Bull. Astr. Soc. India (1994) **22**, 1-19

General motion of N-rigid bodies

K. B. Bhatnagar and L. M. Saha
Department of Mathematics, Zakir Husain College, Delhi 110 002

Received 1993 January 4; accepted 1993 June 3

Abstract. This is a review article on the general motion of N-rigid bodies. The development of analytical qualitative methods in the study of the rigid-body motion are presented in a systematic way starting from early researches of particular solutions to the recent development on the existence of stationary solutions. The motion of two rigid bodies in general and the motion of coupled rigid bodies about a fixed point are discussed in detail.

Key words : Celestial mechanics—rigid body motion

1. Introduction

With the launching of artificial Earth's satellites some old problems of Classical Celestial Mechanics and Gyrodynamics have brought the attention of various scientists. One such problem is the general motion of N-rigid bodies moving under the Newtonian law of gravitation. The difficulty arises on account of the fact that so far we have not been able to separate the translational motion of their mass centres and rotations about the latter.

Euler (1741-1766) studied various problems of Celestial Mechanics and in particular mechanics of rigid bodies. He developed the theory of moments of inertia of a rigid body and obtained the equations of motion for the rotational motion of a body. Euler's work is published in the Memories de l'Academic Royale des sciences et Belles-Letters de Berlin, Vols. 5-16 (1751-1767). This work was followed by the work of Poinsot who wrote the treatise "Theoria Motus Corporum Solidorum Sen Rigidorum". Leimanis (1958) brought about a report 'On some recent advances in the dynamics of rigid bodies and Celestial Mechanics' followed by another book in 1965 on 'The General Problem of the Motion of Coupled Rigid Bodies about a Fixed Point'. Duboshin (1958, 1959a, 1959b) in a series of papers has discussed the above problem somewhat in detail. The same problem has also been discussed by Cid (1984) and Sansaturio (1986). These papers give lot of information.

2. Motion of N-rigid bodies

(a) *Statement* : Suppose there is a system of independent, absolutely N-rigid bodies whose elementary parts attract each other according to Newtonian law of gravitation. These bodies have definite external surfaces and definite physical matter having density at each internal point. To describe the motion of each body is called the 'N-rigid body problem'.

(b) *Position and orientation of the* $N = n + 1$, *bodies* M_i $(i = 0, 1, \dots n)$: The position and orientation of each of the bodies M_i is determined with the help of 6 parameters independent of each other ξ_i, η_i, ζ_i, ψ_i, ϕ_i, θ_i, the first three representing the co-ordinates of the centre of mass of M_i, and the other three representing the Eulerian angles (figure 1).

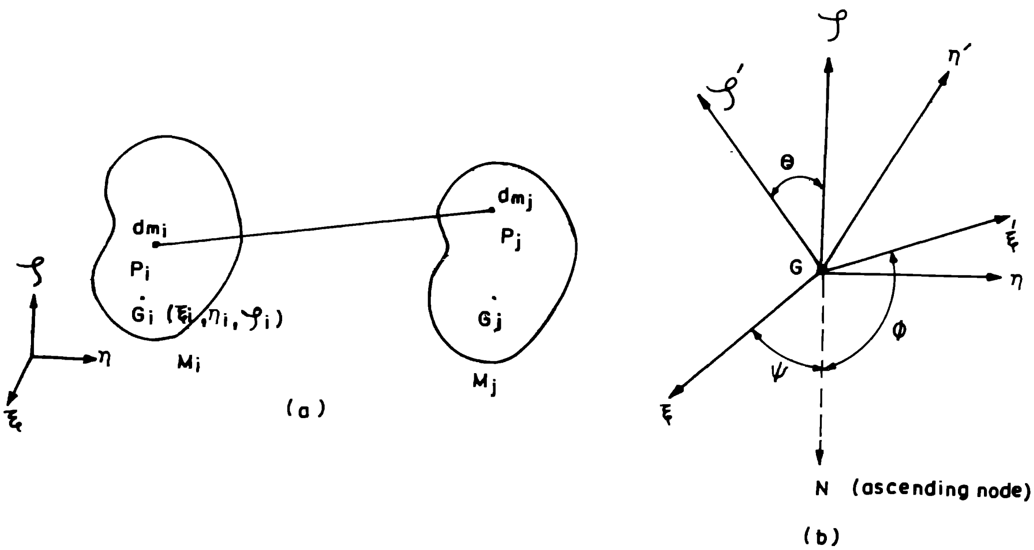

Figure 1. Eulerian angles.

The two systems $G(\xi, \eta, \zeta)$ and $G(\xi', \eta', \zeta')$ are connected in the following manner (Ralph 1963) :

	ξ_i'	η_i'	ζ_i'
ξ_i	$a_{11}(i)$	$a_{12}(i)$	$a_{13}(i)$
η_i	$a_{21}(i)$	$a_{22}(i)$	$a_{23}(i)$
ζ_i	$a_{31}(i)$	$a_{32}(i)$	$a_{33}(i)$

where $a_{11}(i) = \cos\phi_i \cos\psi_i - \sin\phi_i \sin\psi_i \cos\theta_i,$

$$a_{21}(i) = \cos\phi_i \sin\psi_i + \sin\phi_i \cos\psi_i \cos\theta_i,$$

$$a_{31}(i) = \sin\phi_i \sin\theta_i,$$

$$a_{12}(i) = -\sin\phi_i \cos\psi_i - \cos\phi_i \sin\psi_i \cos\theta_i,$$

$$a_{22}(i) = -\sin\phi_i \sin\psi_i + \cos\phi_i \sin\psi_i \cos\theta_i,$$

$$a_{32}(i) = \cos\psi_i \sin\theta_i,$$

$$a_{13}(i) = \sin \psi_i \sin \theta_i,$$

$$a_{23}(i) = -\cos \psi_i \sin \theta_i,$$

$$a_{33}(i) = \cos \theta_i.$$

For detailed study of Eulerian angles, the reader may refer to Leimanis (1965).

(c) *Euler's geometrical equations* : These equations connect the components p_i, q_i, r_i of the angular velocity of the rigid body M_i and the rate of change of the Euler's angles :

$$p_i = \sin \phi_i \sin \theta_i \dot{\psi}_i + \cos \phi_i \dot{\theta}_i ,$$

$$q_i = \cos \phi_i \sin \theta_i \dot{\psi}_i - \sin \phi_i \dot{\theta}_i ,$$

$$r_i = \cos \theta_i \dot{\psi}_i + \dot{\phi}_i . \quad \text{... (2.1)}$$

(d) *Kinetic energy* : Referred to the principal axes of the rigid body M_i, its kinetic energy is

$$T_i = \frac{1}{2} m_i (\xi_i^2 + \eta_i^2 + \zeta_i^2) + \frac{1}{2} (A_i p_i^2 + B_i q_i^2 + c_i r_i^2)$$

and the kinetic energy of the whole system is

$$T = \sum_{i=0}^{n} T_i.$$

(e) *Potential function* : The potential U_{ij} between the rigid body M_i and the rigid body M_j is given by

$$U_{ij} = f \iint \frac{dm_i dm_j}{\Delta_{ij}} ,. \quad \text{(Brouwer \& Clemence 1961)}$$

where f is the gravitational constant and Δ_{ij} is the distance between the two elements of mass dm_i of the body M_i and dm_j of the body M_j. The integrations are to be extended over the masses of both bodies.

The potential U for the system is

$$U = \frac{1}{2} \sum_{i=0}^{n} \sum_{j=0}^{n} U_{ij}, \quad i \# j. \quad \text{... (2.2)}$$

From the above formula Brouwer and Clemence have deduced the potential between two rigid bodies :

$$U = \frac{fM_2}{r^3}\left[\frac{1}{2}(A_1 + B_1 + C_1) - \frac{3}{2}(A_1 a_1^2 + B_1 b_1^2 + C_1 c_1^2)\right]$$

$$+ \frac{fM_1}{r^3}\left[\frac{1}{2}(A_2 + B_2 + C_2) - \frac{3}{2}(A_2 a_2^2 + B_2 b_2^2 + C_2 c_2^2)\right], \qquad \ldots (2.3)$$

where a_1, b_1, c_1, and a_2, b_2, c_2 are the direction cosines of G_1G_2 relative to the principal axes of M_1 and M_2 respectively.

The reader is also advised to read a paper by Paul (1988). The paper deals with the expansion in power series of mutual potential for gravitating body with finite sizes.

(f) *Equations of motion* : Duboshin (1958) has worked out the differential equations of translational and rotational motion of N-rigid bodies in the following form :

$$m_i\ddot{\xi}_i = \frac{\partial U}{\partial \xi_i}, \qquad m_i\ddot{\eta}_i = \frac{\partial U}{\partial \eta_i}, \qquad m_i\ddot{\zeta}_i = \frac{\partial U}{\partial \zeta_i}; \qquad \ldots (2.4)$$

$$A_i\dot{p}_i - (B_i - C_i)\, q_i r_i = \left[\frac{\partial U}{\partial \psi_i}\cos\theta_i + \frac{\partial U}{\partial \phi_i}\right]\frac{\sin\phi_i}{\sin\theta_i} + \cos\phi_i \frac{\partial U}{\partial \theta_i},$$

$$B_i\dot{q}_i - (C_i - A_i)\, r_i p_i = \left[\frac{\partial U}{\partial \psi_i} - \cos\theta_i \frac{\partial U}{\partial \phi_i}\right]\frac{\cos\phi_i}{\sin\theta_i} - \sin\phi_i \frac{\partial U}{\partial \theta_i},$$

$$C_i\dot{r}_i - (A_i - B_i)\, p_i q_i = \frac{\partial U}{\partial \phi_i} \cdot (i = 0, 1, 2, \ldots, n). \qquad \ldots (2.5)$$

The last three equations can also be written as

$$\ddot{\psi}_i = \psi_i^*, \qquad \ddot{\phi}_i = \phi_i^*, \qquad \ddot{\theta}_i = \theta_i^*,$$

where

$$\sin\theta_i \psi_i^* = \dot{\theta}_i\dot{\phi}_i - \cos\theta_i\dot{\theta}_i\dot{\psi}_i + \left(\frac{1}{A_i} - \frac{1}{B_i}\right)\sin\phi_i \cos\phi_i \frac{\partial U}{\partial \theta_i}$$

$$+ \operatorname{cosec}\theta_i\left(\frac{\sin^2\phi_i}{A_i} + \frac{\cos^2\phi_i}{B_i}\right)\left[\frac{\partial U}{\partial \psi_i} - \cos\theta_i\frac{\partial U}{\partial \phi_i}\right]$$

$$+ (\cos\theta_i\dot{\psi}_i + \dot{\phi}_i)\left[\frac{B_i - C_i}{A_i} + \frac{C_i - A_i}{B_i}\sin\phi_i\cos\phi_i\sin\psi_i\right.$$

$$\left. + \left(\frac{C_i - A_i}{B_i}\right)\cos^2\phi_i - \left(\frac{B_i - C_i}{A_i}\sin^2\phi_i\right)\right)\dot{\theta}_i\Bigg],$$

$$\phi_i^* = \frac{1}{C_i}\left[\frac{\partial U}{\partial \phi_i} - \operatorname{cosec}\theta_i\dot{\theta}_i\dot{\psi}_i - \cot\theta_i\psi_i^*\right] + \frac{A_i - B_i}{C_i}$$

$$\times [\sin\theta_i \sec\phi_i\dot{\psi}_i + \cos\phi_i\dot{\theta}_i]\,[\cos\phi_i \sin\theta_i\dot{\psi}_i - \sin\phi_i\dot{\theta}_i],$$

$$\theta_1^* = -\sin\theta_1 \dot{\psi}_i \dot{\phi}_1 + \left(\frac{\cos^2\phi_i}{A_1} + \frac{\sin^2\phi_1}{B_1}\right)\frac{\partial U}{\partial \theta_1} + \left(\frac{1}{A_1} - \frac{1}{B_1}\right)$$

$$\times \frac{\sin\phi_i \cos\phi_1}{\sin\theta_1}\left[\frac{\partial U}{\partial \psi_1} - \cos\theta_1 \frac{\partial U}{\partial \phi_1}\right] + (\cos\theta_1 \dot{\psi}_1 + \dot{\phi}_1)$$

$$\times \left[\frac{B_1 - C_1}{A_1}\cos^2\phi_1 - \frac{C_1 - A_1}{B_1}\sin^2\phi_1\right]\sin\theta_i \dot{\phi}_i$$

$$+ \left(\frac{C_1 - B_1}{A_i} + \frac{A_1 - C_1}{B_1}\right)\sin\phi_1 \cos\phi_1 \dot{\theta}_1. \qquad \ldots (2.6)$$

Chobanov *et al.* (1986, 1989) have determined Lagrange and Appell equations for rigid bodies from the known Euler-dynamical axioms. Malyshev (1988) has shown that Lagrange equations of the first kind enable us to solve the problem of determining the motion for certain mechanical systems of rigid body with a number of operations that is proportional to n-degree of freedom.

(g) *Ist-integrals* : In the case of N-rigid bodies as well (as compared to N-point masses), we have 10 first integrals; 6 from the motion of the centre of mass of the system, 3 from the angular momentum and 1 from energy. If the bodies are completely arbitrary shaped, then the equations of the translational-rotational motion will not have other first-integrals in addition to the above 10 integrals. However in particular cases, more integrals may exist.

(h) *Particular cases* : Some of the rigid bodies $M_1, M_2, \ldots, M_K$ $(K < n)$ are spherical in shape. In this case $A_1 = B_1 = C_1$, $(i = 1, 2, \ldots, K)$ and the force function U does not depend upon the angles ψ_1, ϕ_1, θ_1 $(i = 1, 2, \ldots, K)$, consequently

$$p_1 = p_1^{(0)} = \text{constant}, \quad q_i = q_1^{(0)} = \text{constant}, \quad r_1 = r_1^{(0)} = \text{constant}, \quad (\mathrm{i} = 1, 2, \ldots, K)$$

are supplementary integrals of our equations. One can easily conclude that the angular velocity of each of the body $M_1, M_2, \ldots, M_K$ will remain constant in magnitude and direction relative to the inertial system of coordinates. This means we can determine their rotational motion exclusively by the initial conditions of the problem.

(i) Further it will be possible to reduce the order of the system by $6K$ as the force function U will not depend on the Eulerian angles of the spherical bodies.

In particular when $K = n$, the rotational and translational motion will be independent of each other. And if we take the axis of rotation of each body as the Z-axis, we shall have

$$p_1 = 0, \; q_i = 0, \; r_1 = w_1 = \text{constant}$$

and $$\psi_1 = \psi_1^{(0)} = \text{constant}, \quad \theta_1 = \theta_1^{(0)} = \text{constant}, \quad \phi_1 = w_1 t + \text{constant}.$$

(ii) Some of the rigid bodies $M_0, M_1, \ldots, M_K$ $(K < n)$ are axis symmetric. In this case $A_1 = B_1$ $(i = 0, 1, 2, \ldots, K)$ and the function U will be independent of $\phi_0, \phi_1, \ldots, \phi_K$, we will get $r_1 = r_1^{(0)} = \text{const.}$ $(i = 0, 1, \ldots, K)$. The order of the system, therefore, can be reduced by $2(K + 1)$. We will also have

$$\phi_i = \phi_i^{(0)} + r_i^{(0)} - \int_{t_0}^{t} \dot{\psi}_i \sin\theta_i \, dt \quad (i = 0, 1, 2, \ldots K).$$

(iii) Ferrandiz *et al.* (1989) have shown that the degree of freedom for the motion of a non-spherical rigid body satellite can be reduced with the help of a canonical transformation.

3. Relative motion

The ten classical integrals enable us to reduce the order of the system of translational rotational motion by ten units. For lowering down the order of the system, we take the relative axes instead of the absolute axes. We take G_0 as the origin of the new system. Let us suppose the relative co-ordinates of the body m_i as x_i, y_i, z_i thus

$$x_i = \xi_i - \xi_0, \qquad y_i = \eta_i - \eta_0, \qquad z_i = \zeta_i - \zeta_0 .$$

The translational equations of relative motion of the body M_i ($i = 1, 2, \ldots, n$) can be written in the form

$$\ddot{x}_i = \frac{m_0 + m_i}{m_0 m_i} \frac{U_{10}}{x_i} + \frac{\partial R_i}{\partial x_i}, \qquad \ddot{y}_i = \frac{m_0 + m_i}{m_0 m_i} \frac{U_{10}}{y_i} + \frac{\partial R_i}{\partial y_i},$$

$$\ddot{z}_i = \frac{m_0 + m_i}{m_0 m_i} \frac{U_{i0}}{z_i} + \frac{\partial R_i}{\partial z_i}, \qquad i = 1, 2, \ldots, n,$$

where U_{ij} are given by (2.2) and

$$R_i = \sum_{j=1}^{n} \left[\frac{1}{m_i} U_{ij} + \frac{1}{m_0} \left\{ x_i \frac{\partial U_{j0}}{\partial x_j} + y_i \frac{\partial U_{j0}}{\partial y_j} + z_i \frac{\partial U_{j0}}{\partial z_j} \right\} \right].$$

Equations (2.1) and (3.1) give the relative translational and rotational motion of the system involving $6n + 3$ variables

$$x_i, \quad y_i, \quad z_i, \qquad i = 1, 2, \ldots, n,$$

$$\psi_i, \quad \theta_i, \quad \phi_i, \qquad i = 0, 1, 2, \ldots, n.$$

Thus the order of the whole system is reduced by 6 units. In case M_0 is a sphere, U will not depend upon ψ_0, θ_0, ϕ_0 and the order of the system is further reduced.

4. Canonical form of the equations of motion

Taking for the generalised co-ordinates the absolute co-ordinates of point G and Eulerian angles of the system, the Canonical form of the equation of motion can be written as

$$\dot{\xi}_i = \frac{\partial H}{\partial \xi_i^*}, \quad \dot{\eta}_i = \frac{\partial H}{\partial \eta_i^*}, \quad \dot{\zeta}_i = \frac{\partial H}{\partial \zeta_i^*};$$

$$\dot{\psi}_i = \frac{\partial H}{\partial \psi_i^*}, \quad \dot{\phi}_i = \frac{\partial H}{\partial \phi_i^*}, \quad \dot{\theta}_i = \frac{\partial H}{\partial \theta_i^*};$$

$$\dot{\xi}_i^* = -\frac{\partial H}{\partial \xi_i}, \quad \dot{\eta}_i^* = -\frac{\partial H}{\partial \eta_i}, \quad \dot{\zeta}_i^* = -\frac{\partial H}{\partial \zeta_i},$$

$$\dot{\psi}_i^* = -\frac{\partial H}{\partial \psi_i}, \quad \dot{\phi}_i^* = -\frac{\partial H}{\partial \phi_i}, \quad \dot{\theta}_i^* = -\frac{\partial H}{\partial \theta_i}, \qquad \dots (4.1)$$

where $H = T - U$,

$\xi_i^*, \eta_i^*, \zeta_i^*, \psi_i^*, \phi_i^*, \theta_i^*$ are the generalised momenta corresponding to $\xi_i, \eta_i, \zeta_i, \psi_i, \phi_i, \theta_i$ respectively. These generalised momenta can be determined from the relations

$$\xi_i^* = \frac{\partial T}{\partial \xi_i} \text{ etc.}$$

Equations (4.1) are $12(n + 1)$ differential equations of Ist-order in generalised co-ordinates $\xi_i, \eta_i, \zeta_i, \psi_i, \phi_i, \theta_i$ and their corresponding generalised momenta $\xi_i^*, \eta_i^*, \zeta_i^*, \psi_i^*, \phi_i^*, \theta_i^*$. These equations also possess 10 first-integrals.

5. Motion of two and more than two-rigid bodies

(a) The finite two-body problem is still unsolved due to non-integrable nature of dynamical systems. In the general problem of the translational-rotational motion of two rigid bodies which experience no forces other than mutual gravitational attraction, ten classical integrals exist expressing the conservation of the linear momentum, the angular momentum and the energy. Only when each body is a sphere with spherical density distribution, the separation of the equations into two systems of equations (one for the translational motion and the other for rotational motion) is possible and each system can then be easily solved. In all other cases the equations of motion cannot be integrated generally, since the translational and rotational motions depend on each other. However, when only terms containing second and third powers of the inverse distance remain in the differential equations, the equations of the translational motion can be separated from those of the rotational motion, thus forming an independent system. In this case, the orbital motion is not disturbed by the rotational motion, that is, it can be assumed to be given (Duboshin 1958), whereas the rotational motion is influenced by the orbital motion. This problem is the so-called restricted two-body problem or satellite problem.

Many astronomers and mathematicians have worked on the motion of two rigid bodies. Some of them are Russell (1928), Cowling (1938), Kopal (1938), Sterne (1939), Brouwer (1946), Duboshin (1958, 1959a, 1959b, 1960), Kondurar (1963, 1969), Goodyear (1965), Hori (1967), Johnson & Kane (1969), Lanzano (1969, 1970), Kinoshita (1970a, 1970b, 1972a, 1972b), Choudhry & Misra (1974), Belbruno (1977), Bhatnagar & Gupta (1977), Bhatnagar (1978) and El-Saburi (1978).

Brouwer (1946), Kondurar (1963), Johnson & Kane (1969), Lanzano (1969, 1970) and Bhatnagar & Gupta (1977) have dealt with the motion of two rigid spheroids. Assuming that the eccentricity and the lean angles are small, Brouwer has used a method similar to that of the secular perturbations of elements for planetary theory and has discussed only the motions of the pericentre and the node. Kondurar (1963) has studied the periodic solutions of equations of the forward rotating motion of two spheroids with non-coinciding symmetry planes. Johnson & Kane (1969) have studied the motion of two rigid spheroids having a fixed orbital plane. They have not imposed any restrictions on the eccentricity and the lean angles. Lanzano (1969, 1970) has obtained the solution as a power series with respect to meridional eccentricity of spheroidal bodies and the eccentricity of the orbit. Both Brouwer & Lanzano have dealt with the system of two rigid spheroids in which the axes of rotation of the spheroids are not perpendicular to the orbital plane. Bhatnagar & Gupta (1977) have studied the resonance in the restricted problem of two rigid spheroids, depending on the mean motion and the angular velocity of rotation of each body.

Hori (1967) has discussed the finite two-body problem with the use of suitable canonical variables and by a method of general perturbations based on canonical transformations. Kinoshita (1972b) has found the first order perturbations using Hori-Lie transformation of the two finite body problem, one spherical and the other triaxial which experience no forces other than mutual gravitational attraction. Belbruno (1977) has proved that it is possible to establish an equivalence of the Kepler motion with the geodesic flow on the unit sphere. Choudhry & Misra (1974) and Bhatnagar (1978) have discussed the motion of two rigid bodies having a fixed orbital plane under certain conditions. Both the authors used the Hamilton-Jacobi theory and the method of averaging to determine the motion of the system. Choudhry & Misra have taken one body triaxial and the other spheroidal whereas Bhatnagar has taken both triaxial. El-Saburi (1978) has found the first order perturbations for two-body problem having ellipsoids of inertia with a small difference from spheres and used Delaunay-Andoyer variables in the equations of translatory-rotatory motion. Cid *et al.* (1988) have studied same properties on the rotational motion of a rigid body moving about a fixed point using a canonical transformation. Wang *et al.* (1991) have studied the Hamiltonian dynamics of a rigid body of finite extent moving under the influence of a central gravitational field. A principal motivation behind this paper is to reveal the Hamiltonian structure of the N-rigid body problem. In the spirit of Arnold & Smale, exact models of spin-orbit coupling are formulated with particular attention given to the underline Lie-group frame work. The reader is also advised to consult the work of Duboshin (1984), Ermenko (1983), Elipe & Cid (1985), Elipe & Ferrer (1985). All these authors have studied the Newtonian many rigid body problem. For the basic problem of the dynamics of a rigid body or gyrostat in a central gravitational field, we refer the reader to the work of Beletskii (1966), Duboshin (1958), Robertson (1970), Longman (1971), Meirovitch (1968), Mohan *et al.* (1972), Likins (1965), Sincarsin *et al.* (1983), Pascal (1985) and Sarychev *et al.* (1975). All these authors are concerned with large earth satellites dealing with gravity gradient torque and its effect on the stability of earth-pointing satellite attitude. Bloch *et al.* (1990) while studying the stabilization of rigid body dynamics by the Energy-Casimir method has shown that the angular momentum equations of the rigid body can be stabilized by feedback about the intermediate unstable axis of inertia by a single torque by its major or minor axis. The authors show that the system under feedback is a Lie-Poisson (Hamiltonian) system. The reader may also refer to the works of Grandos Garcia (1991) and Sudakov (1991).

During the past several years, a number of papers have been written on two-body problem besides the above mentioned. Burniston & Siewert (1974), Gaida (1974), Mison (1974), Verhulst (1975), Glikman (1976, 1978), Hut & Verhulst (1976) have further studied the two-body problem. Glikman (1976, 1978) and Verhulst (1975) have discussed the two-body problem with variable mass while Hut & Verhulst (1976) have studied the two-body problem with a decreasing gravitational constant. Gaida (1974) has dealt with the three dimensional Larangian formulation of the relativistic two-body problem in classical mechanics. The reader may also refer to the works of Patrick (1989), Huber (1990), Stronge (1990), Robertson *et al.* (1988) and Belyaev (1988).

Besides Duboshin (1958), papers have been written on the solutions of the differential equations of translational-rotational motion of two rigid bodies by Shinkarik (1971), Barkin (1975a, 1975b, 1977a, 1977b, 1977c, 1977d), Barkin & Zeldakova (1978), and Barkin & El-Saburi (1979). Shinkarik has dealt with the analytical and numerical integration of the equations of motion of a dynamically symmetric body in the gravitational field of a spherical body. Barkin (1977b) has studied the existence of plane periodic Poincare solutions of two solid bodies. Barkin (1977c) has deduced the equations of translational-rotational motion of Celestial bodies in osculating elements using Hamilton-Jacobi theory and variation of the constant method. Barkin & El-Saburi (1979) have examined the approximate analytical solution of the plane motion equations of a rigid body under the attraction of a sphere in the resonance case. Vidyakin (1974, 1975, 1976, 1977) has studied the Lagrangian, near-Lagrangian and Euler solutions in the problem of the translatory-rotatory motion of three rigid bodies, besides the plane restricted circular problem of three spheroids. Barkin (1980) has studied the resonance and periodic motion of a solid satellite relative to its mass centre. Ruzanova (1983) has studied the stability of stationary motions of symmetrical rigid body, Macijewski (1986) and Macijewski *et al.* (1990) have studied the periodic rotation of a rigid body located at the libration point for the restricted three body problem. El-Shaboury (1991) has discussed the equilibrium solutions of the restricted problem of 2 + 2 axis symmetric rigid body. Six of these solutions are located at the colinear point of the restricted problem of three axes symmetric ellipsoid. In a special case he has found sixteen stationary solutions in the neighbourhood of triangular lagrangian points.

The existence and stability of the particular solutions of the restricted two body problem have been investigated by Pringle (1964), Kane (1965), Meirovitch & Wallace (1967), Robertson (1968) and Kinoshita (1970b) and those of the unrestricted two-body problem by Duboshin (1959a, 1959b, 1960) and Kinoshita (1970a, 1972a). Besides this Ucerasnjuk & Eremenko (1974) have studied the stability of motion of a body of variable composition with one stationary point in a Newtonian force field. Recently, Mavraganis (1979) has studied the stationary solutions and their stability in the planar magnetic-binary problem by taking into consideration the oblateness of the primaries. Yehia (1981, 1987) has discussed the stability of the planar motion of a rigid body about a fixed point in a Newtonian field of force. Rsymbetov (1988) has discussed the stability of relative equilibrium on a circular orbit of rigid body with two rods. The reader may also refer to the work of Eprikashvili (1987).

On the restricted two-body problem, more work has been done. Crenshaw and Fitzpatric (1968) have discussed the gravity gradient effects on the rapid rotational motion of an axi-symmetric satellite in the spherical field of the Earth and have investigated the locus of the angular momentum vector. Osipov (1969, 1970) has discussed a solution analogous to the regular motion of the 'float' type for the problem of an axi-symmetric satellite around a spherical earth. Hitzl & Breakwell (1971) have studied the rotational motion of a triaxial satellite around a spherical earth and have investigated the phenomenon of resonance whereas Colombo (1966), Holland & Sperling (1969), Peale (1969) and Giacaglia & Jefferys (1971) have investigated the rotational motion of a triaxial satellite around an axi-symmetric earth. Beletsky (1975) has discussed the motion of a satellite about its centre of mass in a gravitational field. Kammeyer (1976) has studied periodic and quasi-periodic earth satellite orbits. Volkov (1975) has discussed the quasi-periodic solution of the translational-rotational motion of Celestial bodies. For further studies of the motion of a satellite the reader may refer to the works of Gol'tser *et al.* (1990)

Duboshin (1959a) has considered the most simple case of the general two-body problem, when one of the two bodies is a homogeneous material rod. Particular solutions of this two-body problem are investigated. Duboshin (1959b) has dealt with the motion of an artificial satellite around the earth, which is assumed to be a homogeneous sphere, the satellite itself having an arrow-like form and can, dynamically be likened to a rectilinear homogeneous material segment and is described as a rod. Under the attraction of the earth, three special solutions of the satellite, called the regular motions and designated as 'arrow', 'float' and 'spoke' (figure 2) are found and their stability is discussed in Lyapunov's sense. In the 'arrow' case, the rod is always tangential to circular orbit of its centre of inertia, in the 'float' case the rod is always perpendicular to the plane of the circular orbit and in the 'spoke' case the rod is always along the radius of the circular orbit.

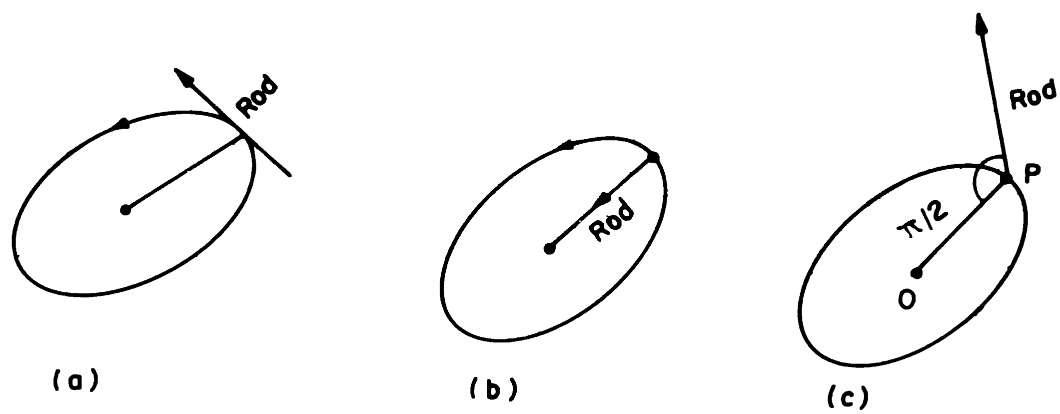

Figure 2. (a) Arrow, (b) spoke, (c) float.

Duboshin (1960) has also studied the problem of the motion of an artificial Celestial body revolving around a central planet and possessing rotational motion about its centre of inertia. He has found three particular solutions analogous to the points 'spoke', 'arrow' and 'float' and has discussed their stability.

Kinoshita (1970a) has discussed the existence and stability of the stationary motions of an axisymmetric body moving around a spherical body. He has found three particular solutions corresponding to the stationary motions and has designated them as 'spoke', 'arrow', and 'float'. He had discussed both secular stability and ordinary stability (Lyapunov's sense). He has defined motion to be secularly stable, if the configuration is such that the total mechanical potential takes an absolute minimum value. Kinoshita (1972a) has discussed a similar problem for a system consisting of two bodies, one spherical and the other triaxial. He has obtained six possible combinations of θ, ϕ and $\delta = \psi - \nu$ for the stationary motions. Here ψ, θ, ϕ are the Eulerian angles of the triaxial body. Further, he has discussed the stability of these stationary solutions.

Bois (1986, 1988) has discussed the first order and second order theory of satellite attitude motion with a dominant solar radiation pressure torque. He has also compared his results with numerical integration based upon a HIPPARCOS model.

(b) *Equation of motion* : Let us consider two rigid bodies M_0 and M with the masses m_0, m (figure 3).

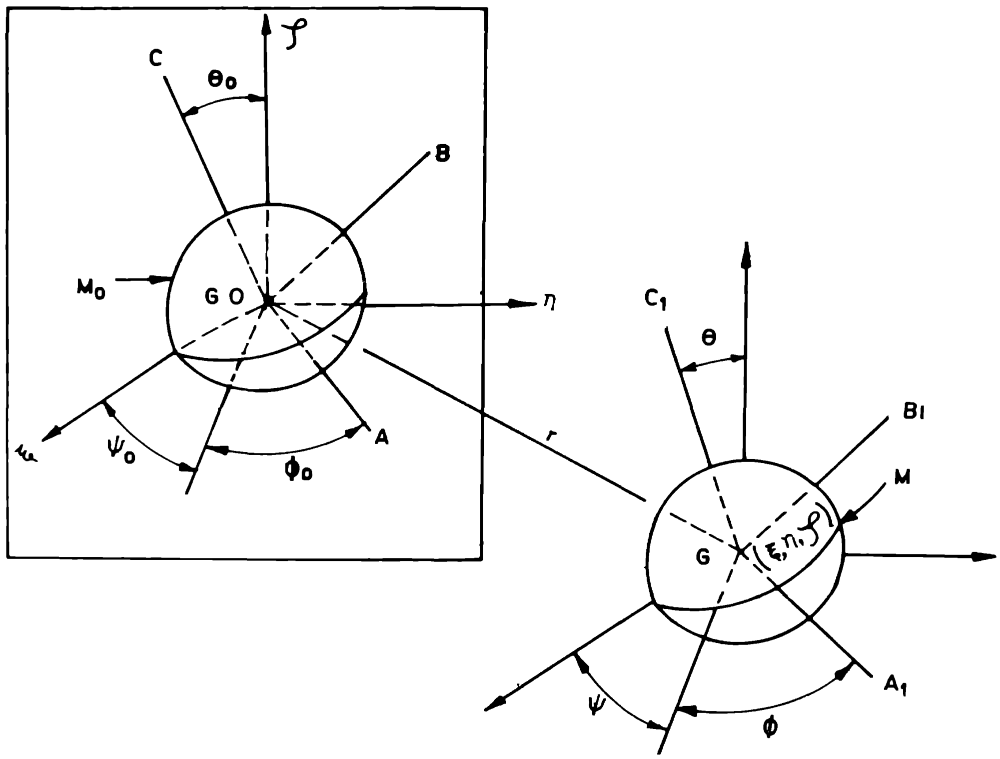

Figure 3.

If ξ, η, ζ be the co-ordinate of the centre of inertia G of the body M referred to an inertia frame at the centre of inertia G_0' of the body M_0; ψ, ϕ, θ; ψ_0, ϕ_0, θ_0, the Eulerian angles of the bodies M and M_0, then the equations of motion of the general two body problem can be written in the form (Duboshin 1959).

$$\ddot{\xi} = \frac{m_0 + m}{m_0 m} \frac{\partial U}{\partial \xi}, \quad \ddot{\eta} = \frac{m_0 + m}{m_0 m} \frac{\partial U}{\partial \eta}, \quad \ddot{\zeta} = \frac{m_0 + m}{m_0 m} \frac{\partial U}{\partial \zeta};$$

$$A\dot{p} - (B - C)qr = \frac{\sin \phi}{\sin \theta} \left[\frac{\partial U}{\partial \psi} - \cos \theta \frac{\partial U}{\partial \phi} \right] + \cos \phi \frac{\partial U}{\partial \theta},$$

$$B\dot{q} - (C - A)rp = \frac{\cos \phi}{\sin \theta} \left[\frac{\partial U}{\partial \psi} - \cos \theta \frac{\partial U}{\partial \phi} \right] - \sin \phi \frac{\partial U}{\partial \theta},$$

$$C\dot{r} - (A - B)pq = \frac{\partial U}{\partial \phi};$$

$$A_0\dot{p}_0 - (B_0 - C_0)q_0r_0 = \frac{\sin\phi_0}{\sin\theta_0}\left[\frac{\partial U}{\partial\psi_0} - \cos\theta_0\frac{\partial U}{\partial\phi_0}\right] + \cos\phi_0\frac{\partial U}{\partial\theta_0},$$

$$B_0\dot{q}_0 - (C_0 - A_0)r_0p_0 = \frac{\cos\phi_0}{\sin\theta_0}\left[\frac{\partial U}{\partial\psi_0} - \cos\theta_0\frac{\partial U}{\partial\phi_0}\right] - \sin\phi_0\frac{\partial U}{\partial\theta_0},$$

$$C_0\dot{r}_0 - (A_0 - B_0)p_0q_0 = \frac{\partial U}{\partial\phi_0};$$

$$p = \sin\phi\sin\theta\dot{\psi} + \cos\phi\dot{\theta}, \qquad p_0 = \sin\phi_0\sin\theta_0\dot{\psi}_0 + \cos\phi_0\dot{\theta}_0,$$

$$q = \cos\phi\sin\theta\dot{\psi} - \sin\phi\dot{\theta}, \qquad q_0 = \cos\phi_0\sin\theta_0\dot{\psi}_0 - \sin\phi_0\dot{\theta}_0,$$

$$r = \cos\theta\dot{\psi} + \dot{\phi}, \qquad r_0 = \cos\theta_0\dot{\psi}_0 + \dot{\phi}_0,$$

where

$$U = f\frac{m_0m}{r} + f\,m\left[\frac{A_0 + B_0 + C_0 - 3I_0}{2r^3}\right] + f\,m_0\left[\frac{A + B + C - 3I}{2r^3}\right] + \ldots\ldots.$$

Here, $r^2 = \xi^2 + \eta^2 + \zeta^2 = (G_0G)^2$

I_0, I = moment of inertia of the body M_0 and M relative to G_0G.

These equations, are very much simplified if the body M is very small in comparison to the body M_0.

(c) *Stationary solutions* : In the case of two rigid bodies a maximum of 36 stationary solutions exist (table 1).

Table 1

Sl. No.	Nature of rigid bodies	No. of stationary solutions	Solutions
1.	Both triaxial (Bhatnagar 1986)	36	L_i^j ; $i = 1, 2, \ldots, 6$ $j = 1, 2, \ldots, 6$
2.	Spheroid, triaxial (Bhatnagar 1986)	18	L_i^j ; $i = 1, 3, 5$ $j = 1, 2, \ldots, 6$
3.	Both spheroid (Bhatnagar 1980a)	09	L_i^j ; $i, j = 1, 3, 5$
4.	Spherical, triaxial (Kinoshita 1972)	06	L_i^j ; $i = 1$ $j = 1, 2, \ldots, 6$
5.	Sphere, spheroid (Kinoshita 1970)	03	L_i^{j+1} ; $L_i^j = L_{i+1}^j$; $\forall\ i = j$ $i, j = 1, 3, 5$

(d) *Special cases* : Bhatnagar and Gupta (1980a, 1980b, 1986) have studied :

(i) The motion of two rigid bodies under the gravitational influence of each other.

(ii) The existence and stability of the libration points of an axi-symmetric body moving around another axi-symmetric body.

(iii) The existence and stability of the libration points of a triaxial body moving around another triaxial body.

These problems have been studied under certain special conditions. The result of Johnson & Kane (1969) and Bhatnagar (1978) are modified. They have found that a, e are constants and Eulerian angles θ_j, ϕ_j, ψ_j ($j = A, B$), τ and w are linear functions of time t.

(e) *Regions of motion* : Chapsiadis *et al.* (1988) have studied the bounded motion in a two body system consisting of a solid body and a material point for different values of the energy or the angular momentum. Sergysels (1988) has studied the regions of motion in configurational space under certain conditions.

(f) *Rigid body dynamics of unidirectional spin* : Bondi (1986) has discussed the spin of a boat-shaped toy in one direction only. Its sophisticated rigid body dynamics is examined in some detail. It is interesting to see how complex a subject the rolling motion of a rigid body is. Yet more general cases, where the rest position does not have a principal axis of inertia vertical, amount examination.

6. Motion of coupled rigid bodies about a fixed point

We can briefly discuss this problem in the following three categories :

(a) It concerns with particular cases of integrability of the equations of motion of a single rigid body moving around a fixed point. Major countributors to this problem are Euler (1707-1783), Lagrange (1736-1813), Poinsot (1777-1859), Kovalevskaya (1850-1891), Mozalevskaya (1988) and many others.

For detailed study, one should refer to Domogarov (1893), Klein & Sommerfeld (1897), Greenhill (1914), Gray (1918), Grammel (1950), Routh (1892, 1897, 1898a, 1898b), Scarborough (1958), Golubev (1960), Starzhinskii (1990), Dovbysh (1990), Zitterbarth (1991), Moshchuk *et al.* (1992) and Lewis *et al.* (1992).

In this category, one can also study the motion of a symmetric as well as an asymmetric self-excited or externally excited rigid body. (*Def.* : A body is said to be self-excited if the torque applied is either fixed in the body or moves in a particular manner). Wiebelitz (1955) has discussed the motion of a rigid body when it is subjected to a periodic torque. It has its application in astronomy and atomic physics. In astronomy we are concerned with the perturbation on Earth's rotation about its axis due to the phenomenon of precession and nutation. This is discussed in detail by Woolard (1953) and Poison (1830). In atomic physics we come across gyroscope which are subjected to periodic torques. Reader is advised to refer to the works of Bloch, Hansen & Packard (1946), Bloch & Siegert (1940), Wangsness & Bloch (1953) and Kirchner (1955).

(b) Research in rotational motion of a rigid body got a new direction with the invention of gyroscopic compass, of gyro horizon and rate gyro. These instruments are used for guidance and control of ships, air-craft, missiles and space-craft. Much work has been done in this direction by Foucault (1819-1878), Krylov (1863-1945), Schuler (1927, 1951), Grammel (1889-1964), Draper (1963) and many others. For detail study one may refer to Ferry (1932), Grammel (1950), Krylov & Krutkov (1932), Richardson (1954), Draper, Wrigley & Grohe (1955), Siff & Emmerich (1960), Bulgakov (1960), Arnold & Maundder (1961), Savet (1961) and Ziegler (1962).

(c) In this category falls the work of Kelvin & Tait (1912). They were concerned with the classification of various types of forces, such as gyroscopic forces, i.e. forces which depend on the generalized velocities q_k. Linear terms containing $\dot{q}_k$ do appear in non-linear equations of motion containing gyroscopes and holonomic systems. Effect of gyroscopic forces are generally studied with the help of a parameter H. Reader may refer to the work of Merkin (1956). For gyroscopic stabilization one may consult the work of Metelieyn.

Yehia (1986) in his two papers I and II has studied the motion of a rigid body about a fixed point under the action of stationary, non symmetric potential and gyroscopic forces. The problem has been modeled by the motion of an electrified, magetized, gyrostat under the action of a combination of Newtonian, Coulomb, and Lorentz forces. This work of Yehia is an extension of his own work of (1985, 1986) and of Beletsky *et al.* (1985). Sansaturio *et al.* (1988) have discussed the translatory and rotatory motion of a system made of two gyrostats attracting one another according to Newton's law using modified canonical variable of Delauny and Serret-Andoyer. Pascal *et al.* (1991) have studied the equilibrium orientation of a gyrostat satellite in the gravity field of a point mass. They have solved the semi-inverse problem when some parameters giving orientation of the satellite are chosen arbitrarily. Burov (1986) has discussed the motion of a gyroscope in a Cardano suspension and the motion of the gyroscope with two degrees of freedom. He proved theorems on the nonexistence of an additional first integral. Sarychev *et al.* (1988) has studied the steady motion of a gyrostat suspended from a string. Guelman (1988) deals with gyrostat trajectory and core energy. This paper deals with energy-sink method to obtain qualitative and quantitative information regarding attitude motions of space-craft. Rubanovskii (1988) has studied branching and stability in a field of Newtonian gravitation. For further studies of the motion of a gyroscope the reader is advised to refer to the works of Grioli (1988), Sidoreko *et al.* (1989), Gashenenko (1990), Kharlamova (1990), Panayotounkos (1990), El-Sabba (1991), Rubanovskii (1991), Storozhenko *et al.* (1991), Saccomandi (1991), Gashenenko (1991), Savchenko (1991), and Kharlamova (1991).

7. Multiple and connected rigid bodies

Blazer (1990) has studied a multibody system consisting of a set of rigid bodies interconnected by joints. He has determined a matrix form of Kane's equations. Tokad (1992) has discussed a network model for rigid body motion. This article is devoted to the derivation of such a mathematical model of a rigid body as a $(k + 1)$-port components. Rodriguez *et al.* (1992) has described a new spatial operator algebra for the dynamics of general-topology rigid multibody system. Reader may also consult the works of Agop *et al.* (1991) and Jean (1991).

8. Free motion of a single rigid body

Fuchs (1991a, 1991b) has discussed in two papers the motion of a rotating ball flying through the Earth's atmosphere. In these papers he also considers applications to games such as volleyball, tennis or golf but not football and compares theoretical and experimental results. Both papers list a number of relative papers and books. The reader may also refer to the works of Mirer *et al.* (1991) and Sidorenko (1991).

Acknowledgement

Our thanks are due to the Department of Science & Technology, Govt. of India, Delhi, for funding this Project. A part of work has been completed during our visit to IUCCA, Pune.

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