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Gravitational collapse of inhomogeneous dust : naked singularities, black hole formation and Hawking evaporation

C. S. Unnikrishnan

Gravitation Expts. Group, Tata Institute of Fundamental Research, Homi Bhabha Road, Bombay 400 005

Abstract. There are several examples of existence of naked singularities in models of spherically symmetric gravitational collapse and they are considered to be counter-examples to the Cosmic Censorship Conjecture (CCC). In fact, there were suggestions and attempts to realize energy output in the form of gamma ray bursts or particle production near the strong naked singularities. We have argued that the counter-examples cited are physically unrealistic and therefore they do not violate a realistic CCC. This also means that attempts to model them as high energy sources are unrealistic. However, we point out that there is a possibility of formation of black holes of mass small enough for quantum Hawking evaporation to become important, in inhomogeneous collapse. But estimate of the energy output and other physical considerations like the need for fine tuning show that it is too speculative to invoke them in any realistic astrophysical situation.

Key words : gravitational collapse—cosmic censorship—Tolman-Bondi models—Hawking evaporation—high energy bursts

It has been shown that the gravitational collapse of spherically symmetric and inhomogeneously distributed pressure free dust results in a singularity which is either weak or when strong, strongly censored by an event horizon (Newman 1986). The strong singularities without an event horizon covering them (naked singularities) found in some of the solutions can be shown to arise from physically unacceptable initial density distributions (Unnikrishnan 1993). Here the singularity is defined to be strong when the quantity $k^2 R_{ab} K^a K^b \neq 0$ as $k \rightarrow k_s$, along a null geodesic where k is the affine parameter and the singularity is at k_s. (R_{ab} is the Ricci tensor and K^i is a tangent to the null geodesic). This is the physically interesting singularity since, when this condition is satisfied the singularity is gravitationally strong enough to crush matter into zero volume (Tipler *et al.* 1980). In the case of spherical homogeneous collapse, all the mass which is undergoing collapse is within the event horizon when the black hole forms and the mass of the black hole is the total collapsed mass. The inhomogeneous collapse differs drastically from this situation in that when the black hole forms first, a substantial fraction of the collapsing mass can be outside an event horizon and therefore the mass of the initial black hole can be a tiny fraction of the total collapsing mass. This initial mass is decided by the degree of inhomogeneity and subsequently the event horizon expands due to infall of the outer massive shells into the black hole, finally saturating at a radius $R = 2M$ where M is the total mass.

This scenario acquires a special significance in the context of Hawking evaporation of black holes by radiation. Though any acceptable fusion between quantum mechanics and General Relativity is not yet achieved, calculations within a semiclassical framework have provided many important results. The discovery by Hawking of the quantum evaporation of black holes is one such important result (Hawking 1974; Novikov & Prolov 1989). This well studied phenomenon can be summarized in a context suitable for order of magnitude estimates in astrophysically interesting situations by stating the expressions for the equivalent blackbody temperature (T) of the radiating black hole, its luminosity (L), and the time (t) it takes (for an asymptotic observer) for the black hole to evaporate completely (Page 1976). These are

$$T = \frac{hc^3}{16\pi^2 Gk}\frac{1}{m} = \frac{h\,g}{4\pi^2 ck} \approx \frac{10^{26}}{m}\,{}^{\circ}\mathrm{K} \qquad \ldots (1)$$

$$L = \frac{1}{4}acT^4 A \approx \frac{3\times 10^{45}}{m^2}\ \mathrm{ergs/sec} \qquad \ldots (2)$$

where

$$a = \frac{\pi^2 k^4}{15h^3c^3} \quad \text{and} \quad A = \frac{16\pi G^2 m^2}{c^4}$$

$$t = \frac{mc^2}{3L} \approx 10^{-25}\, m^3\ \mathrm{sec} \qquad \ldots (3)$$

Here A is the area of the event horizon and g is the surface gravity. c, G, h and k are fundamental constants in standard notation and m is the mass of the central black hole formed, which could be much smaller than the total mass of the collapsing matter. (For numerical estimates indicated in the equations, mass is expressed in grams).

These equations show that when $m \gg 10^{15}\,g$, quantum evaporation is essentially negligible. Even if such black holes are of primordial origin, even today their temperature would be too low to be significant. Primordial black holes with mass close to 10^{15} g would just survive till today and the astrophysical implications were discussed in papers by Hawking (1971), Page (1976) and Novikov *et al.* (1979). The equations also show that when the mass of the black hole is very small, in the range of 10^8 g or less, their luminosity can be astrophysically significant. But then the life time, for the lower masses, is too short to be of any importance except when there is continuous feeding of matter into the evaporating black hole at a rate appropriate to sustain its mass nearly constant for durations much larger than its bare life time. This is exactly the situation which is possible during inhomogeneous gravitational collapse (Of course, questions regarding the stability of the event horizon in the situation when both evaporation and accretion are taking place should be addressed separately).

To focus attention on the main result, we consider the Tolman-Bondi inhomogeneous collapse of pressure free dust. The relevant equations, when physically acceptable pre-collapse density functions are considered, were discussed by Newman (1986). He showed that when the second derivative of the radial distribution of energy density is zero, the resulting singularity is strongly censored. Essentially what happens is that in such a situation, there is some region of non-zero radius in the centre where the density is uniform and the central regions follow the Oppenheimer-Snyder homogeneous collapse (Oppenheimer & Snyder 1939) without interference from the outer collapsing shells; an event horizon forms even before the central singularity forms. In the inhomogeneous case, the initial black hole can have a mass which is a very small fraction of the total mass which is collapsing. In particular, it is possible that a black hole of mass small enough for Hawking process to become important may form at the centre. (A more realistic scenario, when the collapsing matter has pressure, could be significantly different from the above situation in that the black hole which forms may then be massive enough for Hawking evaporation to be unimportant.)

We will assume that the energy output from black hole evaporation should be larger than 10^{30} ergs/sec, lasting for more than a millisecond or so for it to be astrophysically interesting. Then from equations (1)-(3) we get that the mass of the initial black hole should be 5×10^{7} g or less. (This estimate is crude since the observed luminosity will be dependent on what the actual spectrum of radiation is. This is discussed by Page (1976). Also, the amount of radiation that can come out of the contracting region will depend on details like opacity etc. for the various particle components. Further, there is a redshift factor to be included which will depend on the actual density distribution). The quantum life time of the black hole then will be 10^{-2} sec or less. Now we observe that during inhomogeneous collapse outer mass shells are continuously falling in (neglecting any extreme back reaction from the evaporation of the black hole) allowing the interesting possibility of a nearly sustained quantum evaporation and energy output. It is possible to get highly luminous bursts lasting for a millisecond or more.

Even in the case when the singularity is weak and naked initially, there is the possibility that the event horizon forms later as a result of more matter falling into the weak singularity. In this case also the initial black hole can have a mass small enough for quantum evaporation to be significant. Though we have pointed out some interesting possibilities in inhomogeneous gravitational collapse, it is unlikely that these possibilities will be easily realized in physically realistic collapse situations. Therefore we have avoided an explicit discussion on specific applications like gamma ray bursts where the power radiated is in the range of 10^{52} ergs/sec for a millisecond or so which will require incredibly small initial black holes with finely tuned infall rates and possibly other parameters, if the present scenario is invoked.

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