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A maximum entropy method based scheme for the inversion of extended radio images in gravitationally lensed systems

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Abstract. A new scheme is advanced for the process of source reconstruction (or image inversion) from extended images in gravitationally lensed systems which show multiple imaging. We suggest that forward simulations of such systems may be employed as *a priori* information to aid the inversion process. Several advantages are to be gained by this, chief among them being that a simulation can easily include information from more than one sets of observations of the same system, whereas the inversion schemes currently in existence operate on a single image map at a time. We present a formulation of the new scheme in terms of the Maximum Entropy Method, in which *a priori* information can be incorporated in a natural manner.

Key words : maximum entropy—gravitational lensing—galaxy

1. Introduction

A problem that is common to many branches of astronomy is the reconstruction of the two-dimensional intensity distribution of a source in the sky, given observational data that is incomplete or distorted on account of instrumental effects or intervening media. When an extended source is imaged due to the gravitational bending of light by an intervening galaxy, the phenomenon may be regarded as the action of a distorting telescope. The lens galaxy's mass distribution, in projection along the line of sight, sets what may be regarded as the gravitational lens point spread function. In this case, however, one has to reconstruct the source as well as determine the properties of the lens (Kochanek *et al.* 1989). This is actually possible because parts of the source are typically multiply imaged, and can be related to each other through a many-one image to source mapping. In the present work, a new scheme for the process of source reconstruction (image inversion) is advanced, in which *a priori* information regarding the source can be incorporated. The motivation for this scheme and its formulation in terms of the method of maximum entropy will be discussed.

2. The need for *a priori* information

Existing algorithms for carrying out the process of image inversion in the case of extended image systems in gravitational lensing treat a single image map at a time (Kochanek *et al.*

1989; Kochanek & Narayan, 1993), so that it is not guaranteed that an independent reconstruction performed on a map of the same image system at another frequency of observation will yield structures for the lens and source that are consistent with the first set. On the other hand, it is possible to simulate image maps with the help of a model for the source, in which constraints on the structure of the source can be included. Figure 1 shows, by way of example, source structures inferred through simulations, at two frequencies of observation for the gravitational lens system PKS1830-211 (from Nair *et al.* 1993). A knowledge of the physical behaviour of the structure of such radio sources, as a function of frequency, has also been used. The model source structure may be employed to bias the source reconstruction; it contains information regarding the gross morphology of the source, the actual intensity distribution of which can be set through the process of inversion.

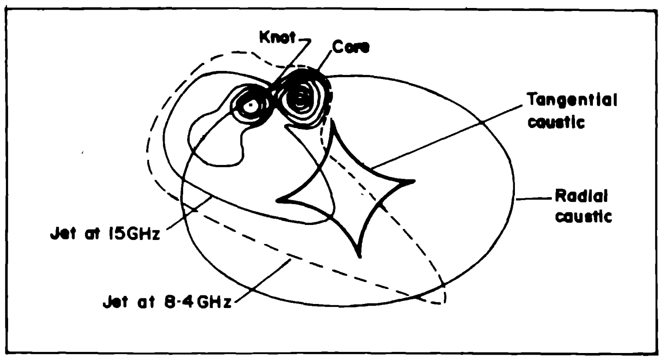

Figure 1. Physically realistic source structure consistent with the observations at two frequencies, as inferred by a simulation of the image maps, for the system PKS1830-211 (from Nair *et al* 1993). This is treated as *a priori* information in the image inversion scheme.

3. The source reconstruction problem

(a) *Applying the method of maximum entropy to the source reconstruction*

For the sake of simplicity in the present discussion, it shall be considered that the image map is deconvolved as regards beam of the man-made telescope (possibly by a standard application of MEM), so that it consists of a uniform grid of $N \times N$ pixels, each with an associated intensity. The ultimate goal is to obtain a uniquely defined source plane intensity map of, say, $M \times M$ pixels ($M > N$). To formulate this problem for MEM, we represent the image plane pixels in terms of the source plane ones. This in turn requires a model for the lens galaxy.

(i) *The lens model* : The typical lens galaxy may be treated as a parametrized elliptical potential or mass distribution, which, in projection, involves at least five or six parameters. The technique of Kochanek *et al.* (1989) is adopted in a 'coarse grained' sense (several image pixels relate to a single source pixel—initially, we take $M = N$). A starting set of lens parameters is chosen (the lens model from simulation). All image pixels that map onto the same source pixel must carry identical intensities, for the correct lens model (since intensity is conserved in gravitational lensing). In general, there will be some degree of mismatch between these intensities, and the lens parameters can then be tuned until the mismatch is minimized ('best match' model).

(ii) *ME applied to the source plane* : With $M > N$, the mapping that is produced by the best match is used to relate the image pixels to the source pixels. In this part of the scheme the mapping is 'fine grained', in which several source pixels are used to effectively represent the mapping of an image plane pixel in the source plane (figures 2(a) and (b)), and the approximations to these mappings in terms of source plane pixels (figure 2(c)), for a typical elliptical lens model.

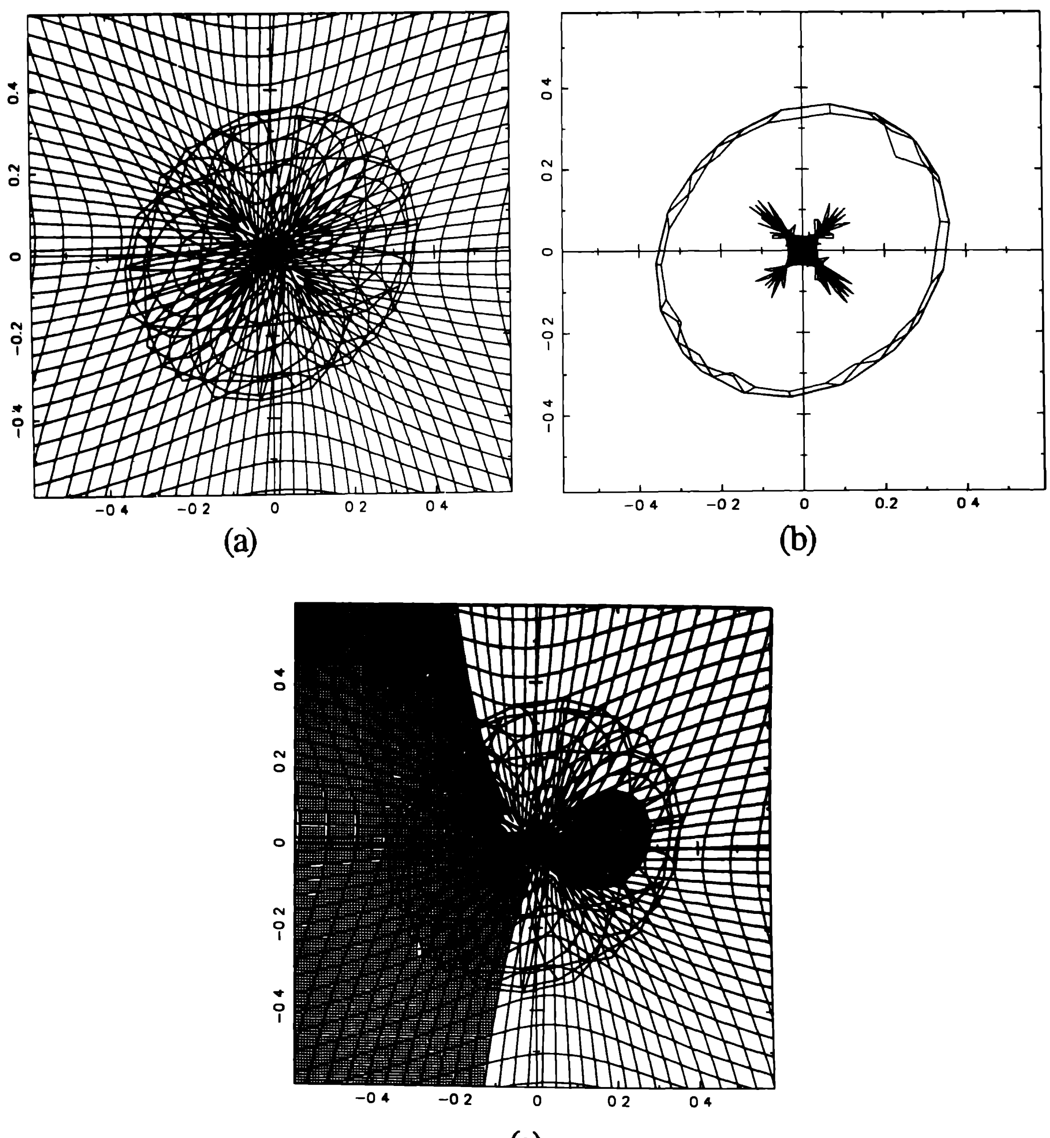

Figure 2. The mapping in source space of a uniform grid in image space, (a) for typical image plane pixels and (b) for those pixels that cross the critical curves. (c) Showing the manner in which the mappings of these pixels are approximated by the source plane pixels (represented as fine dots).

Let the kth image pixel map onto n_k source pixels, which will be denoted by indices p. Then, if the intensity of the image pixel is I_k, it is related to the source pixels' intensities, I_{sp}, via the relation :

$$I_k = \sum_{p\varepsilon\{n_k\}} \frac{I_{sp}}{n_k} \qquad \ldots (1)$$

$\{n_k\}$ denotes the set of source pixels p related to image pixel k. The entropy function to be maximized is in terms of source pixels :

$$S = -\left(\sum_q I_{sq} \ln\left(\frac{I_{sq}}{I^m_{sq}}\right)\right) \qquad \ldots (2)$$

where I^m_{sq} are the source pixel values as in the source *prior*.

The maximization of this entropy is to be performed subject to two constraints, firstly that the source be such that it produces images the intensities of which agree with the observed intensities O_k to within the noise :

$$\sum_k \left[\frac{\left(\sum_{p\varepsilon\{n_k\}} I_{sp}/n_k - O_k\right)^2}{\sigma_k^2}\right] = \Omega \qquad \ldots (3)$$

and secondly, the total intensity in the image plane as obtained from the source mapping must remain fixed :

$$\sum_k I_k = \sum_k \left(\frac{\sum_{p\varepsilon\{n_k\}} I_{sp}}{n_k}\right) = T \qquad \ldots (4)$$

T is the total image intensity and $\Omega \simeq N^2$, the number of independent data, for large N.

4. Conclusions

The present scheme for inverting extended images in gravitationally lensed systems provides a simple means of incorporating information regarding the image morphologies into the process. A knowledge of the lens properties and the source structure that is consistent with a range of observations can be included in a simulation of the system; including this in an inversion scheme combines the advantages of both techniques of modelling lensed systems. The problem has been formulated in terms of a two-stage process, involving the method of maximum entropy.

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