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On naked singularities in spherically symmetric gravitational collapse

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Abstract. We explore the physical nature of the singularity which forms in the gravitational collapse of spherically symmetric pressureless dust with inhomogeneous density distribution (Tolman-Bondi model). We show that in this model the singularity is weak when naked, and it is hidden by an event horizon when strong. Based on these results we arrive at a correct physical interpretation of the strong naked singularities demonstrated in similar collapse scenarios by many authors. We argue that in fact these naked singularities are unphysical and therefore pose no threat to the Cosmic Censorship Conjecture.

Key words : gravitational collapse—cosmic censorship—naked singularities—Tolman-Bondi models

1. Introduction

One of the most important unsolved problems in classical general relativity is the question whether singularities which form as a result of gravitational collapse could be naked (that which has a causal contact with an observer at infinity). The Cosmic Censorship Conjecture (CCC) states that such singularities are always clothed by an event horizon under reasonable and minimal physical restrictions (Penrose 1969). While this conjecture remains unproved in a rigorous framework, there have been many counter examples in the recent past which seem to show that naked singularities arise even within the physical premises in which the conjecture is stated (Christodoulou 1984; Papapetrou 1985; Kuroda 1984; Newman 1986; Ori & Piran 1990; Dwivedi & Joshi 1989). Many of these singularities arise in the case of spherically symmetric gravitational collapse. Since it is reasonable to expect that if there are naked singularities in spherical collapse then they can arise more easily in more complicated (e.g. asymmetric collapse or collapse with rotation etc.) collapse situations, it is important to critically examine the counter examples for their physical content. (Naked singularity arising in the case of imploding radiation is not a realistic threat to CCC even within classical Maxwell theory since the waves cannot be confined to a region smaller than the wavelength in the absence of trapped surfaces.)

To illustrate the generic models investigated by several authors we may take the Tolman-Bondi dust collapse models (Tolman 1934; Bondi 1947). The metric in the comoving coordinates is

$$ds^2 = -dt^2 + \frac{R'^2}{1+f}dr^2 + R^2(d\theta^2 + \sin^2\theta d\phi^2). \quad \ldots (1)$$

The other relevant expressions are

$$T^{ij} = \varepsilon\delta^i_t\delta^j_t\,, \qquad \varepsilon = \varepsilon(r,t) = \frac{M'}{R^2R'} \quad \ldots (2)$$

and $$\dot{R}^2 = \frac{M}{R} + f. \quad \ldots (3)$$

The only nonvanishing component of the energy momentum tensor, T^{ij} is the energy density ε since pressure is zero. $R = R(r, t)$ is the physical radius and from these equations the interpretations of $M = M(r)$ as the total invariant mass within a shell of coordinate radius r and $f(r)$ as the total energy are clear (Tolman 1934). Integration when $f = 0$ gives

$$R(r,t) = r^{3/2} - \frac{3}{2}\sqrt{M}t. \quad \ldots (4)$$

Using this expression we can write (Oppenheimer & Snyder 1939) equation (2) as

$$\varepsilon(r,t) = \frac{4/3}{(t - G/F)(t - G'/F')} \quad \ldots (5)$$

where we have written $G = r^{3/2}$ and $F = \sqrt{M}$.

It is clear that if we choose M' in equation (2) such that it varies slower than r^2, there is a singularity at $(t = 0,\ r = 0)$, in the sense that the energy density and therefore the curvature blows up as $r = 0$ is approached. This singularity can be shown to be naked under a wide range of conditions on M'. Also it can be shown that this singularity is a strong curvature singularity when $M'(0)$ is nonzero, in the sense that the quantity $k^2 R_{ab}K^aK^b$ in the limit $k \to 0$ is nonzero along a null geodesic, where k is an affine parameter (Dwivedi & Joshi 1992). (R_{ab} is the Ricci tensor and K^i is a tangent to the geodesic). From equation (5), the central density, $\varepsilon(0, t)$, is seen to vary as t^{-2}. Then central density is finite on any $t \neq 0$ hypersurface. Therefore the claim was made that the singularity arose from the collapse of a spherically symmetric cloud with its energy density finite everywhere (though non-analytic at $r = 0$ in the case of pressure free dust) in the past ($t < 0$). Similar conclusions and interpretations were arrived at in the case of inhomogeneously distributed fluid with pressure (Ori & Piran 1990; Joshi & Dwivedi 1992). If this interpretation is true then a serious violation of the CCC is suggested. This situation calls for a thorough analysis of all the existing counter examples for their physical content.

A straightforward approach for this analysis is to start with a finite, physically acceptable general density distribution at $t = 0$ hypersurface and follow the collapse forward in time. For the collapsing three dimensional cloud, physically acceptable density distributions are those which are analytic at $r = 0$ and this means that the Taylor expansion about $r = 0$ demands that the density function is

$$\varepsilon(r,0) = \rho(r) = \rho_0 + \rho_0'' r^2 + \ldots \quad \ldots (6)$$

with ρ_0'', which is $\rho(r)''\rfloor_{r=0}$, negative. (Even if we start with a non-analytic density distribution, allowing the first derivative term, the final conclusions on the nature of resulting singularities do not change.) When the collapsing cloud is marginally bound, the density function can be written as in equation (5). With the initial scaling described by $r = R(r, 0)$, we have all the relevant functions ($G(r)$, $F(r)$, $M(r)$ and M') in hand and we can write the density at all times near $r = 0$ (to second order in r) as follows :

$$\varepsilon(r, t) = \frac{4/3}{\left(t - \frac{2}{3}\sqrt{\frac{3}{\rho_0}}\left(1 + \frac{3}{20}\rho_0'' r^2\right)\right)\left(t - \frac{2}{3}\sqrt{\frac{3}{\rho_0}}\left(1 + \frac{13}{20}\rho_0'' r^2\right)\right)}. \quad \ldots (7)$$

More specifically the behaviour of the central density is given as

$$\varepsilon(0, t) = \frac{4/3}{\left(t - \frac{2}{3}\sqrt{\frac{3}{\rho_0}}\right)^2}. \quad \ldots (8)$$

This indicates that the singularity forms in a finite time and the time it takes to form the central singularity is determined by the central density alone and is same as the time it takes for the homogeneous cloud to collapse to $r = 0$ (Oppenheimer & Snyder 1939). There are some important points to be noted : (1) The central density does not vary as t^{-2} as was the case in counter examples, (2) when the central singularity forms, the radial dependence of the density is given for small r as

$$\varepsilon(r, t_s) \sim 1/r^4 \quad \ldots (9)$$

which corresponds to a dependence of $1/R^{12/7}$ and not R^{-2}.

The situation described in this paper was analyzed by Newman for the strength of the singularity in terms of the limiting focusing conditions and it was concluded that the singularity, though naked, is not strong (Newman 1986). It cannot crush infalling matter to zero volume. Also, it can be verified that the singularity is strongly censored when the coefficient of the second term in the expansion of the density function is zero.

So we have the following important conclusion : The singularity which forms during the collapse of pressure free inhomogeneous dust with initially regular and finite density distribution is either weak or, when strong, strongly censored. (Gravitational collapse of fluid with pressure has to obey more restrictive conditions on its density behaviour and it seems plausible that in such cases this conclusion holds in a stronger sense but this needs to be proved.)

This brings us to the following question : What is the physical interpretation of results obtained by several authors on the formation of naked singularities in Tolman-Bondi collapse? Those singularities do not arise from dynamical collapse of physical density functions, but arise because the starting density functions were intrinsically singular. Since the formation of the event horizon cannot proceed faster than the speed of light, the initial point in time ($t = 0$) should be naked if we start at $t = 0$ with a density distribution which varies as $r^{-\alpha}$ with α a positive quantity, and this is what happens in the solutions presented as counter examples. (The central density goes as $1/t^2$ in the counter examples and note that we get this behaviour for the central density in equation (8) if we put the starting central density as infinity). With this correct interpretation it is easy to see why the phenomenon of naked singularities arise in the case of perfect fluid with pressure also, independent of many details of the equation of state and other physical considerations. The problems with the interpretation of dynamical collapse in the counter examples can be foreseen in the expression for the energy density, equation (2). Note that if the density needs to be finite for $t < 0$ at $r = 0$ (where $R = 0$ always) the quantity R' has to be infinite which means that the metric is infinite all throughout the past!

These considerations show that so far there is no single physically sound counter example to the cosmic censorship conjecture as far as spherically symmetric collapse is concerned.

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