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Modelling rotation curves—the turbulent way

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Abstract. Helical hydrodynamic turbulence can magnify perturbation seed eddies and this can lead to the appearance of large scale structures. The evolution of the instability is accompanied by a transfer of energy from small scale to large scale fluid motions. This has motivated us to explore the possibility of explaining the formation of large scale structures in the universe as a result of such an 'inverse cascade'. As a first step in this direction we have modelled the rotation curves of two galaxies by combining the effects of rigid rotation, gravity and turbulence. The model has yielded very good estimates for the parameters like the galaxy-mass, turbulent velocity and the angular velocity of the galaxies. This encourages us to study the velocity fields on larger scales such as those of clusters and superclusters to vindicate the model's predicted spectrum.

Key words : galaxies : kinematics and dynamics—hydrodynamics—turbulence

1. Introduction

The rotation curves of galaxies have been the subject of great speculation in the recent past. If galaxies are considered as solid bodies in rotation then their linear velocity must increase in a linear manner $V \propto r$ where r is the radial distance from the centre of the galaxy. The trouble arises when the picture of a 'falling curve' as predicted by the Newtonian gravity for the outer region of a galaxy doesn't tally with what is observed. We get a flat rotation curve on the outer scales. This has given birth to a lot of models which try to account for rotation curves. The suggestions include (1) a modification of the Newtonian force (Milgrom 1983), (2) the effect of the magnetic stresses (Nelson 1988), (3) the presence of a large amount of hidden mass that does the trick! (Einasto 1990).

The issue of dark matter has kicked considerable dust in this area, and the gravity of the dark matter appears to be a favourite candidate. This must be tested against its alternatives. Verschuur (1991) has revived the old debate of the missing mass vs the missing physics. The appearance of large scale structures in turbulent flows, ter Harr (1989), Moiseev *et al.* (1983), Levich & Tzvetkov (1985), which are stationary, isotropic but not reflection-invariant has become an exciting prospect potential enough to play a major role in the astrophysical context. The major weakness of all structure-formation models (e.g. CDM, Gravitational instability models) till date is their inability to reproduce the large scale structures, observed in the universe (of the order of 100 Mpc). Krishan & Sivaram (1991) showed that the clustering and superclustering of galaxies and clusters respectively could be viewed as the outcome of the 'inverse cascade' process in a turbulent medium. In this paper we model the flat rotation curves of the galaxies by combining the effects of rigid rotation, gravity and turbulence.

2. Modelling of rotation curves

The complete energy spectrum in a helically turbulent medium was derived in Krishan (1991) and Krishan & Sivaram (1991) (see figure 1). Here we model the rotation curves of two galaxies observed by Amram *et al.* (1992) (see figures 2a and 2b). We propose a law of velocities which is of the type :

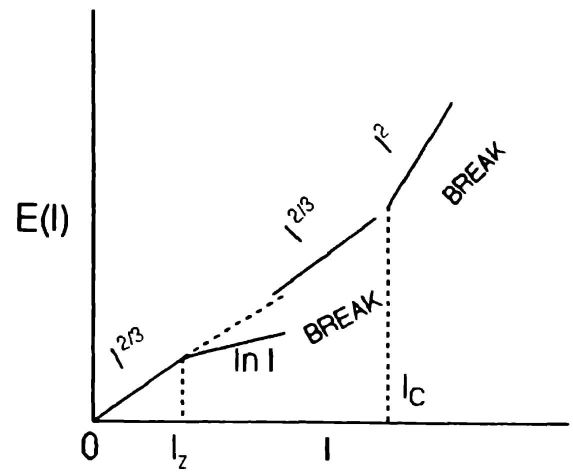

Figure 1. Turbulent energy spectrum. l_z normalizing length; l_c break due to Coriolis force (Krishan 1991).

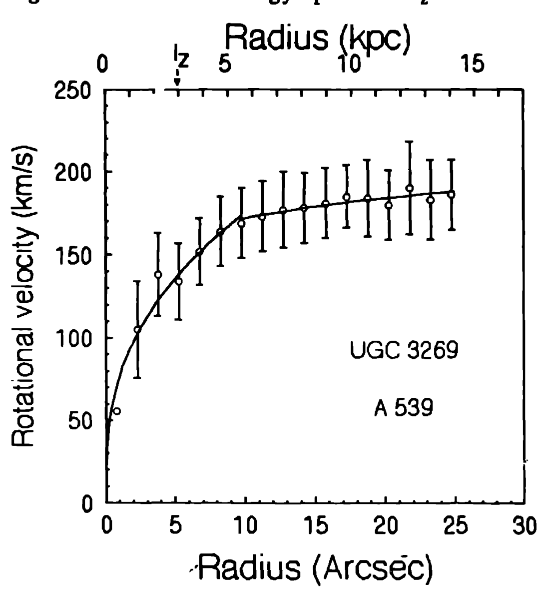

Figure 2a. Rotation curve of UGC 3269.

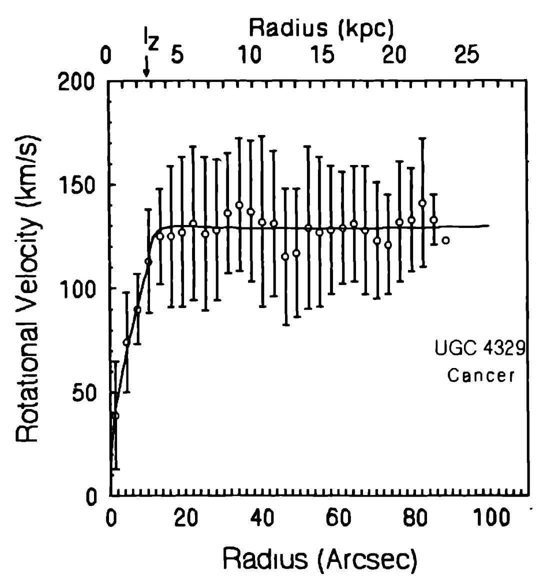

Figure 2b. Rotation curve of UGC 4329.

$$V(l) = Al + Bl^{1/3} \quad \ldots (1)$$

in the inner i.e., $l \le l_z$ (l is the length scale) and

$$V(l) = Cl^{-1/2} + D\sqrt{\ln(l/l_z)} \quad \ldots (2)$$

in the outer regions i.e., $l \ge l_z$ of a galaxy, where A, B, C and D are the coefficients to be determined from the fits, with the observed velocity-fields. The first terms on the right-hand side of equations (1) and (2), correspond to rigid rotation and gravity respectively, therefore

$$A = \omega \quad \ldots (3)$$

the angular velocity of a galaxy and

$$C = \sqrt{GM}, \quad \ldots (4)$$

(where G is the universal gravitational constant), refers to the mass of a galaxy. The second terms on the r.h.s of equations (1) and (2) are due to the turbulence cascading so that

$$B = \varepsilon^{1/3} \quad \ldots (5)$$

and $$D = (\varepsilon^2 l_z \tau)^{1/5} \quad \ldots (6)$$

where, ε = average energy exchange rate between the scales (ergs/gm/sec) ($\varepsilon = V_0^2/\tau$). V_0 is the initial rms velocity on small scales, τ is the duration for which this energy is available) The normalizing length l_z marks the transition from one inertial range to the other. By a judicious choice of l_z we can estimate: V_0, τ, ε, ω and mass M of a galaxy.

3. Results

Our model yields an estimate of

$$V_0 \approx 100 \text{ km/sec}$$

$$\tau \approx 10^{14} \text{ sec}$$

$$\varepsilon \approx 10^{-2} \text{ ergs/gm/sec}$$

$$\omega \approx 10^{-16} \text{ sec}^{-1}$$

$$\text{Mass} \approx 10^{10} M_\odot.$$

One must note that we didn't have to choose any abnormal value of l_z for obtaining the best fits and it lies in the range 2-10 kpc. This tells us that on scales smaller than l_z, the turbulence is isotropic and on the scales equal to and larger than l_z the turbulence becomes more and more anisotropic facilitating the inverse cascade of energy.

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