos\theta$$
$$d\theta\, d\phi \sin\chi\, d\chi\, d\lambda. \qquad \ldots (8)$$

Integration of (8) over the above specified limits gives the total number of captured comets per unit time irrespective of their perihelion distances. In order to find the number of visible captured comets (the ones which come within 3 AU of the Sun), in equation (8) the variable θ is replaced by the variable X using the relation

$$\sin^2\theta = X^2 + V_e^2\,(X - X^2)/V^2 \qquad \ldots (9)$$

where $X = q/R$ and $V_e = (2G\,(m_\odot + m_c)/R)^{1/2} \simeq (2G\, m_\odot/R)^{1/2}$ as mass of comet $m_c << m_\odot$. Then integration of the resulting equation over $0 \le \lambda \le 2\pi$, $0 \le \phi \le 2\pi$, $0 \le \chi \le \pi$, $0 \le V \le V_e$ and finally over X from 0 to X yields (neglecting the terms containing X^2 as $X^2 << X$)

$$N_5(X) = \frac{\pi n R^2 V_e^4 X}{(2\pi\sigma^2)^{5/2}} [a_1 B - a_2 V_0 U' + a_3 V_e U' - a_4 V_e^2] \qquad \ldots (10)$$

where $\bar{l} = \bar{m} = \bar{n} = 1/\sqrt{3}$ i.e. equal direction cosines of U' have been assumed for the sake of simplicity.

$$B = 2\sigma^2 - u'^2 - v'^2 - w'^2 - V_0^2 = 2\sigma^2 - U'^2 - V_0^2,$$

and

$$a_1 = 2\pi^2, \quad a_2 = \pi^3/\sqrt{3}, \quad a_3^* = 64\pi^2/105\sqrt{3}, \quad a_4 = \pi^2. \qquad \ldots (11)$$

It is evident from (10) that the number of captured comets per unit time is proportional

*In fact a_3 is quite a cumbersome function of (V_e, X, R). The value given here is obtained after some approximations in order to visualize the variation of N with these parameters. Hence this value of a_3 should not be taken too strictly. Its value used in actual computations is about 4-5 times the one mentioned here (see Appendix I and II).

to X i.e. proportional to perihelion distance q and inversely proportional to R. The q-proportionality has been verified by plotting cumulative frequency (w.r.t. q) versus q (figure 2) separately for Class I, Class II and all the orbits (Class I and Class II taken together) of 'new' comets computed by Marsden *et al.* (1978) and Everhart & Marsden (1983). Class I orbits are the most precisely determined orbits. On the contrary, Class II orbits are less

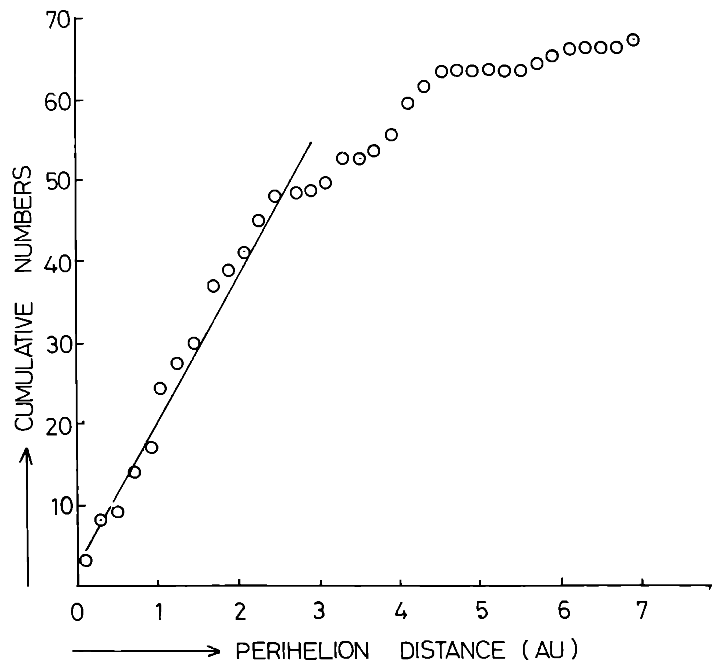

Figure 2a. Cumulative number of new comets (with respect to perihelion distance) vs perihelion distance for class I comets. The solid line represents the theoretical straight line least squares fit to the data (up to q = 3 AU) and dots represent the observed cumulative number corresponding to various perihelion distances.

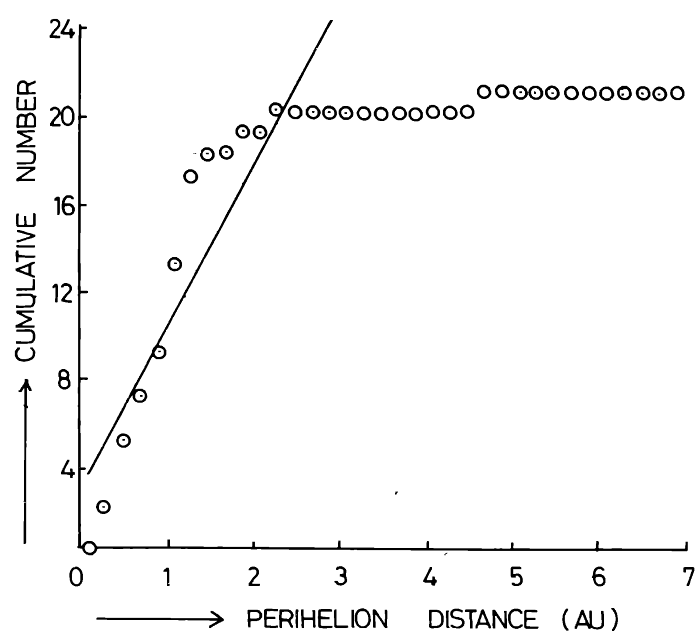

Figure 2b. Same as figure 2a for class II comets.

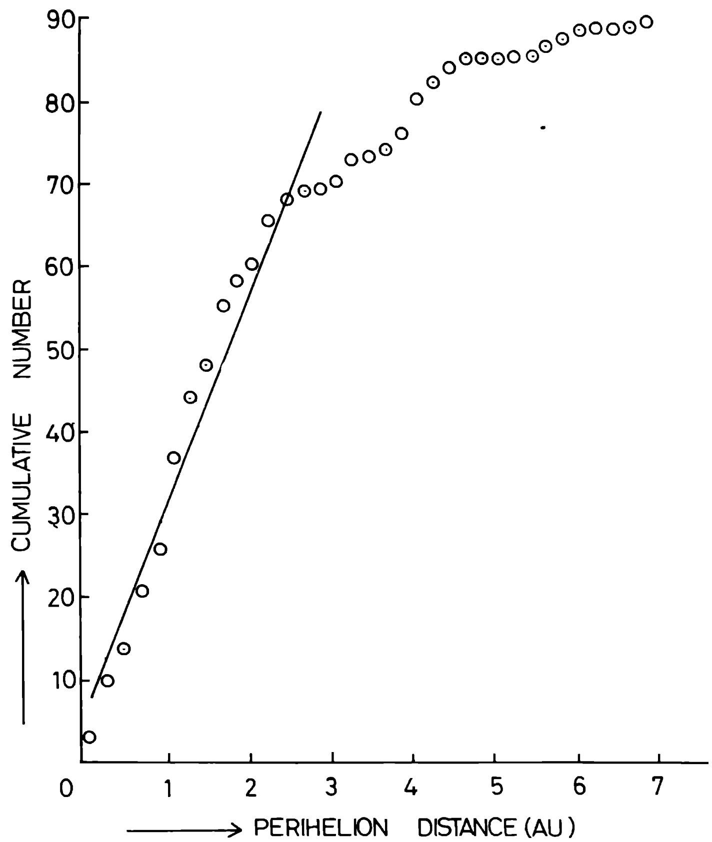

Figure 2c. Same as figure 2a for all comets.

precisely determined. From figures 2a, 2b, 2c it is evident that the relation between q and cumulative number of comets is linear up to 3 AU. It is also verified by the χ^2-test of straight line least square fit to the data (up to q = 3 AU). The observed failure of the above mentioned theoretically predicted correlation beyond 3 AU may be due to the fact that all the comets beyond this perihelion distance are not visible. This relation is found to be the best satisfied in case of Class I orbits and gets worsened for Class II orbits as well as for total number of Class I and Class II orbits. Hence it is inferred that the aforesaid correlation will be better satisfied in the latter cases also if their orbits are also determined very precisely.

In order to find the rate of capture of visible comets, X is taken within $0 \leq X \leq 3/5 \times 10^4$ ($= 6 \times 10^{-5}$). Hence the yearly rate of capture of visible comets can be obtained on multiplying the following expression by number of seconds in a year

$$N = A\,(B - a_2 V_0 U'/a_1 + a_3 V_e U'/a_1 - a_4\, V_e^2/a_1) \qquad \ldots (12)$$

where $A = \sqrt{\pi} R^2 V_e^4 X n / 2^{3/2} \sigma^5 = 9.4 \times 10^{-23} n/\sigma^5$ sec km^{-2} if σ, V_0, U' and V_e are in km/s.

At this stage it is worthwhile discussing the significance of the terms in expression (12). The first two terms depend upon σ, V_0 and U'. $\sqrt{2}\sigma$ is the most probable speed for comets and $(V_0^2 + U'^2 + a_2 V_0 U'/a_1)^{1/2}$ is Sun's overall statistically averaged relative speed (w.r.t. comets) as changed under the influence of perturbations. Evidently these terms yield the rate at which the comets having most probable speed $\sqrt{2}\sigma$ are captured due to encounter between them and the Sun moving with the perturbed relative velocity $\vec{V}_0 - \vec{U}'$. The third term depends upon U' but is independent of V_0. Hence it yields that part of the total capture rate which arises directly from the imposed disturbances only. From its very form and its net positive contribution it is clear that it represents the number of comets captured at heliocentric distance R only due to the imposed disturbance and not due to change in Sun's relative velocity. However, it is found that its contribution in general is very small in comparison with that of first two terms. The fourth term is independent of both V_0 and U' and is always negative. Its contribution represents the number of comets moving away from the sphere of radius R i.e. away from Sun. In the contribution from first three terms there is no provision for probable escape of comets (except when $B < 0$) when their relative speeds are in the domain 0 to Ve. On the contrary the fourth term depends only on escape velocity and because of its negative sign its magnitude represents the number of freely outgoing comets escaping the capture. It can be seen that the total contribution of last two terms is not more than 1% that of the total. Hence the dominant contribution is from first two terms only. From their above discussed nature it is obvious that these two terms can be visualised as a measure for Sun's capture efficiency. Accordingly a captivity parameter has been defined here as

$$\varepsilon = B - a_2 V_0 U'/a_1 = 2\sigma^2 - V_0^2 - U'^2 - a_2 V_0 U'/a_1 \qquad \ldots (13)$$

ε represents the probability for capture of comets (having the most probable speed $\alpha = \sqrt{2}\sigma$) by the Sun when it moves through them with relative velocity $\vec{V}_0 - \vec{U}'$. Hence the yearly capture rate can be obtained by approximating equation (12) as

$$N = A(2\sigma^2 - V_0^2 - U'^2 - a_2 V_0 U'/a_1) = A\varepsilon \qquad \ldots (14)$$

This very concise expression has been used in the further discussions and computations.

3. Anisotropy in capture from apex and antapex groups

The comets approaching the Sun from apex and antapex directions are said to form apex and antapex groups respectively. Valtonen and Innanen (1982) have shown that there exists an anisotropy in the number of observed comets of these groups. They have found that the ratio of comets in antapex group to those in apex groups is ~ 2.5 implying thereby that more capture takes place from antapex direction.

In order to explain observed anisotropy on the basis of present formulation, the number of comets captured from the apex and antapex directions are to be separated according to their relative speeds w.r.t. the Sun. For the comets having their speeds distributed according to assumed Maxwellian law, the overall averaged velocity is zero in the absence of perturbations. On the other hand, under the effect of perturbations although $\vec{U}'$ is perturbative part of cometary velocities yet in the process of overall statistical averaging, U' appears as the speed for ensemble of comets. Hence it is obvious that the magnitude V_{Rel} of Sun's relative velocity with respect to the comets approaching it from apex and antapex directions will be $(V_0 + U')$ and $(V_0 - U')$ respectively. Therefore the number captured from these two directions can be separated by writing equation (14) in the form

$$N = A\ [\{B_1 - a_2\ (V_0 + U')^2/4a_1\} + \{B_2 + a_2\ (V_0 - U')^2/4a_1\}]$$

$$= A\ (\varepsilon_1 + \varepsilon_2) = N_1 + N_2 \qquad \dots (15)$$

where $B_1 = [\sigma^2 - (V_0 + U')^2/2], \quad B_2 = [\sigma^2 - (V_0 - U')^2/2],$

$$B_1 + B_2 = B, \quad \varepsilon_1 + \varepsilon_2 = \varepsilon$$

$$\varepsilon_1 = [B_1 - a_2\ (V_0 + U')^2/4a_1] = [\sigma^2 - 1/2\ (1 + \pi/4\sqrt{3})\ (V_0 + U')^2],$$

$$\varepsilon_2 = [B_2 + a_2\ (V_0 - U')^2/4a_1] = [\sigma^2 - 1/2\ (1 - \pi/4\sqrt{3})\ (V_0 - U')^2],$$

$$N_1 = A\varepsilon_1 \qquad N_2 = A\varepsilon_2. \qquad \dots (16)$$

It may be noted that in the statistical averaging there appears only two streams of comets with relative speeds ($V_0 \pm U'$) w.r.t. the Sun. Hence the total probability of capture is split into partial probabilities for capture from these two directions. Thus ε_1 and ε_2 can be referred to as partial captivity parameters which give measure of Sun's efficiency for capture from apex and antapex directions respectively. It may be pointed out that the total captivity parameter ε (= $\varepsilon_1 + \varepsilon_2$) is a measure of Sun's total efficiency for capture from all the directions. In case V_0 and U' are both zero, $2\sigma^2$ is a measure of Sun's total capture efficiency. For non-zero V_0 and $U' = 0$, its capture efficiency for apex and antapex directions will be same and is given by $\frac{1}{2}(2\sigma^2 - V_0^2)$. Whenever the disturbance U' is effective the respective capture efficiencies for the two directions are unequal and are measured by ε_1 and ε_2. From equations (16) it is clear that as soon as V_0 changes due to perturbations, ε_1 decreases and ε_2 increases as compared to $\frac{1}{2}(2\sigma^2 - V_0^2)$ thereby decreasing and increasing the number of comets captured from the respective directions. In the present formulation, the ratio of comets captured from antapex direction to that from apex direction is given by

$$r = \frac{N_2}{N_1} = \frac{\varepsilon_2}{\varepsilon_1} = \frac{\sigma^2 - \frac{1}{2}(1 - \pi/4\sqrt{3})\ (V_0 - U')^2}{\sigma^2 - \frac{1}{2}(1 + \pi/4\sqrt{3})\ (V_0 + U')^2}. \qquad \dots (17)$$

If a_1 and a_2 are taken to be equal (as $a_2/a_1 \simeq 0.97$) then

$$r = \frac{\varepsilon_2}{\varepsilon_1} = \frac{\sigma^2 - \frac{1}{4}\ (V_0 - U')^2}{\sigma^2 - \frac{3}{4}\ (V_0 + U')^2}. \qquad \dots (18)$$

From the above equation it follows that the anisotropy is meaningful only when

$$V_{\mathrm{Rel}} < \sqrt{2}\alpha \text{ for antapex group} \qquad \dots (19a)$$

and

$$V_{\mathrm{Rel}} < \sqrt{\tfrac{2}{3}}\alpha \text{ for apex group.} \qquad \dots (19b)$$

Hence in the assumed Maxwellian distribution, more velocity domain is permissible for capture from antapex direction than for capture from apex direction. The relative speed of the Sun with respect to the antapex group of comets being smaller permits more time for their mutual gravitational interaction. At the same time the number of interacting comets in the permissible velocity domain is larger for antapex group [conditions (19a) and (19b)]. Hence the capture from antapex direction is favoured over that from apex direction. In section 4 formulae derived in sections 2 and 3 have been used to compute capture rate and anisotropy.

4. Results

From expressions (14) and (18) it is evident that capture rate, and hence anisotropy, depends upon the magnitudes of σ, V_0, U' and n. Hence the parameter hyperspace has been scanned here for locating the parameters σ, V_0, U', n that can explain the observed capture rate and the observed anisotropy. From the papers of Marsden *et al.* (1978) and Everhart & Marsden (1983) it is obvious that 48 'new' comets [(i.e. the comets with $1/a \leq 100 \times 10^{-6}$ AU^{-1}) (Marsden *et al.*) with $e \leq 1$ have visited the solar system with their perihelia within 3 AU in the last 180 years or so. Hence the total capture rate is ~ 0.27 per year. The corresponding observed anisotropy is ~ 2.4. For finding the best parameters σ, V_0, U', n with regard to the total capture rate and anisotropy only the comets with $q \leq 3$ AU have been considered here since these are very well within visible range and hence are least liable to be masked by q-dependent observational selection (if any). In order to obtain the set of best-fit parameters σ, V_0, U', n, the observed anisotropy (~ 2.4) has been fitted first by scanning the hyperspace of σ, V_0, U' only since it is independent of n. In table 1, the anisotropy and capture rate (in terms of n) has been given for only those sets of parameters σ, V_0, U' for which the theoretically predicted anisotropy is not very much deviated from 2.4. These computations have been

Table 1. Yearly capture rate (N) in units of n (interstellar comet density per pc^3) and anisotropy (r) for different sets of parameters σ ($\sqrt{2}\alpha$, being the most probable cometary speed), V_0 (Sun's relative speed w.r.t. cometary medium) and U' (the cumulative change in cometary speeds due to external perturbations).

V_0 (km/s)	σ (km/s)	U' (km/s) 0		0.10		0.20		0.40	
		N	r	N	r	N	r	N	r
0.50	1.00	$5.1 \times 10^{-15}n$	1.1	$5.0 \times 10^{-15}n$	1.3	$4.7 \times 10^{-15}n$	1.5	$4.1 \times 10^{-15}n$	2.4
	1.25	$2.8 \times 10^{-15}n$	1.1	$2.7 \times 10^{-15}n$	1.2	$2.7 \times 10^{-15}n$	1.3	$2.5 \times 10^{-15}n$	1.6
	1.50	$1.6 \times 10^{-15}n$	1.0	$1.6 \times 10^{-15}n$	1.1	$1.6 \times 10^{-15}n$	1.2	$1.5 \times 10^{-15}n$	1.4
	1.75	$1.1 \times 10^{-15}n$	1.0	$1.0 \times 10^{-15}n$	1.1	$1.0 \times 10^{-15}n$	1.1	$1.0 \times 10^{-15}n$	1.2
	2.00	$7.2 \times 10^{-16}n$	1.0	$7.1 \times 10^{-16}n$	1.1	$7.0 \times 10^{-16}n$	1.1	$6.8 \times 10^{-16}n$	1.2
1.00	1.25	$2.0 \times 10^{-15}n$	1.5	$2.0 \times 10^{-15}n$	2.0	$1.8 \times 10^{-15}n$	2.7	$1.5 \times 10^{-15}n$	10.6
	1.50	$1.4 \times 10^{-15}n$	1.3	$1.3 \times 10^{-15}n$	1.5	$1.2 \times 10^{-15}n$	1.7	$1.1 \times 10^{-15}n$	2.6
	1.75	$9.2 \times 10^{-16}n$	1.2	$9.0 \times 10^{-16}n$	1.3	$8.9 \times 10^{-16}n$	1.4	$8.2 \times 10^{-16}n$	1.8
	2.00	$6.4 \times 10^{-16}n$	1.1	$6.4 \times 10^{-16}n$	1.2	$6.4 \times 10^{-16}n$	1.3	$6.0 \times 10^{-16}n$	1.5
1.50	1.75	$7.0 \times 10^{-16}n$	1.7	$6.7 \times 10^{-16}n$	2.1	$6.4 \times 10^{-16}n$	2.7	$5.7 \times 10^{-16}n$	6.2
	2.00	$5.3 \times 10^{-16}n$	1.4	$5.2 \times 10^{-16}n$	1.6	$5.0 \times 10^{-16}n$	1.9	$4.6 \times 10^{-16}n$	2.7

done keeping in view that neither the capture conditions (19a) and (19b) nor the condition $(V_e + V_0 - U')^2 < 2\sigma^2$ (which is in accordance with the expression (7) for $V = V_e$ i.e. maximum V) are violated. As already discussed in section 3 in the present treatment U' appears as the overall averaged speed for the ensemble of comets. Hence for 'new' comets U' should be $\sim V_e$ since their orbits are nearly parabolic. U' as estimated by Biermann (1978) and Bailey (1983) is also $\sim V_e(R)$. It readily follows from the table that there are only two sets of such parameters that not only reproduce the anisotropy but also fulfill this requirement. These are $\sigma = 1.25$, $V_0 = 1.00$, $U' = 0.20$ and $\sigma = 1.75$, $V_0 = 1.50$, $U' = 0.20$*. With these sets an optimum fit to their observed capture rate (~ 0.27) can be obtained by varying n. It is evident from table 1 that $n = 1.5 \times 10^{14}$ yields 0.27/year as the capture rate with the former set while for the latter set, capture rate ~ 0.26/year is obtained with $n = 4.0 \times 10^{14}$. Out of these two sets the former one seems to be more justified on physical grounds. For this set Sun's relative velocity is smaller and hence the capture process is more probable. Although the capture rate is well reproduced by the other set also yet this conformity with the observations is achieved by increasing the number density n (in comparison with that for the former set) as a compensation for relatively smaller capture probability in this case. Hence the former set has been taken as the best-fit set in the following discussion.

5. Discussion

The present formulation is an improvement over Hasegawa's treatment of capture hypothesis (1976) in the sense that it specifically includes in a quantitative way the external perturbative effect on cometary speeds. Computations reveal that in the capture process although there is no direct effective role of external perturbations yet their main contribution is manifested through the change produced by them in relative velocity of the Sun w.r.t. comets. Anisotropy has also been explained and computed here in terms of Sun's relative velocity with respect to the comets approaching it from antapex and apex directions. It may be added here that in the anisotropy computed here the integrated effect of all the directions of approach of comets has been included in the two main streams i.e. apexial and antapexial groups. In order to visualize the mechanisms undergoing the partial capture processes from the two directions, the total captivity parameter ε has been split into partial captivity parameters ε_1 (for apex direction) and ε_2 (for antapex direction) as shown in equations (15) and (16). It is obvious that the relative speed of comets approaching the Sun from antapex direction is always less than that for the ones approaching it from apex direction i.e.

$$(V_0 - U') < (V_0 + U').$$

Thus the antapexial comets spend more time with sun and hence the probability of their capture is more as compared to apexial comets. Besides this the overall statistical contribution from all directions to the two groups of comets is such that the effective relative speed (i.e. $V_{\text{rel eff}}$) for antapexial comets is further reduced while that for apexial comets is further increased by factors $(1 \mp \pi/4\sqrt{3})^{1/2}$ respectively i.e.

$$(1 - \pi/4\sqrt{3})^{1/2} (V_0 - U') \ll (1 + \pi/4\sqrt{3})^{1/2} (V_0 + U').$$

*σ, V_0 and U' are in units of km/s.

Thus the individual and combined effect of these mechanisms is to render anisotropy in favour of antapex direction i.e. always

$$r = \frac{\varepsilon_2}{\varepsilon_1} = \frac{\sigma^2 - \frac{1}{2}(1 - \pi/4\sqrt{3})\,(V_0 - U')^2}{\sigma^2 - \frac{1}{2}(1 + \pi/4\sqrt{3})\,(V_0 + U')^2} > 1.$$

It may be added here that it is only non-zero magnitude of U' that gives rise to considerable anisotropy (see table 1). Hence it is inferred that external perturbations play a significant role in introducing anisotropy. It will not be out of place to mention here that Hasegawa's approach cannot explain anisotropy as perturbations in cometary velocities have not been taken into account in it. Although no such consideration is given to external perturbations even by Valtonen & Innanen (1982) in their treatment yet later on Valtonen (1983) has asserted that the capture process discussed by them earlier is most efficient if the Sun has an unseen companion star.

The parameter hyperspace has been scanned here for σ, V_0, U' and n so that the computed capture rate and anisotropy conform to their observed values. As is clear from table 1 and the discussion in the above sections, the best reasonable set is with $n = 1.5 \times 10^{14}$, $\sigma = 1.25$ km/s, $V_0 = 1.00$ km/s, $U' = 0.20$ km/s. The predicted capture rate is ~ 0.27 per year for comets with $0 \leq q \leq 3$ AU. The capture rate obtained for 48 'new' comets observed in the last 180 years since 1800, with elliptic orbits and $0 \leq q \leq 3$ AU is 0.27 per year. Obviously the agreement between the theoretically predicted and observed capture rates is fairly good. It may also be indicated here that the predicted yearly capture rate for the comets with $0 \leq q \leq 1.5$ AU is 0.27/2 (i.e. ~ 0.14) by virtue of its q-proportionality in this region [equation (10)]. The corresponding observed capture rate is 30/180 i.e. 0.17 per year while the one predicted by Biermann *et al.* (1983) is ~ 0.16 per year. The above mentioned best-fit parameters also reproduce well the observed anisotropy in capture. The magnitudes of σ and V_0 obtained here are in good agreement with the ones obtained by Hasegawa (1976) and Valtonen & Innanen (1982). Further $U' = 0.20$ km/s fulfills the condition that $U' \leq V_e$ (at 5×10^4 AU) so that the comets have bound energies. Moreover Biermann (1978), Weissman (1980) and Bailey (1983) have also estimated that the cumulative effect of external perturbations is ~ $V_e(R)$. Also $n = 1.5 \times 10^{14}$ lies well within the estimates given for it as ~ 1.1×10^{13} and 10^{15} respectively by Hasegawa (1976), Valtonen & Innanen (1982) and Valtonen (1983). Hence it is concluded that $n = 1.5 \times 10^{14}$, $\sigma = 1.25$ km/s, $V_0 = 1.0$ km/s and $U' = 0.20$ km/s are reasonable parameters conforming to physical requirements of the process (i.e. capture of comets in presence of external perturbations) and observed total rate and anisotropy in capture of 'new' comets.

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