STUDIES ON THE VARIATION OF ROTATION ON THE SURFACE AND IN DEPTH ON THE SUN IN RELATION TO THE PHOTOSPHERIC MAGNETIC FIELDS

A Thesis
submitted for the degree of
Doctor of Philosophy in Physics of
PANDIT RAVI SHANKAR SHUKLA UNIVERSITY
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RAIPUR - 492010

by

S. SURENDRA GUPTA

INDIAN INSTITUTE OF ASTROPHYSICS
BANGALORE - 560034

July, 1994

DECLARATION BY THE CANDIDATE

I , Mr. S. SURENDRA GUPTA , certify that as outlined in the synopsis, I have measured sunspot areas and solar diameter on nearly 19,000 photoheliogram plates of the Sun obtained over a period of 84 years from 1904 to 1987 myself. Nearly 8,50,000 positional measurements for sunspots and solar diameter were made completely by me to determine the solar differential rotation rate and other related results presented in this thesis.

1/7/94

Bangalore - 560034 (S. Surendra Gupta)

Date: July 1, 1994 Candidate

CERTIFICATE FROM THE SUPERVISOR

I certify

a) that the thesis embodies the work of the candidate, Mr.S. Surendra Gupta himself,

b) that the candidate worked under me for the period required under para 8 and

c) that in my opinion the thesis fulfils the requirements of the ordinance relating to the Ph.D., degree of the University,

d) and that the thesis on the research topic "Studies on the variation of rotation on the surface and in depth on the sun in relation to the photospheric magnetic fields" is an original work of the candidate.

I further certify that

i) he has measured neaarly 19,000 photoheliograms of the sun covering the period 1904 to 1987, which forms the basic data from which the results presented in the thesis have been arrived at,

ii) the complete organisation, meaurement and analysis, interpretation of the results are the work of the candidate, Mr.S. Surendra Gupta.

This work has not been submitted for the award of any Degree, Diploma, Fellowship, Associateship, etc., of any other University or Institute.

(Prof. K.R. Sivaraman)
Supervisor

Bangalore – 560034
Date: July 1, 1994

DECLARATION BY THE SUPERVISOR

I hereby certify that Mr.S. SURENDRA GUPTA , has worked on the research topic:

> "Studies on the variation of rotation on the surface and in depth on the sun in relation to the photospheric magnetic fields"

under my supervision. I further certify that

(i) he has measured nearly 19,000 photoheliograms of the sun covering the period from 1904 to 1987, which forms the basic data from which the results presented in the thesis have been arrived at,

(ii) the complete data organisation, measurement and analysis, interpretation of the results are the work of the candidate, Mr.S. Surendra Gupta,

(iii) that Mr.S. Surendra Gupta has worked under my supervision for the period required as per the University Ordinance.

(iv) and in my opinion, the thesis fulfils the requirements of the ordinance relating to the Ph. D. Degree of the university.

This work has not been submitted for the award of any Degree, Diploma, Fellowship, Associateship, etc., of any other University or Institute.

Bangalore - 560034
Date: July 1, 1994

(Prof. K.R. Sivaraman)
Supervisor

To

My Late Parents

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SUMMARY

The measurement and determination of the solar rotation rate has a long histroy. But in the recent decades this topic has assumed increased importance because of the necessity to measure the rotation rate and its variation with latitude on the solar surface and depth inside the sun with high accuracies. This is the result of the wider acceptance that the causal agency that gives rise to and supports the dynamo action (which in turn maintains the solar cycle) is the interaction between the rotation and convection. Since the basic solar cycle mechanism is a subsurface phenomenon, this adds further importance to the determination of the differential rotation which holds several clues for understanding and interpreting the solar cycle phenomenon. All rotation tracers like sunspots, plages, surface magnetic fields are believed to be linked to subsurface layers by magnetic field lines and hence the rotation rates derived from monitoring them represent the rotation rates of the depths at which these tracers are anchored.

In this thesis, the rotation rate as a function of latitude has been computed from the measurements of the positions and areas of sunspots from the Kodaikanal data with an accuracy hitherto not attempted. These data in the form of photographic images of the sun or the photoheliograms obtained at the Kodaikanal observatory since 1904 forn one of the longest uninterrupted data series in the world. The measurements of the positions and areas of —— 332620 sunspots were done from 18,888 Kodaikanal photoheliograms that cover the period 1906 - 1987.

In Chapter I a brief review of the present status of the solar rotation measurements relevant to this thesis and the aim of the present study are presented.

In Chapter II we describe the details of the Kodaikanal programme under which this long data bank was created, and the measurement techniques adopted for measuring the positions and areas of the sunspots from the daily photoheliograms.

In Chapter III the results that have come out of this study have been presented. The rotation rates using sunspot groups, individual spots divided

into three groups accordidng to their areas (small, medium and large spots) and all the spots in the three groups put together (all spots) have been computed. The rotation rate derived from the small spots is highest and the rotation rate progressively decreases with increasing size of the spots and the rotation rate of the spot groups is the lowest. The rotation rates for the latitudes of the sunspot zone in the two hemispheres (i.e., 0° - 30° latitude and also all spots poleward of 30° in the north and south hemispheres) have been calculated. The north - south asymmetry has been studied and it is seen, generally, the rotation rates at all latitudes within the sunspot zone in the southern hemisphere are higher than in the northern hemisphere.

In addition, the meridional motions measured from the proper motions of the spots have been computed and these results are presented. These show a poleward motion in the northern hemisphere and an equatorward motion in the mid latitudes and a reversed motion (i.e., towards the direction of the pole) at latitudes beyond 20° in the southern hemisphere. We have looked for solar diameter changes during the period of the analysis. No evidence in the data for any long term changes in the solar diameter was seen.

In Chapter IV the variation in the rotation rate with the sunspot cycle has been studied and the results are presented. It is seen that the rotation rates are higher during solar minimum and lower at maximum. This difference has been interpreted atleast partly as a consequence of two different samples of sunspots being used as tracers for rotation rate determination at the two extreme epochs of the solar cycle.

In Chapter V a summary of the conclusions of the study has been presented.

LIST OF FIGURES

LIST OF TABLES

CONTENTS

CHAPTER I

REVIEW ON THE STATUS OF SOLAR ROTATION MEASUREMENTS

1.1 Introduction

The Sun is the most unique of all stars due to its proximity to the earth, that its surface details can be observed at high spatial resolution. Besides the large radiative flux, the sun permits us to study the morphology and dynamics of its atmospheric inhomogenities and thereby understand the physical processes involved in their creation, support and evolution.The study of the rotation and its dependence on latitude and depth in the sun is itself an important problem; besides, it has vast implications in the study of the late type main sequence stars in view of the immense possibility of applying the rotation - magnetic activity - age connection known for the sun to all of them.

The origin of the solar rotation studies can be traced back to the sunspot observation of Galileo Galilei in 1610. Following Galileo, a number of astronomers carefully monitored the movements of the sunspots across the solar disk day after day. Their studies led to the first set of papers on solar rotation (Scheiner, 1630; Hevelius, 1647; Wolf,1862). Christoph Scheiner mentions in his "Rosa Ursina", published in 1630, that sunspots close to the solar equator rotate at a faster rate than the spots farther away from the equator. This led to the detection of the differential rotation on the sun. The determination of the differential rotation of the sun is an important and difficult problem. It is now widely accepted that the activity cycle is a magnetic dynamo process which depends on the convection and the differential rotation. The details of the physical mechanism are unclear as are the details of the maintenance of the differential rotation with latitude. The basic problem is that we can see only a thin surface layer of the sun, and much of the large-scale motions associated with the dynamo take place in the layers below the photosphere and cannot be observed. One way is to study carefully the surface motions spectroscopically and another is by using any one of the several features on the sun as tracers, such as, sunspots, faculae, photospheric magnetic fields, supergranulation cells, the Calcium K network, etc. Yet another method is to study the frequency shifts of high frequency solar acoustic oscillations.

Thus the methods of measurements of the rotation and differential rotation rates of the sun fall broadly into three classes:

(1) Spectroscopic method
(2) Tracer method
(3) Helioseismological method

1.2 Spectroscopic methods

In the spectroscopic method the spectral lines are recorded on the east and the west limbs of the sun at a number of latitutes and from the Doppler shifts of the lines the rotational velocities are derived. The first results were published more than a century ago (Vogel, 1872; Young, 1876, Duner, 1890). These observations although were made mainly to verify the Doppler effect to start with also yielded the solar rotation rate. With the availability of high-dispersion spectrographs in the beginning of this century, spectroscopic techniques were used to determine the solar rotation rate (e.g., Adams, 1911; J.S.Plaskett, 1915; H.H.Plaskett, 1916; Storey, 1932; St.John, 1932). The results from these studies were mutually discordant and these were attributed to various errors (De Lury, 1939). These are listed below:

i) Systematic errors from the measurement of small displacements of the spectral lines.
ii) Blending of spectral lines chosen for the study resulted in reduced measures of the displacements of spectral lines.
iii) Accidental errors due to velocities in the pores which were hypothetically and observationally found to have convective system, and this contributed to the increase in the wavelengths of many spectral lines at the solar limbs.
iv) Scintillation caused by the earth's atmosphere and imperfect optics resulted in the scattering of light in the telescope in a complicated manner. Also the scattered light, seeing conditions and state of the optics all vary with time. Coupled to these were the imperfect guiding of the image and the consequent uncertainitics in the slit position with reference to the point of observation on the image.
v) Errors due to the approximations in the computations of the rotation rates. In these methods, correction for the components of the earth's velocity was done only to the limb measurements , whereas for points within the solar

limb and at high latitudes, no corrections were done and this resulted in errors.

vi) The interpretation of the spectroscopic measurements depends on the values of i and Ω (Carrington, 1863). i, the inclination of the sun's polar axis to the ecliptic and Ω, the longitude of the ascending node of the solar equator on the ecliptic. Hence any errors in the values of i and Ω will be reflected in the rotation rate determination.

All these errors lower the values of rotation rates. To minimize these errors, the present day spectroscopic observations use photoelectric systems. Several reviews on solar rotation (Gilman, 1974, 1980; Howard, 1978, 1984; Schröter and Wöhl, 1978; Schröter, 1985; Bogart, 1987; Stix, 1989; Libbrecht and Morrow, 1991) summarize the problems of measurements of solar rotation with historical background and discuss the merits and sources of errors in their measurements. Recent developments in this field have been reviewed by Howard et al., (1991).

1.2.1 Recent Measurements

In the latter half of the century, Doppler measurements with the photoelectric systems replaced the cumbersome photographic method, and this resulted in velocity data of higher accuracy (Howard and Harvey, 1970). At about the same period, several theoretical models of the solar differential rotation were constructed (e.g., Kippenhahn, 1963; Köhler, 1970; Durney and Roxburg, 1971). This gave further motivation to the observers to vastly improve the quality of observations and reach higher and higher accuracies, so that these could provide as effective checks on the various models. Well known reviews on theoretical models are by Gilman (1974, 1976) and Dicke (1970).

Photoelectric detectors measure the Doppler shift of a spectral line via a pair of exit slits while the entrance slit of the spectrograph scans over the solar disc. With the installation of magnetographs, the line - of - sight velocity data were obtained regularly at many observatories like Mt.Wilson, Stanford, Kitt Peak, Crimea, Locarno, Capri, etc., by operating the magnetograph in the velocity mode. The large amount of velocity measurements across the full disc allowed the rotation determination to be done with an accuracy level much higher than so far possible. The systematic errors caused by the scattered light were mostly eliminated by omitting the data near the limbs. Howard and Harvey (1970) using the velocity data of Mt.Wilson for the period 1966 to 1968, derived the sidereal rotation rates for the full disc.

Day-to-day scatter in their data was nearly 10%, but improvements in the instrumentation brought down the daily variations down to $\sim$ 1%. These fluctuations however remain and these could be of solar origin (Howard 1984). Ideally using a typical medium strong Fraunhofer line it should be possible to realise an accuracy of $\approx$ 5 m/sec in the velocity determinations. But, in practice this is never achieved due to several uncertainties both pertaining to the instrumentation as well as of solar origin for which corrections cannot be done perfectly. These errors are briefly discussed below.

1.2.2 Uncertainities of Solar Origin

Resolved motions result mainly in noise and unresolved motions result in systematic errors.

A. Sources of noise in the data: Granular motions, supergranulation flow pattern and oscillations are of solar origin and these give rise to noise. Beckers and Canfield (1975), and Beckers (1981) have discussed the source of errors of solar origin and methods to reduce their influence on the rotation data. Even after these procedures, they found that the noise level could not be brought down to $<$ 10 m/sec level from day-to-day observations.

B. Systematic errors: These are produced by
i. incorrectly eliminated limb shifts,
ii. variation of the line-asymmetry across the solar disc,
iii. line - asymmetry differences between quiet and acive regions.

C. Instrumental effects:
i. Thermal inhomogenities of the air inside the spectrograph, thermal pressure on its parts, long term variations of temperature, air pressure and water vapour give rise to noise and spurious signals in the data. These are estimated to go upto $\sim$ 40 m/sec (Howard and Harvey, 1970; Livingston and Duvall, 1979; Howard, Boyden and LaBonte, 1980).

ii. Grating illumination: Change of spectral line intensity due to improper reimaging of the entrance slit of the spectrograph coupled with errors in grating ruling leads to systematic wavelength shifts. Limb darkening also gives rise to the same effect. (Livingston, 1968; Brandt et al., 1978, Howard, Boyden and LaBonte, 1980).

iii. Interference fringes: These are produced in the spectrum due to one of the optical components acting as a Fabry - Perot etalon. Room temperature variations produce change in the fringe pattern, leading to incorrect centering of the line on the exit slits. These lead to systematic errors (La Bonte and Howard, 1981).

iv. Detectors: 0.1% non - linearity of the two detectors can give rise to a spurious signal leading to an error in the range of 25 m/sec (Brandt et al., 1978). Lowering the intensity signals by a factor of 100, would lead to an error of 60 m/sec (Howard, Boyden and LaBonte, 1980).

v. Scattered light: Svalgaard, Scherrer and Wilcox, (1978), showed that instrumental and atmospheric scattered light effects contributed to the observed short term variations in the rotation rate.

1.3 Method using tracers

By monitoring the day-to-day positions of stable and long - lived tracers such as sunspots, faculae, filaments, magnetic regions, chromospheric plages, coronal holes, etc., solar rotation can be derived. All long - lived solar features are of magnetic origin. To study the surface motions, the most obvious choice is the motion of sunspots. An advantage of studying sunspot motions is that the strong magnetic fields of sunspots are possibly linked to the sub - surface fields and thus we are seeing the effects of the motions of these sub - surface layers. In principle, by tracing solar features, one can measure two components of the velocity field. From these, surface Reynold's stresses can be measured (Ward, 1965; Belvedere et al.,1976; Schröter and Wöhl, 1976).

1.3.1 Coordinate system to measure the solar rotation

In the case of sunspot the day-to-day positions are measured and from the change in positions between two successive days and the actual time interval between the measurements of these positions the rotation rate is calculated. An important prerequisite for measuring daily positions which can be used subsequently to compute the rotation rates is a coordinate system against which these position measurements can be referred to. An obvious choice is to use the heliographic coordinates. These are based on the position of the solar rotation axis within the ecliptic plane and specified by the two coordinates called rotational elements: Ω - the longitude of the ascending node of the solar equator on the ecliptic and i - the inclination of the sun's equator to the ecliptic plane. Ω and i, determine the annual variations of the angle P which is the angle between the north direction on the sky and the solar north and B_o, the annual tipping of the solar axis towards or away from the observer.

The rotational elements Ω and i derived by Carrington have been used by the subsequent observers. Stark and Wöhl (1981) showed an error of 0.5° - 3.0° in Ω and 0.3° in i in the values given by Carrington. Inaccuracies in Ω and i can result in spurious annual periodicity in the results. Many investigators have tried to determine the correct values of Ω and i (Schröter and Wöhl, 1975; Wöhl, 1978; Clark et al., 1979; Stark and Wöhl 1981; La Bonte, 1981) which would enable the calculation of the correct values for P and B_o.

It has become customary to represent the latitude dependence of the solar angular rotation ω by the formula

$$\omega(\phi) = A + B \ Sin^2 \ \phi \ + C \ Sin^4 \ \phi \ ---1$$

where ϕ is the latitude on the solar surface and A. B, and C are the coefficients which are determined from measurements through a least square fit. A, gives the equatorial rate, B and C give the differential rotation. For sunspot position measurements C is assumed to be 0. Howard and Yoshimura (1976) have argued that the above polynomial have terms that are not orthogonal, but still serve as a convenient form for deriving solar rotation from measurements. Howard, Boyden and LaBonte (1980) showed that the conversion of the line-of-sight velocity to angular velocity requires terms which introduce non-orthogonality. By orthonormalisation with respect to the solar disc of the rotation function, limb shift and meridional flow, the crosstalk between the coefficients A, B and C for plasma rotation velocities get totally removed (Snodgraas, 1984).

1.3.2 Sunspot as tracers

The advantage of using sunspots as tracers is that the available data have been independently obtained either in the form of white-light images on plates or as drawings that span many solar cycles. Besides faculae, sunspots are the only indicators of solar motion field that were available prior to the middle of this century. The first indication that the sun rotates was found from day to day observations of sunspots across the disc. Sunspots are the ones which can be observed on the simple projection of white light image with even a very small aperture telescope. This shows the simplicity of the data. In the 17th and 18th centuries, many investigators monitored the sunspots to determine the solar rotation. But Carrington (1863) made a systematic study of spots from 1853 to 1861 and found that the sun rotates differentially. Carrington from his observations determined the orientation of the solar rotation axis and this value is still used for ephemeris calculations. The old data has recently been analysed by many investigators (e.g., Eddy, Gilman, and Trotter, 1976; Herr, 1978; Abarbanell and Wöhl, 1981; Arevalo et al, 1982; Yallop et al, 1982).

In the middle of the last century, the programme to obtain white light images of the sun was started at Greenwich, Cape town (South Africa), Mauritius and Dehradun. Greenwich sunspot records span over a century from 1874 to 1976 and the sunspots are sketched on Stonyhurst charts. These constitute the oldest continuous sunspot records that have been extensively used for solar rotation studies. Also, there were three other observatories in the world, namely, Mt. Wilson, Meudon and Kodaikanal where a similar programme was started from the early part of this century and of these Kodaikanal data has the longest duration.

Greenwich data was digitised by Ward (1966) to study the rotation rates. Many of the published results do not refer to sunspot data, excepting Kearns (1979), who used individual-spot data over a short interval. Mt. Wilson data has been digitised and analysed by Howard, Gilman, and Gilman (1984). Their results based on sunspot data, cover the period 1921 - 1982 and are of better accuracy than those from Greenwich records.

Sunspots are relatively good tracers of rotation, as they are compact, well defined, unchanging and often with life times of several days or more. Sometimes spots live longer to cross the central meridian more than once. Number of studies of solar rotation and its variation with the latitude (i.e., differential rotation) using sunspots as tracers are available in the literature: Newton and Nunn, 1951, Ward, 1966; Godoli and Mazzucconi, 1979; Balthasar and Wöhl, 1980; Arevalo et al., 1982; Lustig and Dvorak, 1984; Balthasar, Lustig and Wöhl, 1984; Howard, Gilman and Gilman, 1984 and Howard et al., 1991; Touminen and Virtanen, 1987; Snodgrass, 1992. The Greenwich photoheliographic results covering the period 1874 to 1976 were the main source of data for the study of solar rotation so far. Howard, Gilman and Gilman (1984) devised a different approach to find the spot positions to determine $\omega \ (\phi)$ using Mt. Wilson photoheliograms for the period 1921 to 1984. Recently Ribes, Ferreira and Mein (1993) have used the sunspot data of Meudon and calculated the rotation and differential rotation. In their classical work Newton and Nunn (1951) using 136 long-lived individual spots that transited the solar disc in the years 1934 - 1944, derived angular velocity of solar rotation as

$$(\omega) = 14.38 \ (\pm 0.01) \ - 2.96 \ (\pm 0.09) \ Sin^2 \ (\phi) \ deg./day \quad ---- \quad (2)$$

This value is still taken as a standard of measure of the solar rotation. Balthasar et al. (1982) reported that younger spots tend to show a rotation rate that is faster than that derived from older spots. This can be explained from the observations of Howard, Gilman and Gilman (1984) that small spots rotate approximately 2% faster than large spots. Since large spots live for several rotations represent the older spots and the small ones last only for one rotation or less are relatively young. Ribes (1986) also found that young spots show a more rigid rotation profile than older spots. Howard, Gilman and Gilman (1984) showed that spot groups rotate at an intermediate rate between small spots and large spots which agrees well with the recurrent spot rate of Newton and Nunn (1951); this would however depend to some extent on how a spot group was defined. Balthasar et al., (1986), have done a comprehensive study of the spot groups rotation from the Greenwich data. Dependence of rotation rate on spot age has been confirmed. They also confirm the result of Balthasar and Wöhl (1980) and those of Gilman and Howard (1984) that spot rotation rate is highest near solar minimum. They further confirm the correlation of rotation rate and meridional motion found by Howard and Gilman (1986). This effect was discovered by Ward (1965) from Greenwich data and was taken as an evidence of angular momentum transport towards the solar equator. Touminen and Virtanen (1987) found similar results from the same Greenwich data set. Hathaway and Wilson (1990), using Mt. Wilson data found that solar cycles with less sunspot area show more rapid rotation.

1.3.3 Depth dependence of rotation

Sunspots seen at the photospheric level are rooted to the interior through the magnetic lines of force associted with them. Different spot sizes presumably are anchored to different depths in the convection zone, (Foukal, 1972; Stix, 1976; Gilman and Foukal, 1979). So the rotation rates deriverd by using spots of various sizes would not be the same (Howard, 1984; Newton & Nunn, 1951). Again, sunspots and other magnetic field related structures show a rotation faster than that of the photospheric plasma. Clearly, the photospheric plasma cannot rotate at the same rate as the spots who reflect the internal rotation rates; also it is clear that the photospheric plasma cannot rotate at the same rates shown by all spots of different sizes.

An understanding of the variation in the sun's rotation rate with latitude and depth are necessary to evolve the correct picture of the sub-surface flows which is essential to understand the complicated dynamics of the solar convection zone. This will lead to a better understanding of the interaction of rotation and convection and the regeneration of magnetic fields in the interior of the sun and the stars.

1.3.4 Photospheric magnetic fields as tracers

The pattern of non-spot magnetic fields has been used as a tracer to derive the rotation by various investigators. The source of data are the full disc magnetogram maps of the longitudinal component of the Zeeman effect recorded daily at the Mt. Wilson since 1959 and at the Kitt Peak National Observatory since 1976. The Mt. Wilson data were used to determine the rotation rate for latitudes below $\sim 50°$ by Wilcox and Howard (1970), Wilcox et al (1970) and

Stenflo (1974) where the time series of the daily field measurements was auto correlated. Subsequently a similar analysis of the combined data from the Mt. Wilson and the Kitt Peak Observatories carried out by Stenflo (1989) for all latitudes confirmed the earlier results at the lower latitudes, but the rotation profile became strikingly flatter showing a more rigid-body-like rotation at high latitudes. A similar flattening of the rotation curves was also found by Sheeley, Nash and Wang (1987), from their analysis where they cross - correlated successive synoptic or Carrington maps of the magnetic field. Whereas, the rotation values derived by Snodgrass (1983) using the same Mt. Wilson data by cross-correlating the magnetograms taken 1 - 4 days apart showed that the sideral rotation vs latitude profile was essentially parallel to the rotation profile from Mt. Wilson Doppler measurements with a rate $\sim$ 2% faster at all latitudes with no evidence of a rigid-body-like rotation. Thus there is an anomally that two entirely different rotation laws emerge from the two methods of analysis of the same data: the individual magnetograms were cross-correlated by Snodgrass (1983) with short time lags (of the order 1 - 4 days) while in the other analysis, both the auto correlation and the Carrington cross correlation, match features after time lags of one or more solar rotations (27 days or more).

Thus the two analysis use two different time scales for correlation. This difference in the rotation laws by the two methods of analysis has been explained (Stenflo 1989) on the hypothesis that the magnetic field is regenerated over a time scale that is shorter than 27 days and larger than 4 days. Thus the pattern that recurs at the central meridian after one or more solar rotation is not the same magnetic fluxes, but are the newly emerged fluxes. Thus the rotation rate derived from the auto correlation pattern of one rotation or more would be characteristic of the pattern of phase velocity of the source region of these new fluxes and would differ vastly from the phase velocity pattern in the photosphere. Whereas Snodgrass (1992) explains this difference as a selection effect introduced by the choice of different time scales for the correlation.

1.4 Possible accuracy levels from tracer measurements

The measurement of solar rotation rate as a function of latitude and depth ω (ϕ , h) may seem as a comparatively easy observational task. After making corrections for several sources of possible errors listed earlier. it is possible to obtain a unique value for ω (ϕ) from the spectroscopic as well as from the tracer measurements, at a 3% accuracy level (Figure I.1). This picture changes completely when an accuracy level of 1% or better is demanded. There is distinct divergence among the values of ω (ϕ) not only between the one derived from spectroscopic and the one from tracer measurements, but this divergence persists even within anyone tracer used. It is seen that the ω (ϕ) values depend on the kind of tracer used, and even within any one tracer, the size, age, amount of magnetic flux associated with the tracer, etc., seem to significantly influence the final values of ω (ϕ). Considerable efforts have been spent during the last three decades to derive the rotation rates and differencial rotation by measuring sunspot positioins. The values of the coefficients A and B from these studies are presented in Table I.1 along with the actual period to which the data used pertain to.

By tracing solar features like sunspots and using a heliographic coordinate system, an internal accuracy of better than 20 m/sec (0.14 deg./day) in rotation rates can be attained. By tracing hundreds of spots, an accuracy of better than 4 m/sec (0.03 deg./day) can be achieved (Newton and Nunn, 1951; Ward, 1965, 1966). Accurate position of solar features can be determined by noting the intensity maximum or minimum. This helps to realise a positional accuracy of $<$ 1 arc sec. Using this method to determine sunspot positions on the disc passage on 4 consecutive days it is possible to realise an accuracy of $\leq$ 2 m/sec. or $\leq$ 0.1% (Koch et al., 1981) and for features like calcium fine - mottles, an accuracy of $\leq$ 4.5 m/sec. (Schröter and Wöhl, 1976; Koch et al., 1981). Tracing prominences position to the accuracy of 10 arc/sec. on the east and west limbs would lead to an accuracy of $\leq$ 10 m/sec. in rotation determination.

1.5 Helioseismology methods

Helioseismology is a new field that has emerged in recent years and can be very effectively used to study the solar interior. Acoustic oscillations of 5 min. period have already given valuable results on internal rotation. These oscillations are the surface manifestation of trapped, standing sound waves inside the sun. These standing waves are the normal modes of the sun, with different modes sampling different regions of the solar interior. The frequencies of the waves are modified by internal dynamical effects like rotation and magnetism.

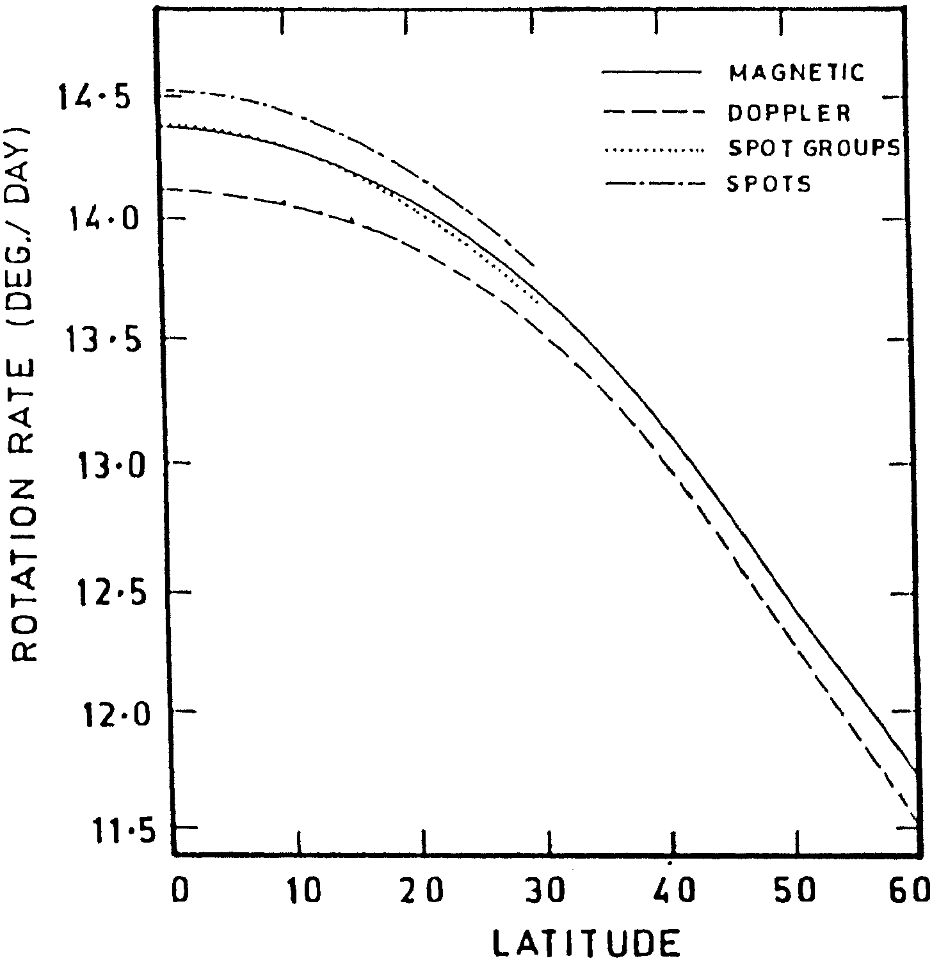


Fig.I.1 Sidereal rotation rates in degrees/day vs latitude from various tracers. Spots and spot groups results from Howard, Gilman and Gilman (1984) for the period 1921-1982. Doppler results from Howard et al (1983)for the period 1967-1982. Magnetic results from Snodgrass (1983) for the period 1967-1982.

Table I.1

Coefficients for the differential rotation (sidereal in deg./day) from ALL SUNSPOTS data

Reference	A	B	Data Period	Total No. of years	Source
Ward (1966)	14.523(± 0.006)	– 2.69 (± 0.06)	1905-1954	50	
Godoli and Mazzuccon: (1979)	14.58	– 2.84	1944-1954	11	
Balthasar and Wöhl (1980)	14.525 (± 0.009)	– 2.83 (± 0.08)	1940-1968	29	
Arevalo et al. (1982)	14.626 (± 0.014)	– 2.70 (± 0.16)	1872-1902	31	
Howard et al. (1984)	14.522 (± 0.004)	– 2.840(± 0.043)	1921-1982	62	Mt.Wilson
Balthasar et al. (1986)	14.551 (± 0.006)	– 2.87 (± 0.06)	1874-1976	102	Greenwich
Hanslmeier and Lustig (1986)	14.397	– 2.64	1947-1985	39	Kanzelhöhe
Present study	14.601(± 0.003)	– 2.872 (± 0.030)	1906-1987	82	Kodaikanal
- - ,, - -	14.596 (± 0.003)	– 2.854 (± 0.032)	1921-1982	62	- - ,, - -
- - ,, - -	14.602 (± 0.003)	– 2.883 (± 0.031)	1917-1987	71	- - ,, - -
- - ,, - -	14.602 (± 0.003)	– 2.885 (± 0.032)	1906-1979	74	- - ,, - -

The oscillation spectrum of the sun is characterised by multiplets labelled by n and ℓ, the radial order and angular degree of the oscillation. Fine structure in each multiplet denoted as m, the azimuthal order, is caused by rotation and magnetism. The lower the degree of the oscillation, the deeper it samples inside the sun. Oscillations of degree $\ell = 1000$ or higher are required to study the region near the solar surface. Recently, the helioseismological data have accrued to a level from which the rotation rates in the solar interior i.e., the rotation rates of the deep radiative zone upto the bottom of the convection zone and upto 0.7 to 0.9 of $R_{\odot}$ can be inferred from the frequency splitting corresponding to modes of degree $\ell \leq 600$. But, to study the solar interior from the surface, or 1 $R_{\odot}$ to below 0.9 $R_{\odot}$ an ℓ value of around 1000 would be desirable. At these high values of ℓ the data is very noisy, and it is difficult to obtain reliable results.

Duvall et al., (1984) found that, near the equatorial plane, the sun rotates rigidly throughout the convection zone with the outer part of the radiative zone rotating some what slowly. Also, the results indicate a rapidly rotating core. Duvall, Harvey and Pomerantz (1986) reported that the whole convection zone rotates with the surface differential rate (i.e., there is no sizeable radial gradient in rotation). This has been observationally verified by Rhodes et al., (1987), Brown and Morrow (1987) and Libbrecht (1989).

The depths to which the sunspots - at least the smaller spots - are anchored are so shallow that very high degrees of ℓ in the acoustic oscillations are needed to study these regions. These are not within reach at least from measurement of acoustic oscillations from ground based telescopes. Thus the only reliable class of tracers which can help to infer the internal rotation rates at such shallow depths below the surface layers are the sunspots. By using a large amount of data on sunspot positions and by accurate measurement of these positions rotation rates and changes in them can be established with high accuracy.

1.6 Meridional motions

Solar differential rotation can be explained by knowing the mechanisms of angular momentum transport towards the equator. Gilman (1980) points out that meridional motion towards the equator in photospheric layer and towards the pole in a deeper layer may cause equatorial acceleration. The meridional circulations can be maintained either by an anisotropic viscosity or by a latitude dependent convective energy transpot (Kippenhahn, 1963; Köhler, 1970; Durney and Roxburg, 1971; Belvedere and Paterno, 1977; Belvedere, Paterno and Stix 1980; Schmidt, 1982). The theories estimate that meridional motions of a few m/sec. are enough to maintain the observed differential rotation.

Meridional motions can be determined either by spectroscopic method or by using sunspots as tracers. Spectroscopially the line - of - sight component of the meridional motion has been measured and it is seen to be poleward with an average magnitude of 10 - 20 m/sec. (Duvall, 1979; La Bonte and Howard, 1982; Snodgrass 1984; Anderson, 1987; Lustig and Wöhl, 1990). The uncertainities in the measured values are of the same order, namely a few m/sec. These may be affected by scattered light, umbrae to - limb variations of turbulent broadening and spectral line asymmetry.

Many investigators determined meridional motions using sunspots as tracers (e.g., Tuominen et al, 1983; Richardson and Schwarzschild, 1953; Balthasar and Wöhl, 1980). Most of these studies have used the Greenwich data. The Greenwich data refer to spot group positions which are eye estimated only. Howard, Gilman and Gilman (1984), have determined the meridional motions using individual area-weighted spot positions.

1.7 Solar radius and its variation

The solar radius is a unit of measure in the astrophysical literature. The solar radius can be defind in two ways: Sun's radius is determined in terms of stellar structure equations using quantities such as luminosity, density, gravitational potential and optical depth at some specified wavelength. The second one which is the apparent radius, is found by measurements using one or the other techniques. Usually, in measurements the sun's apparent diameter is the one measured rather than the radius. So, in the discussion here, we refer to this observed or measured quantity namely the apparent semidiameter, which is equivalent to apparent radius ($R_\odot$). The observed solar diameter at certain wavelength is defined as the angular distance between the centre of the disc and the inflection point (limb).

Solar radius is considered to be constant on time scales short compared to nuclear processes. Solar radius variations are important because of their astrophysical significance and of their possible influence on the earth's climate. Past records of solar radius have shown that if at all there are any changes on time scales of decades to centuries, they could only be of the order of 1% or less. Evidence for both a cycle - dependence and a long term variation of the solar diameter have been presented by Gilliland, 1981; Sofia et al., 1983; Bachurin, 1984; Delache, Laclare, and Sadsaoud, 1985. However the question of solar radius variation is considered as unresolved yet.

Broadly there are two methods to determine the solar semi- diameter ($R_\odot$). One is by measuring the angle subtended by the two opposite limbs. This is rather difficult as one has to observe two contacts simultaneously. The other is by timing the duration between successive contacts of opposite limbs with fiducial lines in the sky. Here one has to make corrections for the solar limb blurring due to earth's atmosphere and also for personal errors in the observed data. These errors can sometimes be as high as 0.5 arc sec. Visual as well as photoelectric techniques are employed in the solar diameter determinations.

Visual observations are based on noting the timing of transit of the solar image, using meridian circle or by noting the transit of a planet like Mercury across the disc. The meridian circles were used to time the passage of the east limb and the west limb of the sun across the meridian. From the average of the two passage times the value of the diameter can be calculated. These observations are usually contaminated with systematic errors due to i) personal bias between observers in noting the transit time of the true solar limb, ii) change of optics in the telescope, discontinuity in the measurement, and iii) the exact timing of the transit duration. These errors can be minimised if observations are obtained by a single observer using the same telescope over long periods. Few such long period data that are available, are those of LaHire (1683 - 1718), covering the period of Maunder Minimum (1645 - 1705), meridian circle observations at the Royal Greenwich observatory from 1836 to 1953 and at the U.S.Naval observatory from 1846 to 1944, Mercury transit records from 1723 - 1973 and 7 timings of solar eclipses between 1715 and 1966.

The ancient solar diameter determinations have been publised by deLalande (1771) and Zach (1795). The results obtained during the 17th and 18th centruries were 4ör 5̈larger than the present adopted value for $R_\odot$ = (959.63 ± 0̈.05) due to Auwers (1891). Auwers results were based on 2849 single heliometer measurements. The currently adopted correction for irradiation is 1.55 making the value for $R_\odot$ = 960.18 (Cullen, 1926). Wittmann, Alge and Bianda (1991), using drift scan technique determined the semi-diameter (in sept. 1990) to be (960.51 ± 0.03̈) from 472 observations made at Izana and (960.59 ± 0.04") from 650 observations made at Locarno. They further analysed 1773 observations made in May and June 1981 and obtained a value of (960.32 ± 0.02̈). Comparing these two they find a difference in solar semi-diameter of about 0.25̈in 10 years, but declared that the results were inconclusive. Ribes et al (1991) reported solar mean radius as 959.44 ± 0.08 arc sec. (CERGA data) and 959.321 ± 0.024 arc sec. (HAO data).

Gilliland (1981) by analysing five data sets comprising of meridian circle observations of the transit of Mercury and solar eclipse observations derived the value for $R_\odot$ = 959:8 ± 0:1 . Using these data sets, Gilliland (1981) found the possible presence of a 76 year modulation in the solar radius. According to him, there was a suggestion of a secular trend showing a shrinkage in the solar radius of the order of 0.1 to 0.2 arc sec per century which although is observationally very small, has important implications. Ribes et al (1991) from a detailed analysis of both modern and historical records found evidence that the apparent radius is not constant in time. They attribute the short-term variations detected in the radius measurements to the change at the limb due to the presence of active regions. Because of this the shape of the solar limb shows variations over the solar cycle. The historical long period records show a semi-annual periodicity. This might be due to limb shape affected by solar activity.

1.8 Aim of the present study

As has been seen from the above, although there are many studies already on rotation and differential rotation basically most of them have used the Greenwich data either in pieces or in whole since that was the only source of such data available for this study. The next one with sufficient length which has been used is the Mt. Wilson photoheliograms data (Howard, Gilman and Gilman 1984). As has been briefly pointed out, the only other data longer than that at Mt.

Wilson are the ones at Kodaikanal. Thus the present study was undertaken with the aim of making measurements of the Kodaikanal data with an accuracy far higher than that in the Greenwich studies and of duration longer than of the Mt. Wilson data. From such measurements, we have been able to arrive at rotation rates with accuracy in the range of a few meters/sec. The high internal consistancy in the results and small error bars bear testimony to the fact that the accuracies achieved are higher than any so far.

Many interesting observations of solar velocity fields revealing flows with velocities of a few metres per second have been seen in recent years like meridional flows on the sun, torsional oscillations, etc. It is only through more and more accurate measurements using a variety of methods and tracers the reality of such flows which are on the very fringe of sensitivity of the measuring devices used for detecting them could be established. This would of course need a prior understanding how these flows would manifest in the behaviour of the tracers employed to detect the flows themselves.

It is with these aims, the present study involving the measurement of positions and areas of sunspots from photoheliogram plates covering nearly 8 solar cycles from 1904 - 1982 was undertaken. In chapter II we have discussed at length the measurement techniques used for measuring the positions and areas of the sunspots and discussed the accuracy levels reached in the present study.

In chapter III we present the results from these measurements. These consists of the following:

(i) Rotation rates vs latitude from spot groups, and individual spots.

(ii) Next we have divided the individual spots into 3 groups area wise (small, medium and large size spots) and derived rotation rates for each of these groups and the asymmetry between the northern and southern hemispheres on the sun.

(iii) We have then derived the meridional motions from the proper motions of sunspots.

(iv) Measurements of solar diameter for this period.

In chapter IV we have presented the dependence of the rotation rates on the solar cycle. Finally in Chapter V, the results from the present study and the conclusions from chapters III and IV are summarised.

CHAPTER II

SOURCE OF DATA AND MEASUREMENT TECHNIQUES

2.1 Introduction

Sunspots, plages, filaments are solar features with vastly differing structures and dimensions and with different extensions within the solar atmosphere. Also they are anchored to different depths within the sub-surface layers. Hence the rotation rates derived from these tracers with such dissimilar parameters although may show agreement amongst themselves in a broad sense, would disagree when compared with high accuracy levels in mind.

While attempting to measure rotation using tracers, the obvious choice would be sunspots as these features have many overwhelming advantages over other tracers. The white light images which are the source of data on sunspots are the easiest of observations, while other features require spectroheliograms which are acquired only in few of the solar observatories in the world. All these solar features have magnetic fields associated with them although of different strengths. The spots are compact and have generally large magnetic fields associated with them; whereas plages have larger extensions and have weaker fields. The topology of the two structures are also vastly different although they may have many other features in common. The difference in magnetic pressure that exists within a spot and a plage will decide the depths to which the magnetic tubes of force are anchored below the surface layers. These may in turn would decide the relative drag the spot and a plage would experience in the plasma medium during rotation. Even within a single tracer, there are differences like, size of spot (or area), age which would in turn alter the geometry of the magnetic fields associated with them. These differences would obviously get reflected in the rotation values derived from monitoring them. Inspite of these differences, the rotation values derived using any one tracer say sunspots from similar data series even though drawn from different observatories should show agreement within 1% level provided the same kind of measuring techniques and subsequent corrections to the data are done in the same way.

The aim of the present study is to determine the sidereal rotation rate of the sun at different latitudes. In doing this although only the sunspot will be used as tracer , the differences among the multitude of sunspots on the sun, would yield different values for the rotational velocity; but these would show difference much less than the rotation derived from two tracers with different properties.

2.2 Kodaikanal White Light Images

2.2.1 The Telescope and Observations

The plate vaults of the Kodaikanal observatory has an excellent collection of white light images of the sun (also called the photoheliograms) that dates back to 1904 thus spanning over nearly 8 solar cycles. Direct photography of the sun commenced at Kodaikanal in August 1903 when the photoheliograph Dallmeyer No.4 was overhauled and put into operation. This used a 4-inch (10cm) aperture, 5-foot (1.5m) focal length objective lens (made by Dallmeyer), modified to give an 8-inch (20cm) diameter image of the Sun similar to the photoheliographs operated from Dehra Dun (India), Mauritius and Greenwich. Photography on a systematic, daily basis started in 1904, using this photoheliograph on Lantern plates of size 10 x 10-inches (25.4 x 25.4 cm) and continued until 31 July, 1912. In 1908 the objective lens was replaced by a new one of superior quality. Starting on 31 July, 1912 the direct photography was carried out with the Lerebours and Secretan equatorial telescope having an aperture of 6 inches (15cm) and a focal length of 8 feet (2.44 m). This, one of the oldest of the Kodaikanal telescopes, was brought to the site in 1898 from Madras and had been in use since 1901, with an enlarged 8-inch diameter image, for visual observations of the solar disc. In 1912 the original objective was replaced by John Evershed with a Cooke photo-visual objective of the same aperture, and it was installed in its present location and adapted for direct photography, using an 8-inch diameter solar image in addition to the visual observations.

Photography of the sun continued regularly with this setup until June 1915, when this telescope was dismantled and the objective and auxiliary optical components were moved to Kashmir, where John Evershed used them for solar photography during 1916. The telescope was reinstalled at Kodaikanal and regular observations as before were resumed starting in February 1917. In June 1918, the 6-inch photo-visual lens, used until that time, was replaced by a visual achromat of the same diameter and focal lengh, and a green filter was also added to the telescope. This gave very good quality images of the Sun, and regular photography was resumed. Since then, the photoheliograms have been obtained with this telescope using the same procedures upto the present time.

The camera used to photograph the solar image has a focal plane shutter which is a metal plate in the form of a sector with a filter mount on it for mounting a broad-band filter. The shutter is activated by releasing a metal spring, and the shutter then slides across an aperture, providing an exposure with a duration of about 0.001 sec. In 1975 the availability of plates (Ilford special Lantern, 10x10-inch size) became irregular and these were replaced by high-contrast film of the same size.

Generally, the observations are made in the morning when the seeing is the best for the day as in all hill stations. Even during the monsoon months of June, July, August, October and November the early mornings may be clear for a very short duration. On occasions when the mornings are overcast, the photograph is obtained on some part of the day when it clears or on some occasions through intermittent gaps in the clouds. Thus the observational data consist of the daily white light photograph of the full solar disc. The image has a diameter of 8 inches (20cm).

This continuous data of Kodaikanal from 1904 to the present day form the largest data next to Greenwich observatory whose data date back to 1874, but the programme of solar photography at Greenwich was stopped in 1976. Thus the Kodaikanal data form a valuable treasure for a study of the kind that has been undertaken here. For the present study white light images for the years 1906 to 1987 were measured. There are intermittant gaps in the data in the years 1904 and 1905 and hence these were not included for measuments. The plates/films containing the images are stored individually in paper envelopes, and they have been carefully preserved over the years in the plate vault at Kodaikanal under good conditions for preservation of the materials. The observing logs for each day of the observations are also well preserved.

2.3 Measurement Procedure

2.3.1 General

Sunspots generally appear in groups. A group may contain two or several spots. Individual small spots may last a day or even less, big spots and spot groups may last for few days to months. Due to solar rotation, the long lived spots transit the disc two or three times in the course of their lives. It is with such long lived spots as tracers, that we can study the solar rotation. Spot areas range from about 2000 km to as high as 40,000 km or so. The dark centre of a spot is called the umbra and umbrae have typical diameters ranging from 10,000 to 20,000 km. The umbral intensity is about 15% of the photosphere. The penumbra surrounds the umbra like a moat and consists of fibril structures or filaments that are typically 5000 to 7000 km long and 300 to 400 km wide. Sometimes, in large spots the penumbra is incomplete; pores are small spots with no penumbra and have typical diameters ranging from 1″to 5″or 700 to 4000 km. Sunspots at the beginning of the solar cycle, appear first at high latitudes of about 30° - 35° in the north or south and the zone of appearance of sunspots gradually migrate towards the equator as the solar cycle progresses. The distribution of sunspots in latitude in the course of a sunspot cycle is described by the butterfly diagram. Thus measuments on spot positions will yield solar differential rotation rates.

2.3.2 Digitiser

The positions and areas of sunspots from the photoheliograms were measured using a Calcomp digitiser. The digitiser consists of a translucent pad of area 24-inch x 36-inch, illuminated from below and where digitising can be done. The digitiser frame contains a network of wires in the horizontal (x) and vertical (y) directions and an internal controller which is used to locate and identify coordinate data pairs, relative to the digitiser origin (point 0.0). This local origin is located at the lower left hand corner of the pad. The coordinates of any point on the pad within the illuminated area (also called as the active area) of the translucent pad, is measured with reference to the origin using a hand-held cursor which forms part of the digitising pad. This cursor has etched on it two very fine hairs at right angles to each other forming a cross hair. These hairs are etched on the bottom of the lighting lens for accurate point selection. It also has 16 buttons which act as a "user definable" key board. To measure the coordinates of any point on the pad, the intersection of the cross hair is placed over that point and one of the buttons (which is assigned this function) is pressed. Then the cursor gives an output which are x and y representing the position of that point directly below the intersection of the cross hair with reference to the local origin. The digitiser has a resolution of 1/1000 inch or 1/50mm. i.e., 0.001 inch or 0.02 mm. With a solar image of 8-inches in diameter, this would correspond to $\approx$ 0.2 arc sec. on the image.

The calcomp digitiser delivers its output to a personal computer (PC/AT 286). The output from the digitiser are stored in the PC and the data are finally transferred on to floppy discs. Both the digitiser and the computer were installed in a room close to the plate vault with adequate protection for stablising the temperature. The temperature and humidity within this room were monitored with a thermometer and a hygrometer.

2.3.3 Preliminaries

The measurement of a photoheliogram plate was done as follows.

Every photoheliogram of Kodaikanal has the image of a thin wire across the solar image. The wire is kept streched within the telescope in the focal plane. This image represents the East - West direction on the sky. The centre of the solar image as well as the position of the solar equator which serve as the reference for all measurements of the sunspsots, are fixed with the help of a template. The template is made of a thin cellophane film and has a circle of nearly 8-inches diameter representing the image of the sun. On this circle are marked the two orthogonal directions representing the East - West and North - South directions starting from the North and also every 45° positions along the circumference. The template is kept firmly fixed on the translucent pad by a thin cellophane tape.

The photoheliograms of each year were cleaned and numbered. The first step in the measurement procedure was to feed from the log book of that year information on plate serial number, date, time of the observation, seeing index and sky condition index. Since the temperature and pressure values at the time of observation were not available annual mean values of these two parameters for Kodaikanal were used. A programme calculates the 'P' angle for each day and visually displays the 'P' angle value on the terminal screen. 'P' is the position angle of the northern extremity of the axis of rotation, measured eastwards from the north point of the disc. This value was used in the calculations and also for aligning the photoheliogram plate with reference to the template on the digitiser pad.

The diameters along the East - West and the North - South directions on the template were measured using the cursor. This was done by placing the cursor successively on the East and West positions and the North and South positions on the circumference of the circle on the template. These diameters designated as δ x and δ y are fed to the computer and used in the programme to determine the centre of the circle on the template and inturn of the solar image. With respect to the centre, the positional coordinates of any point on the image can be calculated further.

To make measurements, the photoheliogram plate was placed on the illuminated digitiser pad so that the emulsion faces the person making the measurements. This eliminates the refraction errors in the measurements caused by the thickness of the glass plate. The plate was now aligned with respect to the template in such a way that the E - W line on the photoheliogram (represented by the image of the wire) coincided with the E - W reference line of the template. Now the photoheliogram plate was rotated about the centre through the 'P' angle with reference to E - W on the template. This was cross checked with the help of the sunchart (i.e., the chart on which the heliographic coordinate grid is printed) on which the sunspots are sketched. As per the practice at Kodaikanal the same observer who obtains the photoheliogram also makes the sketch of the sunspots on the sunchart pertaining to that day immediately after obtaining the photoheliogram. Thus for every photoheliogram there exists a corresponding sunchart with sunspots sketch on it in the plate vaults of Kodaikanal.

2.3.4 Measurements

Now the plate is ready for making measurements. The measurements consisted of two parts, i) measurement of solar diameter on the plate and ii) measurements of positions and areas of sunspots. The hand held cursor was placed successively at every 45° positions on the solar limb starting from the North through West. This gave eight, x and y values. These were paired and the horizontal and vertical diameters and the central meridian were defined. This procedure also defined the precise position of the solar disc in the coordinates of the digitising pad.

To measure the position and area of a sunspot, the cursor crosswire was aligned at two points A and B (Figure II.1), such that the area included in the quadrilateral formed by the two orthogonal cross hair positions represented the area of the umbra of the spot. The cross hair at A and B, was positioned by me to equal the spot area as best as I could estimate it. The cursor cross hair was aligned such that one of the hairs of the cross hair pointed to the centre of the solar disc. The mean of positions A and B gave the position of the spot in the digitiser coordinate system. As shown in Figure II.1 the area enclosed by the quadrilatral, gives the area of the umbra of the spot. The penumbral areas of the spots were not measured as the boundary of the penumbra cannot be decided as unambigously as in the case of umbra. The penumbrae contain much less magnetic flux than in the umbrae and the fast changing inclination of the lines of force makes the penumbra - photosphere interface uncertain. From the practical point of view in measurements of this kind the decision of the boundary is based on the contrast with which the boundary can be recognised. The contrast between the penumbral region and the photosphere being far less than that between the umbra and the penumbra, the boundary of the penumbrae cannot be fixed unambigously. Even moderate seeing conditions can bring down the contrast further causing more uncertainty in fixing the boundary. Occasionally spots have been seen with penumbra only on one side. Addition of the penumbral area in such cases would add to errors. For these reasons it was decided to measure the areas of umbrae only and use them for further analysis.

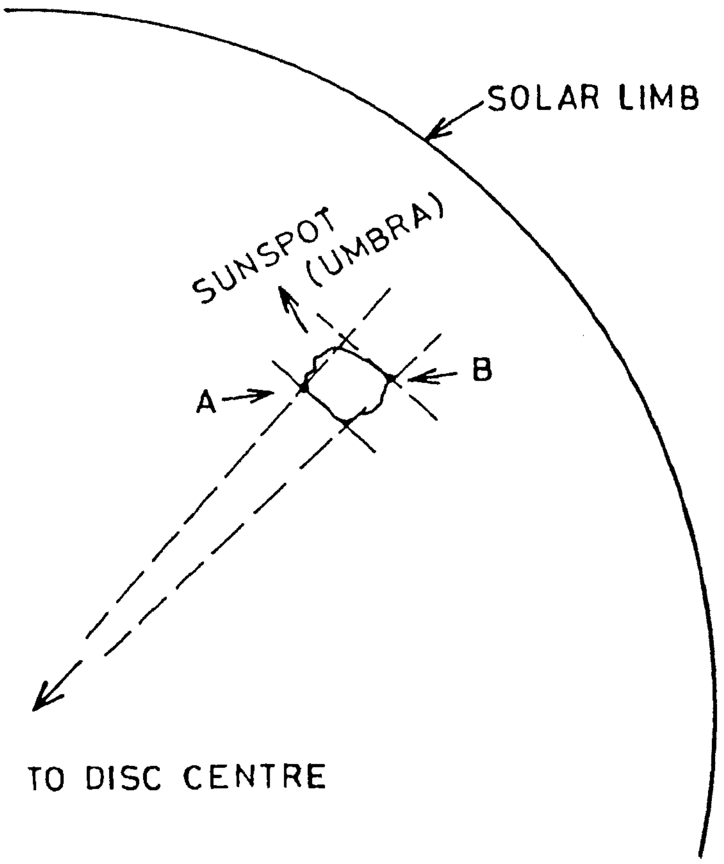


Fig.II.1 Schematic representation of the technique of measuring sunspot positions and areas. A and B are the two successive positions of the cross-hairs of the handheld cursor enclosing the umbra of the spot. One arm of the cross-hair points to the center of the solar disc. The area of the spot umbra is approximately the area enclosed by the cross-hairs.

From the log book it is seen that these photoheliograms have been obtained when the seeing was $\sim 1 - 2$ arc sec. The 0.02 mm positional accuracy of the digitiser corresponds to ≈ 0.2 arc sec. on the 200 mm diameter Kodaikanal solar image. But this accuracy will be limited to 1 - 2 arc sec. by the seeing conditions at the time of observation. This accuracy further depends on the shape and size of the spot. For Mt. Wilson white-light images of 170 mm diameter, Howard, Gilman and Gilman (1984) estimated an error of about 30% in individual small spot area measurements. Our white-light images have a diameter of about 200 mm, and the digitiser has an accuracy nearly five times that used by Howard and so the random errors in the measured area of smallest spots may be far less than 30%.

The software for data acquisition and reduction procedure were kindly made available by Dr.R.F.Howard, of the NSO, Tucson, USA for the present study. The programmes were modified to the extent necessary to accommodate the date, time of observations, the size of the solar image and the latitude of Kodaikanal.The programmes after these modifications, were thoroughly tested by measuring the photoheliograms of four solar maximum years (i.e., nearly 1200 plates or nearly 120,000 positions measurements, assuming 50 spots on each plate). The areas of umbre of spots were independently measured with the grid which is used regularly by all observers for reporting the areas of spots. The agreement between the digitised areas and the grid values was very good. In addition few years' measurements were repeated to check repeatability and internal consistency in the measurement.

Once these were established the measurements of all the photoheliograms of the years 1906-1987 were started. Spots within longitudes 60° E and 60° W were only measured, to avoid the large sec. θ correction for the areas for the spots beyond these limits. The measurements within a year were made in the chronological order; but the order in which the years were measured were made intentionally random to minimize the bias in the measurements. In all, 18,888 photoheliograms covering the 82 year period (1906 - 1987) were measured and the positions and areas of 3,32,620 spots and spot groups were measured from these photoheliograms.

2.4 Reduction Procedure

The eight limb position measurements were corrected along a vertical circle for atmospheric refraction, using terms accurate to better than 0.1" (Saastamoinen, 1979) down to a few degrees from the horizon. Standard values were assumed for the temperature and pressure at the time of observations required for the refraction calculations as these were not recorded in the log book. Using a least squares solution for the circle, the values of the radius and centre position of the disc were derived. The same refraction correction procedure was applied to each one of the spot measurements. From this the spot area in millionths of the solar hemisphere, spot central meridian distance and solar latitude corresponding to the time of the observations were determined for each spot for each day.

To find the spot returns on the next day the first step was to group the measured spots into appropriate spot groups. A spot was included in a group if it lay within 3° in latitude and 5° in longitude of another spot in the group. These criteria being arbitrary might have at times, grouped spots which would have been grouped differently by visual estimates. To verify this method of grouping, two years' data (600 observations or nearly 30,000 spots measurements) were measured and the results were found to be very nearly the same as those from visual grouping at Kodaikanal.

Having classified the spots into groups, the return of the group on the following day was recognised if both the area weighted central meridian distance and latitude were within 3° of the previous day's position. After this identification of the returned group, the next step is to calculate the rotation rate. This rotation rate is simply the central meridian distances expressed in degrees for the two consecutive days divided by the time elapsed between the two observations. If the time lapsed between the two consecutive observations was not exactly one day or twentyfour hours, then these rates were further converted to degrees/day. From these, the rotation rates of the spot groups were computed using the synodic rotation rate (Howard, Gilman and Gilman 1984).

$$\omega(\phi) = 13.44 - 3.13\ Sin^2\ \phi\quad deg/day\ ----(3)$$

where ϕ is the heliographic latitude.

Now, to identify the individual spot returns from one day to the next day, the following criteria was adopted. By using the above eq.(3), the expected position of the spot for the next day was calculated. At this stage, if the spot in question fell within three 1° steps in latitude and five 1° steps in longitude, it was taken as the return of the same spot. To find the number of returned spots, we adopted to call the spots as returned on the next day if it lay within 1/2° in longitude and latitude. With this method if the group showed maximum number of spots within the group as returned on two consecutive days, then these spots were taken as spot returns. From the differences in the central meridian distances of the spot and time elapsed between the two days, the spot rotation was calculated.

This procedure allowed us to determine the synodic rotation rate which is independent of assumed rotation rate. From the daily latitude positions of each spot, the latitude drift was also calculated. The above criteria adopted for recognising the spot group and indivudual spots within the group, is arbitrary and so might have omitted some of the real spot returns. But these numbers would be insignificant.

The 82 years' data (1906 to 1987) were contained in a total of 18,888 plates (taking on an average 230 plates per year) and this involved 1,96,000 limb position measurements and nearly 6,65,240 spot position (332,620 spots x 2 positions per spot) measurements. From the adopted group identification, a total of 78,498 spot groups were found. 41,537 groups contained returned spots out of 43,603 returned groups. A total of 122,426 spots were found to be returned out of 332,620 spots. The "spots" means spot measured on a single day. One spot would be counted more than once as it rotates across the disc.

The synodic rates of spot groups and spots were converted to sidereal rates, after applying appropriate corrections for the varying speed of the earth in its orbit during the year and for the varying inclination of the solar rotation axis. The presentation and the discussion of the results on the rotation rates in the following chapter refer to the sidereal rotations calculated in the way narrated above.

CHAPTER III

SOLAR ROTATION AND DIFFERENTIAL ROTATION MEASURED FROM KODAIKANAL PHOTOHELIOGRAMS

3.1 Introduction

In this chapter, the results of the analysis of the 82 years' data will be narrated. The sidereal rotation rates were derived for three different sizes of spots (< 5 millionths; 5 - 15 millionths and > 15 millionths of the hemisphere) and for spot groups, separately and for latitudes fom 30° N to 30° S in 5° bins. The rotation rates are seen to depend on the size of the spot used as the tracer: the rotation rates are highest with small spots and lower with larger spots and still lower with spot groups. Apart from this the meridional motion on the solar surface has been measured from the proper motions of the spots. Finally, the solar diameter measurements from the solar images are presented.

3.2 Results

3.2.1 Rotation Rates

The sidereal rotation rates have been derived for the following:

(i) Spot groups
(ii) All spots (spots of all sizes/areas i.e., spots in the three groups: < 5, 5 - 15 and > 15millionths all put together)
(iii) Spots of areas < 5 millionths of the visible hemisphere
(iv) Spots of areas 5 - 15 millionths of the visible hemisphere
(v) Spots of areas > 15 millionths of the visible hemisphere

For each of the above five classes, the rotation rates in every 5° latitude bins such as $0 - 5°$, 5° - 10° and so on up to 25° - 30° (6 bins for each hemisphere) were computed. All spots that were beyond 30° latitudes were classified into a 7th latitude bin as $> 30°$ N or $> 30°$ S. Thus we have seven latitude bins in the northern hemisphere and similarly seven in the southern hemisphere.

3.2.1.1 Spot groups and all spots

Table III . 1 gives the rotation rates in degrees per day in the 14 latitude zones for the spot groups, all spots and spots in the three area intervals from the entire measurements covering 82 years. In Figure III. 1, are plotted these rotations rates for spot groups and for all spots. Also shown in the same figure, is the rotation vs latitude curve from the Mt. Wilson Doppler rotation results for comparison. It can be seen that the rotation rates in all latitude zones from all spots are faster by 0.75% than the rotation rates of spot groups and this difference is most in the equatorial zones. Whereas the rotation rates of the photospheric plasma (i.e., the rotation from the Doppler measurements) are much slower than even the rotation rates of spots groups. This difference is pronounced in latitude (~ 2.7% slower) belt 10° N to 10° S and beyond this limit the rotation rates of spot groups and spots fall steeply and these narrow down the differences progressively to a value of ~ 1.9% at 30° latitude.

A least squares fit to the standard formula for the siderial rate,

$$\omega(\phi) = A + B \ Sin^2 \ \phi$$

to the spot groups and for all spots were done.

For the spot groups,

$$\omega(\phi) = 14.528 \ (\pm 0.007) - 3.098 \ (\pm 0.067) \ Sin^2 \ \phi \ \ deg/day$$

and for all spots,

$$\omega(\phi) = 14.601 \ (\pm 0.003) - 2.872 \ (\pm 0.030) \ Sin^2 \ \phi \ \ deg/day$$

The standard deviation refer to the mean values.

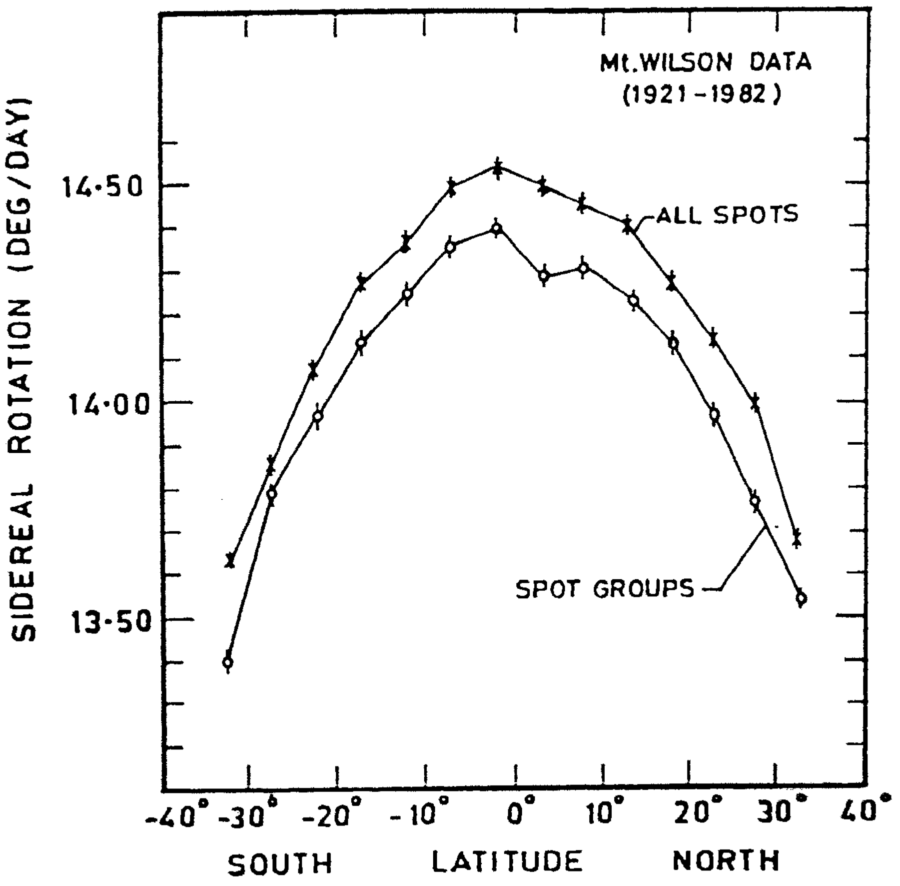
3.2.1.2 Comparison with earlier measurements

As mentioned earlier, the Greenwich photoheliographic data cover a long period (1874 - 1976) and give the daily positions of the centres of spot groups. These positions are measured with the help of a grid of heliographic coordinates placed over the solar image and the position of the centre of a spot group is read

Table III.1

Equatorial rotation rates in deg./day from spot groups from spots of areas < 5 millionths; 5-15 millionths and > 15 millionths of the visible hemisphere and from all spots put together in 5° latitude bins.
Kodaikanal data 1906-1987

		SOUTH					LATITUDE(In Deg.)				NORTH				
		> 30	30-25	25-20	20-15	15-10	10-05	05-00	00-05	05-10	10-15	15-20	20-25	25-30	> 30
Sunspot groups	Rotation Rate	13.63	13.93	14.11	14.26	14.38	14.49	14.53	14.48	14.44	14.36	14.23	14.11	13.84	13.54
	Standard Deviation	0.035	0.027	0.019	0.014	0.014	0.015	0.027	0.024	0.014	0.013	0.014	0.017	0.026	0.037
	No. of groups	649	1331	2719	4988	5592	4448	1229	1418	4603	5958	4807	3537	1560	764
All Spots	Rotation rate	13.77	14.05	14.23	14.37	14.47	14.58	14.59	14.61	14.55	14.42	14.33	14.16	13.98	13.69
	Standard Deviation	0.022	0.014	0.009	0.006	0.006	0.007	0.012	0.011	0.006	0.005	0.006	0.007	0.012	0.020
	No. of spots	1378	3102	6868	13034	16295	11969	3341	4179	13902	18444	14166	10066	4123	1559
Spots area < 5 millionths	Rotation rate	13.82	14.08	14.27	14.40	14.50	14.61	14.62	14.65	14.59	14.45	14.35	14.18	14.01	13.70
	Standard deviation	0.028	0.017	0.011	0.008	0.007	0.008	0.015	0.013	0.007	0.006	0.007	0.009	0.014	0.026
	No. of spots	895	2249	5012	9728	12718	9284	2619	3361	10921	14420	10903	7735	3065	1007
Spots area 5 to 15 millionths	Rotation rate	13.70	13.99	14.16	14.29	14.41	14.49	14.54	14.47	14.44	14.36	14.26	14.12	13.92	13.71
	Standard deviation	0.038	0.022	0.015	0.011	0.010	0.012	0.021	0.020	0.011	0.010	0.011	0.014	0.022	0.034
	No. of spots	387	705	1515	2658	2906	2127	549	624	2357	3184	2606	1903	854	459
Spots area > 15 millionths	Rotation rate	13.57	13.85	13.99	14.16	14.23	14.33	14.39	14.38	14.29	14.25	14.13	13.97	13.74	13.49
	Standard deviation	0.044	0.041	0.022	0.014	0.012	0.014	0.026	0.018	0.012	0.011	0.014	0.016	0.024	0.043
	No. of spots	96	148	341	648	671	558	173	194	624	840	657	428	204	93


Mt.WILSON DATA
(1921-1982)
ALL SPOTS
SPOT GROUPS
SIDEREAL ROTATION (DEG/DAY)
14·50
14·00
13·50
-40°
-30°
-20°
-10°
0°
10°
20°
30°
40°
SOUTH
LATITUDE
NORTH

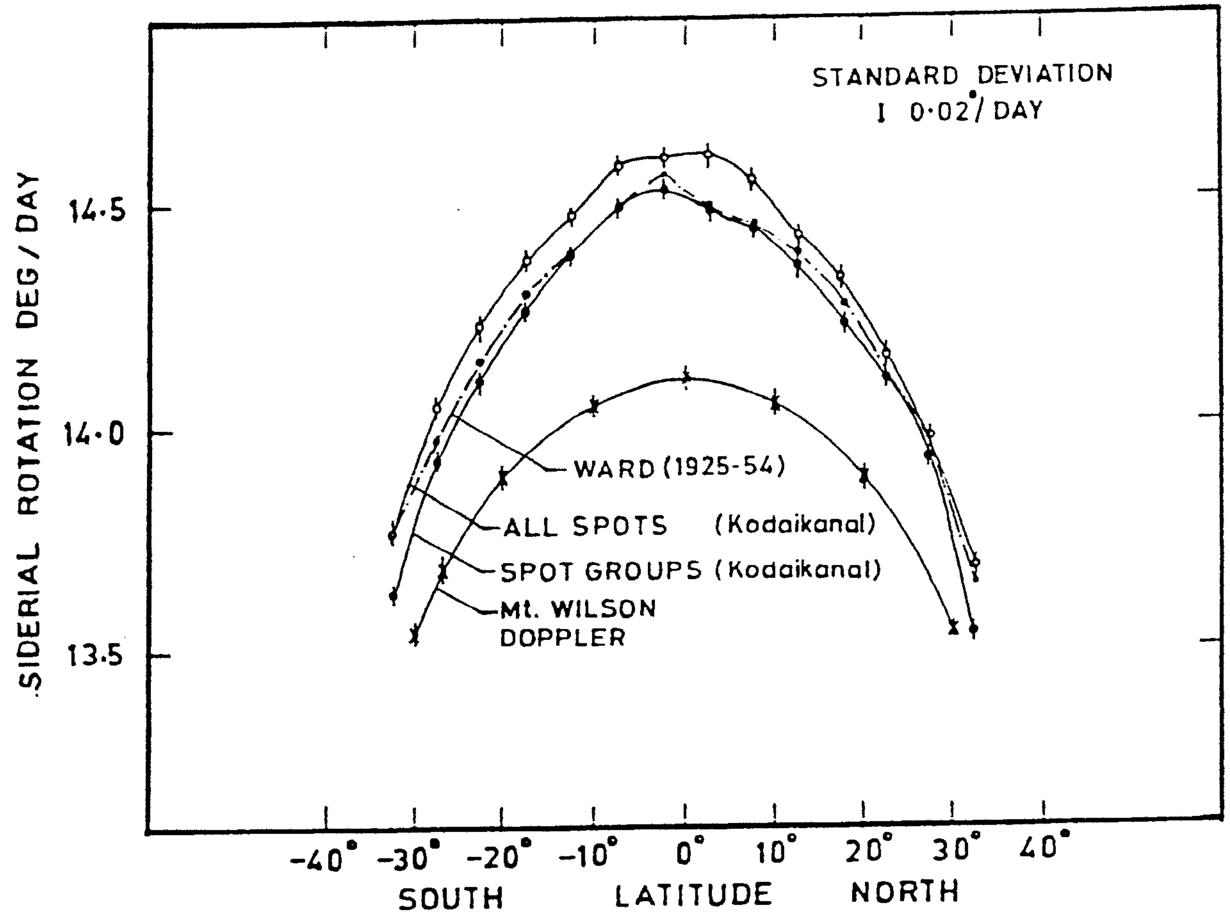


Fig.III.1 Rotation rates from spot groups and all spots vs latitude in the north and south hemisphere (Kodaikanal data for the period 1906-1987). The rotation rates from spot groups of the Greenwich data 1925-1954 (Ward, 1965) are plotted for comparison. Also plotted is the smoothed rotation rates from the Doppler data of Mt. Wilson for the period 1967-1982 (Howard et al 1983). The rotation rates from Mt. Wilson data (Howard, Gilman and Gilman 1984) are plotted on the transparency. This can be placed on the Kodaikanal results for comparison.

off from the grid by eye estimates. These positions have an accuracy of only 0.1 deg in heliographic coordinates. Ward (1965) has used these data and derived sidereal rotation rates. These rotation rates vs latitudes have been plotted along with the rotation rates for spot groups and all spots from the present study for easy comparison (Figure III.1). The agreement between Ward's rotation rates and ours for spot groups is good. The rotation rates from similar data (all spots and spot groups) from Mt. Wilson have also been shown for comparison (Howard, Gilman and Gilman 1984). The Mt. Wilson curves are drawn on a transparent sheet so that this can be overlaid on Figure III. 1 for comparison. It is seen that the rotation curve of Mt. Wilson for all spots is very close to our rotation curve for spot groups, whereas the Mt. Wilson values of rotation rates from spot groups are slower than ours as well as that of Ward by ~ 0.95% and ~ 1.0% respectively. In the case of all spots, our rotation curve is faster than the rotation curve of Mt. Wilson by 0.35% (or 0.05 deg./day) in the northern hemisphere and by 0.84% (0.12 deg./day) in the southern hemisphere. In Table III.2 our results are compared with those of Balthasar, Vazqucz and Wöhl (1986) who have derived rotation rates from the same Greenwich data as Ward (1966) but for the longest period possible (1874 - 1976). Since they have given only the rotation values averaged over the two hemispheres (Table 1 of Balthasar, Vazqucz and Wöhl 1986) we have also done similar averaging to make the comparison possible. These are presented in Figure III.2 along with Ward's values similarly averaged over the two hemispheres. Our rotation curve for spot group agrees fairly well with those of Ward's and Balthasar. The differences noticed should primarily be due to the different levels of accuracies in fixing the positions of spot groups by them and in our study.

3.2.1.3 Rotation rates from individual spots

Individual spots appear on the photosphere in a wide spectrum of sizes or areas and also in widely varying numbers as a function of the phase of the solar cycle. We have already seen that the rotation rates derived from all spots and spot groups significantly differ from each other. It is logical to presume that the rotation rates derived from spots of different sizes would not be the same. With this in mind all the individual spots that were measured were classified into three area groups, namely spots with areas < 5 millionths, spots with areas between 5 and 15 millionths and spots with areas > 15 millionths of the

Table III.2

Rotation velocities averaged for both hemispheres in 5° latitude bins for the period 1874-1976 and 1902-1976 (Balthasar, 1986), Ward, (1966) for the period 1905-1954 and Kodaikanal (present study) 1906-1987

Motions of more than 3 deg/day in longitude or 2 deg./day in latitude are excluded (Balthasar,1986)

Investigator	Balthasar et al (1986)	Ward (1966)	PRESENT STUDY	
Latitude (deg.)	1874-1976	1905-1954	1906-1987	
			For All Spots	For Groups
0 - 5	14.549 ± 0.012	14.52 ± 0.03	14.60 ± 0.011	14.50 ± 0.025
5 - 10	14.478 ± 0.007	14.47 ± 0.02	14.56 ± 0.006	14.46 ± 0.014
10 - 15	14.398 ± 0.006	14.39 ± 0.02	14.44 ± 0.005	14.37 ± 0.013
15 - 20	14.296 ± 0.007	14.29 ± 0.02	14.35 ± 0.006	14.24 ± 0.014
20 - 25	14.142 ± 0.009	14.14 ± 0.02	14.19 ± 0.008	14.11 ± 0.018
25 - 30	13.970 ± 0.014	13.94 ± 0.04	14.01 ± 0.013	13.88 ± 0.026

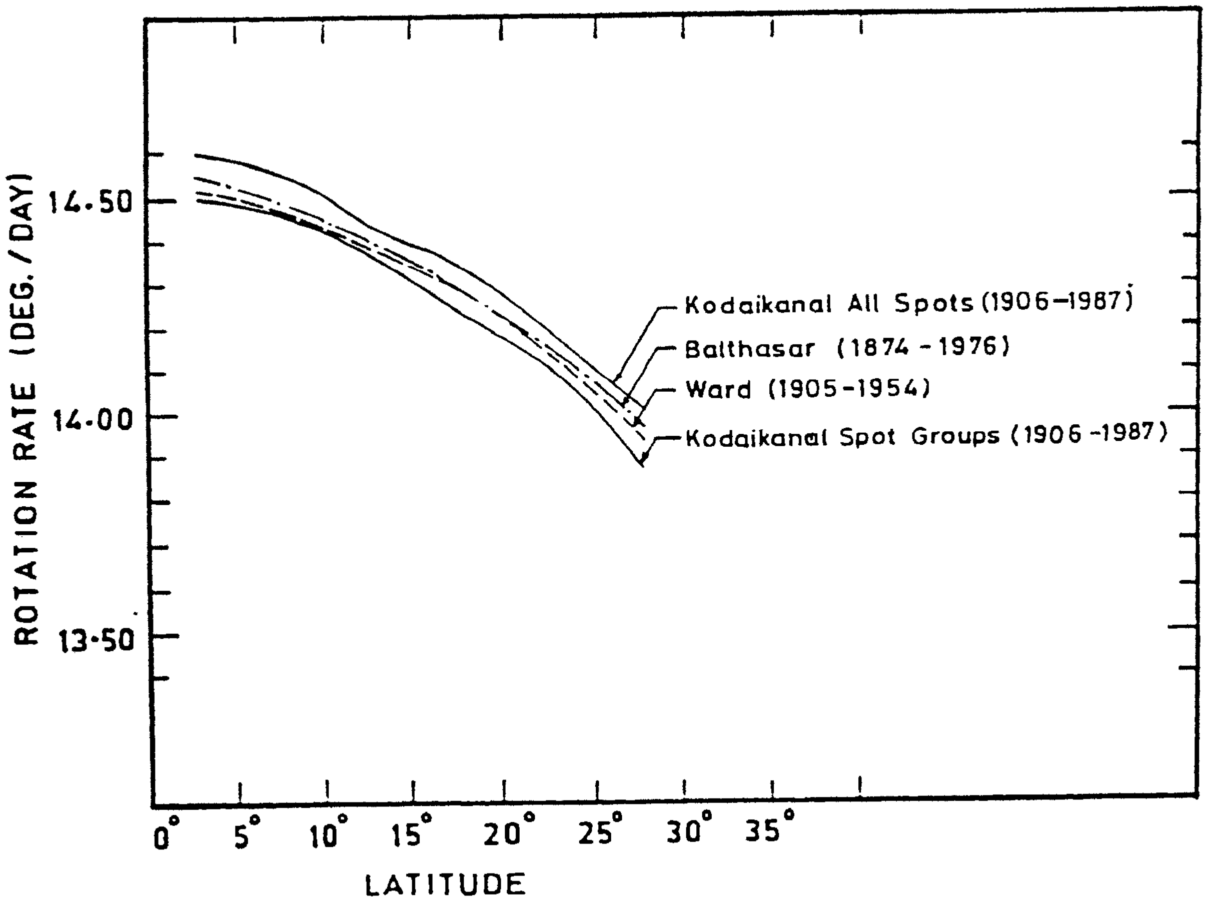


Fig.III.2 Rotation rates from spot groups and all spots vs latitude with north and south hemispheres folded (Kodaikanal data 1906-1987). Also plotted are the results of Balthasar et al (1986) from the Greenwich data (1874-1976) and Ward's results (1966) from the Greenwich data (1905-1954).

visible solar hemisphere. Qualitatively these would correspond to the small, medium and large spots.In the total of 18,888 photoheliograms measured, there were 93,917 spots in the first group ($<$ 5 millionths), 22,834 spots in the group 5-15 millionths and 5675 spots in the group $>$ 15 millionths. The rotation rates from the three groups and for each 5° latitude bins for the north and south hemispheres were computed. These rotation rates are presented in Table III.1. These are plotted in Figure III.3. It is seen that the rotation rates derived from the smallest size spots (i.e. $<$ 5 millionths) are the highest and the rotation rates from the second group (5 - 15 millionths) is lower and those from the biggest spots ($>$ 15 millionths) are the lowest. These differences are most obvious near the equatorial zones (10° N to 10° S) although in all latitudes, the rotation rates of the smallest size group are higher than those from the other size groups. On an average the rotation rates from the spots $<$ 5 millionths are higher by 0.65% than from 5 - 15 millionths group and the latter rotation rates are higher than from $>$ 15 millionths by 1.0%.

3.2.1.4 Comparison with other results

It would be interesting to compare these results with those of Howard, Gilman and Gilman (1984) who have rotation rates for three similar size groups from the Mt. Wilson data. To facilitate this comparison the rotation rates of Howard, Gilman and Gilman (1984) were drawn on a transparent paper and this can be placed over Figure III.3. The rotation rates from Kodaikanal data are higher by 0.09 degrees/day or by 0.65% than Howard, Gilman and Gilman (1984) values for the corresponding sizes. This difference is larger in the 10° N and 10° S zone. Their rotation rates from the $>$ 15 millionths size differ from ours by 0.75%. This impression is magnified to some extent by the zigzagness in their rotation curve for this size group; a smoothed curve would have shown less differences as is the case at higher latitudes. The slopes on either hemispheres for the three curves are similar showing that the differential rotation is similar for the 3 size groups. In an earlier investigation (Sivaraman, Gupta and Howard 1993) a comparison of the rotation rates was done using 36 years' measurements from Kodaikanal and Mt. Wilson. There too, the rotation rates from Kodaikanal data were higher than those by Mt. Wilson. This was explained, as possibly due to the larger number of smaller spots seen in the Kodaikanal data than in

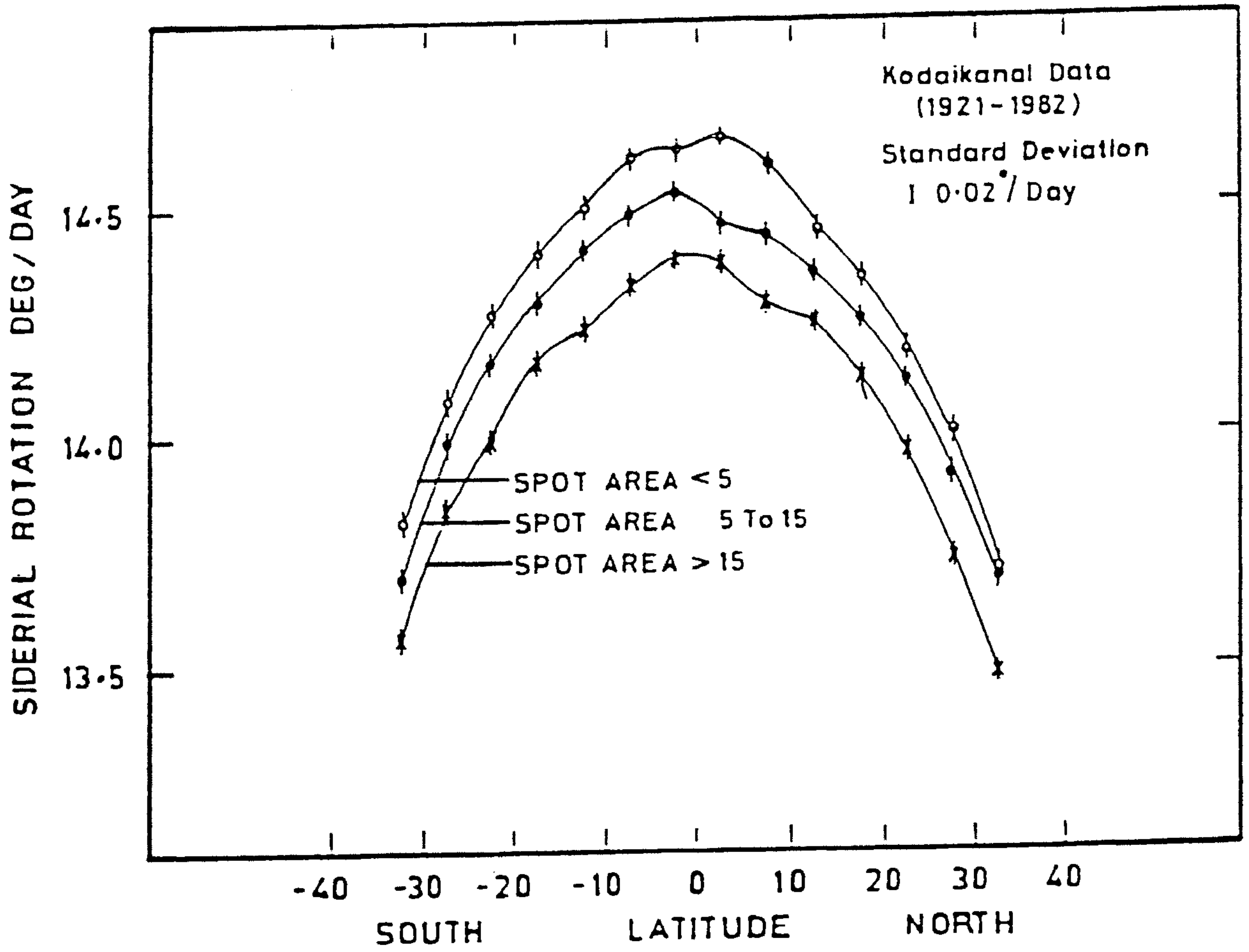


Fig.III.3 Rotation rates vs latitude in 5° bins from spots classified into 3 area groups (area < 5 millionths, 5-15 millionths and > 15 millionths of the visible hemisphere). The results from Mt. Wilson data (Howard et al 1984) are plotted on the transparency. This can be placed over the Kodaikanal results for comparison.

the Mt. Wilson and since the rotation rate from smaller spots is highest, this excess number in the Kodaikanal data could have increased the rotation rate due to higher weightage.

Indeed, the Kodaikanal data has more number of smaller spots. This is because the size of the solar image in the Kodaikanal photoheliograms is 1.2 times the value in Mt. Wilson photoheliograms and this increses the visibility and enables easier identification of smaller spots which might be on the threshold of visibility in the Mt. Wilson plates with an image of smaller size. To verify this arguement, we computed the rotation rates using the reduction procedures for four sets of Kodaikanal data: one with the spots for 62 years (1921-'82); second one with spots for 71 years (1917 - 1987); third one with spots for 74 years (1906 - 1979) and the fourth set with spots for all the 82 years (1906 - 1987). The rotation rates for the three sizes for all the latitudes in 5° bins alongwith the standard deviation are given in Tables III.3, III.4, and III.5. The total number of spots in each zone which have gone into the rotation rate determination are also given in these tables. It can be seen that the rotation rates have high degree of internal agreement. The addition of even as high as 20% of the spots to the data does not alter the rotation rate values. Thus the Kodaikanal measurements done with a larger solar image coupled with the higher ($\sim$ 5 times) least count of the digitising equipment would enable more accurate measurements and so the rotation rates which are higher are intrinsic to the sun.

3.2.2 Asymmetry in the rotation rates between the northern and southern hemispheres

3.2.2.1 Spot groups and all spots

From Figure III.1 it is seen that for the spot groups in the equatorial belt upto 10° latitude, the southern hemisphere rotates faster than the north by $\sim$ 0.3% . Beyond 10° till 35° the rotation rates in the north and the south are the same. In the case of all spots, upto 10° latitude, the rotation rate is the same in the two hemispheres but beyond 10° , the rotation rate in the south is consistently faster than in the north at all latitudes till about 35° . The differences in the equatorial belt ($0 - 10°$) between the two hemispheres would not have been so conspicous if the rotation plots had been smoothed. The

Table III.3

Sidereal Rotation (Deg./Day) derived from spots of areas < 5.0 millionths of the visible hemisphere in 5° latitude bins

Years		SOUTH							NORTH						
		> 30	30 - 25	25 - 20	20 - 15	15 - 10	10 - 05	05 - 00	00 - 05	05 - 10	10 - 15	15 - 20	20 - 25	25 - 30	> 30
1906-'87 (82 years)	Rot. SDS. No.of spots	13.82 .03 895	14.08 .02 2249	14.27 .01 5012	14.40 .01 9728	14.50 .01 12718	14.61 .01 9284	14.62 .01 2619	14.65 .01 3361	14.59 .01 10921	14.45 .00 14420	14.35 .01 10903	14.18 .01 7735	14.01 .01 3065	13.70 .02 1007
1906-'79 (74 years)	Rot. SDS. No. of spots	13.83 .03 811	14.06 .02 2043	14.26 .01 4672	14.41 .01 8718	14.50 .01 11339	14.61 .01 8217	14.61 .01 2347	14.65 .01 2903	14.59 .01 9913	14.45 .00 13428	14.35 .01 10256	14.18 .01 7380	14.02 .01 2959	13.70 .02 995
1917-'87 (71 years)	Rot. SDS. No. of spots	13.82 .03 891	14.07 .02 2190	14.26 .01 4650	14.41 .01 8804	14.50 .01 11645	14.61 .01 8491	14.62 .01 2315	14.64 .01 3106	14.59 .01 9796	14.45 .00 13273	14.36 .01 10280	14.18 .01 7402	14.01 .01 2975	13.70 .02 1004
1921-'82 (62 years)	Rot. SDS. No. of spots	13.82 .03 860	14.07 .02 2042	14.27 .01 4081	14.40 .01 7567	14.49 .01 10105	14.60 .01 7675	14.61 .02 2075	14.64 .01 2815	14.59 .01 8821	14.45 .01 12155	14.36 .01 9433	14.17 .01 7054	14.01 .01 2869	13.70 .02 999

Table III.4

Sidereal rotation (Deg./day) derived from spots of areas 5 to 15 millionths of the visible hemisphere in 5° latitude bins

Data Period		SOUTH							NORTH						
		> 30	30-25	25-20	20-15	15-10	10-05	05-00	00-05	05-10	10-15	15-20	20-25	25-30	> 30
1906-87 (82 years)	Rotation	13.70	13.99	14.16	14.29	14.41	14.49	14.54	14.47	14.44	14.36	14.26	14.12	13.92	13.71
	Standard deviation	0.038	0.022	0.015	0.011	0.010	0.012	0.021	0.020	0.011	0.010	0.011	0.014	0.022	0.034
	No. of spots	387	705	1515	2658	2906	2127	549	624	2357	3184	2606	1903	854	459
1906-79 (74 years)	Rotation	13.75	13.97	14.15	14.28	14.41	14.50	14.54	14.48	14.44	14.36	24.26	14.12	13.92	13.71
	Standard deviation	0.040	0.023	0.016	0.012	0.011	0.013	0.021	0.022	0.011	0.010	0.012	0.014	0.022	0.034
	No. of spots	349	655	1430	2414	2543	1918	504	539	2173	2924	2413	1839	813	454
1917-87 (71 years)	Rotation	13.71	13.99	14.16	14.29	14.41	14.49	14.56	14.47	14.44	14.36	14.26	14.12	13.92	13.71
	Standard deviation	0.038	0.023	0.015	0.011	0.010	0.012	0.022	0.021	0.011	0.010	0.011	0.014	0.022	0.034
	No. of spots	386	700	1450	2585	2784	2021	485	603	2256	3090	2535	1844	826	459
1921-82 (62 years)	Rotation	13.71	13.99	14.16	14.28	14.41	14.50	14.55	14.47	14.45	14.37	14.27	14.12	13.92	13.71
	Standard deviation	0.039	0.023	0.016	0.012	0.011	0.014	0.023	0.022	0.012	0.010	0.012	0.014	0.022	0.034
	No. of spots	376	678	1349	2302	2475	1737	446	538	1997	2820	2277	1788	806	455

Table III.5

Sidereal rotation (Deg./day) derived from spots of areas
> 15 millionths of the visible hemisphere in 5°
latitude bins

Data Period		SOUTH							NORTH						
		< 30	30-25	25-20	20-15	15-10	10-05	05-00	00-05	05-10	10-15	15-20	20-25	25-30	> 30
1906-87 (82 years)	Rot. SDS No. of spots	13.57 0.044 96	13.85 0.041 148	13.99 0.022 341	14.16 0.014 648	14.23 0.012 671	14.33 0.014 558	14.39 0.026 173	14.38 0.018 194	14.29 0.012 624	14.25 0.011 840	14.13 0.014 657	13.97 0.016 428	13.74 0.024 204	13.49 0.043 93
1906-79 (76 years)	Rot. SDS No. of spots	13.59 0.048 86	13.85 0.043 132	13.98 0.023 329	14.16 0.015 584	14.23 0.013 582	14.32 0.014 515	14.38 0.025 165	14.38 0.020 161	14.29 0.012 558	14.25 0.012 767	14.13 0.015 605	13.97 0.016 424	13.74 0.026 194	13.49 0.043 93
1917-87 (71 years)	Rot. SDS No.of spots	13.57 0.044 96	13.85 0.041 148	13.99 0.022 340	14.16 0.015 615	14.24 0.012 641	14.33 0.015 520	14.37 0.029 148	14.38 0.018 191	14.30 0.012 595	14.24 0.011 806	14.14 0.014 636	13.97 0.016 422	13.73 0.024 201	13.49 0.043 93
1921-82 (62 years)	Rot. SDS No. of spots	13.58 0.047 89	13.86 0.040 146	13.98 0.023 318	14.15 0.015 549	14.23 0.013 587	14.33 0.016 454	14.37 0.028 131	14.38 0.019 158	14.30 0.013 501	14.24 0.012 747	14.13 0.014 599	13.96 0.016 419	13.73 0.024 197	13.49 0.043 93

rotation rate plots of the Mt. Wilson data (Howard, Gilman and Gilman, 1984) show that for the spot groups, the rotation rate for the south is higher in the latitude $0 - 10°$ and beyond 10° the rotation rates are nearly the same in the north and south showing that there is no N - S asymmetry. This is the same as our results. Whereas in the case of all spots the rotation rates derived by them are the same for the N - S hemispheres in the latitude belt $0 - 15°$; but beyond 15° till 35° , the rotation rate for the north is higher than for the south. This is the opposite to what we have observed.

Hathaway and Wilson (1990) using the Mt. Wilson data for all spots of Howard, Gilman and Gilman (1984) have shown that the southern hemisphere rotate significantly faster than the northern hemisphere. In Table III.6 the A and B values for the northern and southern hemispheres are presented to show the faster rotation rates in the south for all spots whereas the rotation rates from spot groups are the same in both the hemispheres.

3.2.2.2 Individual spots of the three area groups

Figure III.3 shows the rotation rates for all the latitude bins separately for the three size groups. In the case of the spots of the first two classes (< 5 millionths and 5 - 15 millionths) the rotation rates for the southern hemisphere is faster than the northern hemisphere on an average by $\sim$ 0.3% or by 0.04 deg/day. In the case of large spots (area > 15 millionths) the asymmetry is not significant. The rotation rate plots of Howard, Gilman and Gilman (1984) show that for all the three size groups (< 5, 5 - 15 and > 15 millionths) the rotation rates are higher in the north compared to the south beyond 10° latitude. This is again, the opposite of what we find from our data. Recently, Ribes, Ferreira and Mein (1993) have calculated the rotation rates in 5° latitude bins using sunspots from the Meudon (Paris) observatory $Ca^{+}K1$ spectroheliograms. Although this study covers only a limited period of solar cycle 21 (1977-84) the accuracy of their measurements on individual spots would make the results worthy of comparison with ours. They however do not have the rotation rates for the different sizes of spots and have not given any tables of the rotation rate values for the 5° latitudes and their figure showing the rotation rate plot vs latitude (their Figure 2, Ribes, Ferreira and Mein 1993) is drawn on such a small scale, the rotation rates cannot be read off to the accuracy needed for a direct comparison with our

Table III.6

Coefficients A and B for spot groups and all spots (< 5, 5 - 15, and > 15 millionths put together) for north and south hemispheres from Kodaikanal data for the period 1906-1987

		South hemisphere	North hemisphere
Spot groups	A	14.542(± 0.010)	14.513(± 0.010)
	B	−3.061(± 0.098)	−3.123(±0.091)
All spots(< 5, 5-15 and > 15 millionths)	A	14.612(± 0.005)	14.590(± 0.004)
	B	−2.729(± 0.046)	−2.979 (± 0.040)

results. However, they have compared their rotation rates with those of Howard, Gilman and Gilman (1984) and so we can make a qualitative comparison of our results with those of Ribes, Ferreira and Mein (1993) via the results of Howard, Gilman and Gilman (1984). There too, the rotation rates are higher than those found by Howard, Gilman and Gilman (1984) particularly for the southern hemisphere and between the two hemispheres, the rotation rates for the south are higher at least upto latitude ~ 30° , (Ribes, Ferreira and Mein 1993).

3.2.3 Comparison of rotation rates from spots and from Doppler measurements

In Figure III.1 the smoothed values of the rotation rates from the Mt. Wilson Doppler measurements (Howard and Gilman, 1984) have also been plotted. These are much lower than those for all spots and spot groups. Considerable thought has been given to explain this large difference in the rotation rates. One important point that should be kept in mind while seeking to explain this difference is that the rotation rates from Doppler measurements of spectral lines pertain to the rates of flow of the solar plasma in the layers where these spectral lines originate, whereas the spots which are anchored to different depths in the subsurface layers through the magnetic lines of force, cannot be imagined to yield the same values of rotation rates. They in their motion, through the solar plasma would experience a drag which again would depend on their sizes (areas), shapes and the topology of the magnetic field lines associated with them. Clearly, the photospheric plasma cannot have the rotation rate of the spots.

Again, even within the individual spot class, we have three sub divisions namely small spots (< 5 millionths) medium size spots (5 - 15 millionths) and large size spots (> 15 millionths). We found (Figure III.3) that the rotation rates progressively increase as we go from the large size spots to the small size spots. Gilman and Foukal (1979) have suggested that the spots of different sizes (areas) are anchored to different depths inside the sun; the small spots being anchored to shallower depths and larger ones to deeper depths. If this is so, then the different spot sizes would yield definitely different rotation values, and none of these would match with those from the Doppler measurements.

It would be interesting to verify the hypothesis of Gilman and Foukal (1979). This has become possible now with the availability of the results of the rotation rates in the solar interior from helioseismology data. The rotation rate at the equator as a function of depth inside the sun inferred from rotational frequency splitting measured for degree $\ell = 5$ to 600 has been given by Korzennik (1990), illustrated in a paper by Gilman (1992). These rotation rates are given in nano H_z and depths in fraction of the radius of the sun. We have plotted our values of the sidereal rotation rates for the three spot size groups (< 5 , 5 - 15 and > 15 millionths) on this rotation profile and estimated the depths to which the spots of the three sizes are anchored (Figure III.4). The intersections of the horizontal lines projected from the rotation values for spots of 5 - 15 and > 15 millionths (marked as A & a and B& b respectively) would give the approximate depths at which these are anchored. The intercepts A and B are the valid ones. These depths are: for 5 - 15 millionths class: 0.9 $R\odot$ corresponding to A and for $>$ 15 millionths class: 0.81 $R_\odot$ corresponding to B; for small spots of area < 5 millionths, our rotation value does not intersect the profile and so the depth value cannot be estimated.

3.2.4 Meridional Motions

3.2.4.1 Data and reduction procedure

Meriodional motions were calculated from the latitude differences between consecutive days and the actual time interval between these two positions. The daily positions of sunspots measured for the period 1906 - 1987 for rotation calculations were used for the calculation of meridional motions. This consists of 43,603 returns of spot groups and 122,426 returns of total spots. Of the total number of spots, 93917spots were of size < 5 millionths, 22,834 spots were of size 5 to 15 millionths and 5675 spots were of size > 15 millionths. As the number of spots and spot groups for which positions were measured is much larger than any of the data measured so far and the positional measurements are of higher accuracy, we were able to arrive at the meridional motions correct to a few meters per second. The meridional drifts have been calculated for the three spot sizes, for all spots and for groups in 5° latitude zones from 0° to 30° in both the hemispheres. Spots within 60° east to 60° west only were included in the spot position measurement.

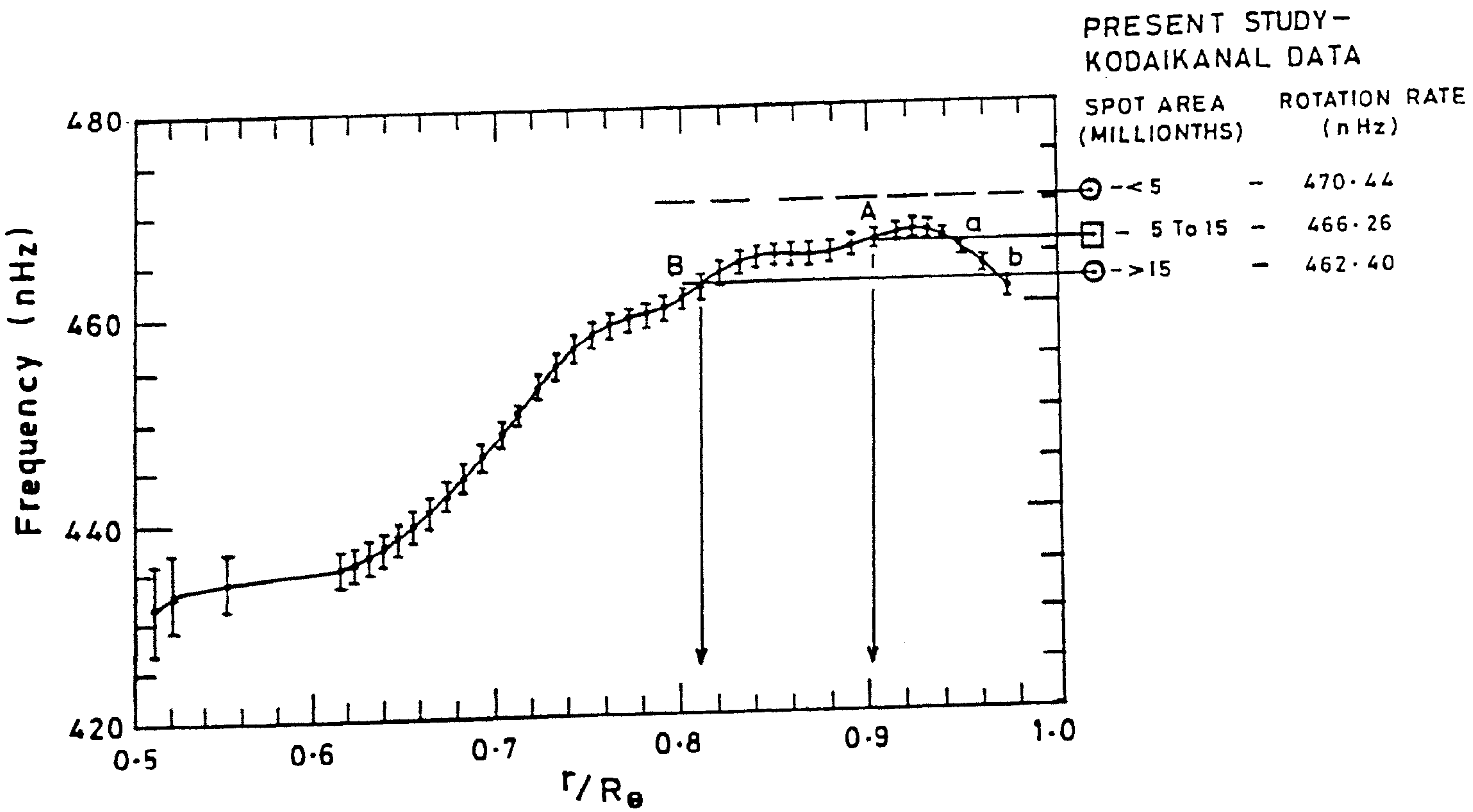


Fig.III.4 The angular velocity profile at the solar equator inferred from rotational frequency splittings measured for degrees $\ell = 5$ to 600. Rotation units along the vertical axis are in nano Hertz (nHz) and the radii values in the horizontal axis are in fractions of the solar radius $R\odot$. Results of S.Korzennik extracted from Gilman (1992). On this are superposed the rotation rates (converted into nHz) from spots of the 3 area grups of the Kodaikanal data (locations A and B) to determine the depths to which these spots are anchored.

3.2.4.2 Results

For measuring the latitude drifts of either spot groups or individual spots, the first step was to decide to which latitude zone a particular spot group or spot belonged to, to start with. One way to do this was to assign a particular group to a particular latitude zone if the area-weighted centroid of the group fell within that zone. In this study, the groups were assigned those latitude zones in which the average of the coordinates of the first and second day of the two days used for calculating the drift happened to fall. In the case of the spots, all the spots located in a latitude zone were considered as part of the group which lay in this latitude zone. These criteria being orbitrary to some extent might have included a few more spots in a particular group although these spots did not strictly fall within this group but lay in the immediate neighbourhood. Similarly a few spots would have been excluded from a group to which they should have justifiably belonged to. However since the gradient of the latitude drift with latitude is very small, such mistakes in assignments and consequent errors in latitude determinations would be insignificant.

The latitude motions of all the spots as well as of the spots in the three size groups were computed in degrees per day. These latitude motions are simply the difference between the latitude positions on two consecutive days divided by the actual time interval between the epochs at which the observations of the spots were made. If this time interval was not exactly one full day (or 24 hours) the latitude motions were converted into degrees per 24 hours or degrees per day. These latitude motions were computed for all the 5° latitude zones. The plots of latitude motions (or drifts) vs latitudes for the spot groups, for all spots (spots of area < 5 , 5 - 15 and > 15 millionths all put together) and for the three size groups individually are shown in Figure III.5.

All spots poleward of 30° have been absorbed in the zone of 30° - 35° in the Figure III.5. In northern and southern hemispheres, positive values denote poleward motion and negative values denote equatorward motion. For the all spots class in the northern hemisphere at all latitudes the meridional drift is towards the pole and in the southern hemisphere the motion is towards the equator at mid latitudes and away from the equator towards the direction of the pole at latitudes beyond 20°. The meridional motion curve for spot size

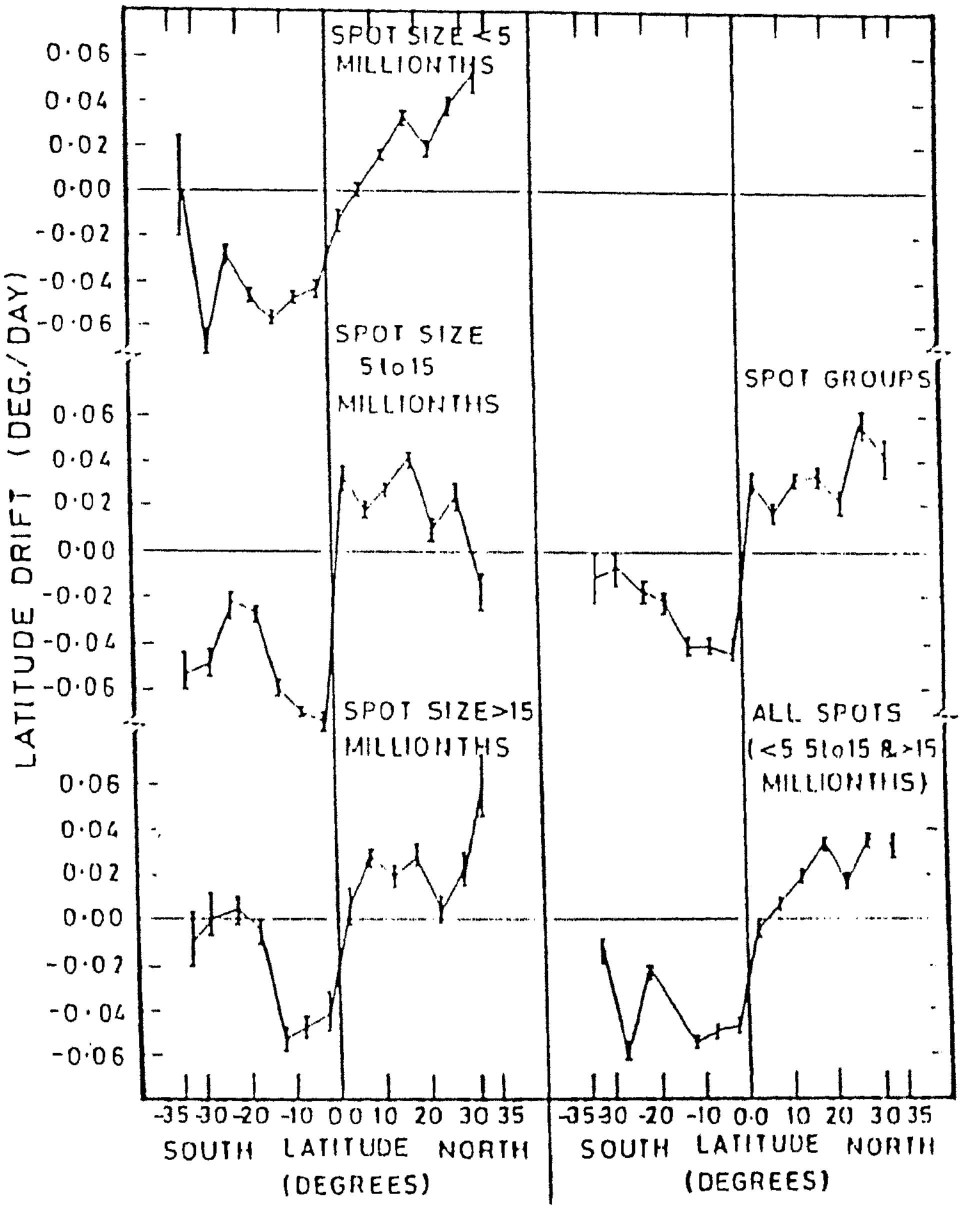


Fig III 5 Latitude motions as function of latitude measured on the Kodaikanal data (1906 1987) for individual spots of three size groups (spots of area < 5 millionths, 5-15 millionths and > 15 millionths of the visible hemisphere); for all the spots combined; and for the spot groups Positive values in drift represent poleward motion and negative values represent equatorward motion in each hemisphere.

$<$ 5 millionths, closely resembles the curve for all spots. That is, spots of size $<$ 5 millionths, which predominate in number decide the pattern of the curve for meridional motions for all spots. Spots of area 5 to 15 millionths show a meridional flow which is all poleward in the northern hemisphere, and in the southern hemisphere it is equatorward near the equator but away from the equator (representing a poleward flow) at latitudes beyond 10° . In the case of the large spots ($>$ 15 millionths) the drift is poleward in the northern hemisphere for all latitudes and in the southern hemisphere, the drift is equatorward till 20° latitude and poleward at higher latitudes. Meridional motion curve for spot groups and spots of size 5 to 15 millionths are similar.

We do not find a large positive peak in any of the five classes as was noticed by Howard and Gilman (1986). From the plots for spots of three sizes, all spots and spot groups, we find that the maximum value of meridional motion is 0.07 deg/day (or 9.8 m/sec.) and a minimum value of 0.025 deg/day (or 3.5 m/sec.). These values are in good agreement with the published results. The maximum value compares well with the mean value of the meridional drift noticed by Makarov and Sivaraman (1989) from the study of migration of $H\alpha$ filaments using Kodaikanal spectroheliograms for almost the same period as in this study.

In order to demonstrate that direction of meridional flow is also poleward in the southern hemisphere for all the classes in Figure III.5 , we have made a plot by symmetrically folding the southern hemisphere values on to the north i.e., for each latitude drift in the north and south are averaged. In the symmetrically folded plot (Figure III.6) positive values represent poleward motion. All the plots except the one for spot size 5 - 15 millionths show a poleward motion at latitudes higher than 25°

3.2.4.3 Solar cycle variations

In order to verify whether the pattern of the meridional motion with latitude shows changes between one solar cycle to another, we calculated the mean of the annual values of the meridioinal drifts for all spots (i.e., $<$ 5, 5-15 and $>$ 15 millionths put together) over each solar cycle period. These are shown in Figure III.7. We find that the general pattern of the flow is maintained from cycle to cycle. In the northern hemisphere, it is a poleward flow beyond 10° N, and

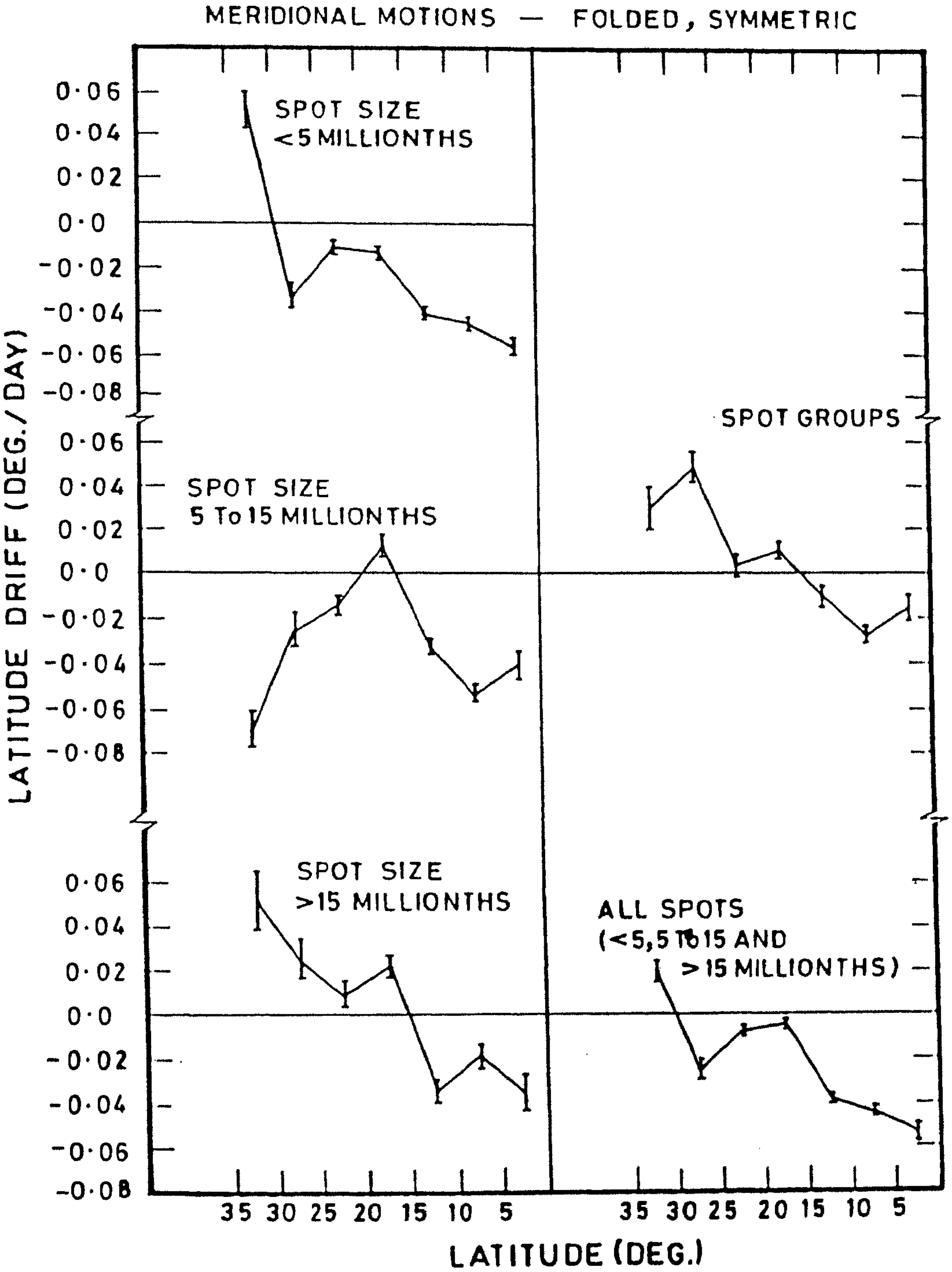


Fig.III.6 Latitude motions with the south folded over the north about the equator for spots of the 3 size groups, all spots together and spot groups. Positive values in drift represent poleward motion and negative values represent equatorward motion.

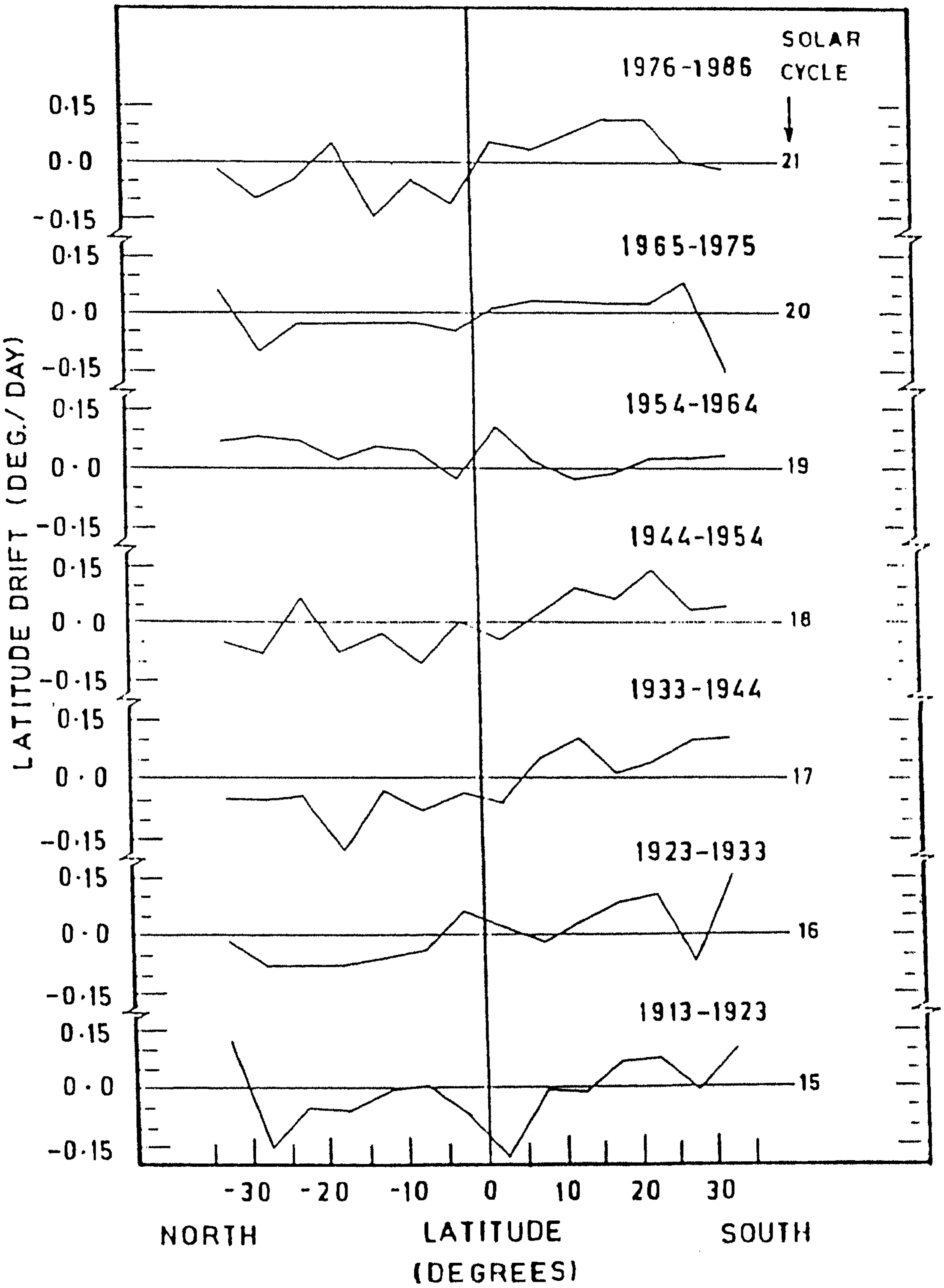


Fig III.7 Latitude motions for all spots as a function of latitude for solar cycles 15-21 to illustrate the pattern of the drift for each cycle.

in the southern hemisphere, it is equator ward at low latitudes and poleward thereon.

Meridional motions determined from sunspot data is more reliable than those derived from spectroscopic results. But while using spots as tracers, one has the inherent disadvantage that the study will be confined between latitudes ± 35° and not beyond. Also, it should be borne in mind that sunspots proper motion do not represent the motions in the photosphere. This point was discussed in chapter III while discussing the solar rotation results. Even with these differences the determination of meridional motions from sunspot is important in itself.

3.2.5 Solar diameter from photoheliograms

From the literature, it is clear that all these years, solar apparent diameter has been studied using meridian circle, transit of Mercury, solar eclipse or micrometer measurements. To study the diameter changes, we need a long observational data base. Here, an attempt is made to look for solar diameter changes using the Kodaikanal photoheliograms which span over nearly 7 solar cycles.

Solar apparent diameter on each day's photoheliogram were measured using the digitiser following the procedure described in chapter II. In all 18,888 photoheliogram solar images were measured. On each of the photoheliograms, four measurements on East, West (horizontal) and North, South (vertical) points on the limb were measured. The four individual measured values were then corrected for atmospheric refraction. From these, the values for the horizontal and vertical solar radii for each day were obtained. From the daily solar radii values, the mean horizontal and vertical radii (in arc sec.) were obtained for each year. The value for $R_\odot$ of 960.0̈was adopted as the zero value and hence the values plotted on the Y - axis in Figure (III.8) are the residuals with respect to this zero value.

As can be seen from Figure III.8, the annual mean radii values have a range in scatter. This scatter in the $R_\odot$ value may be due to

i) changes in local seeing conditions

ii) changes in the photographic emulsion response,

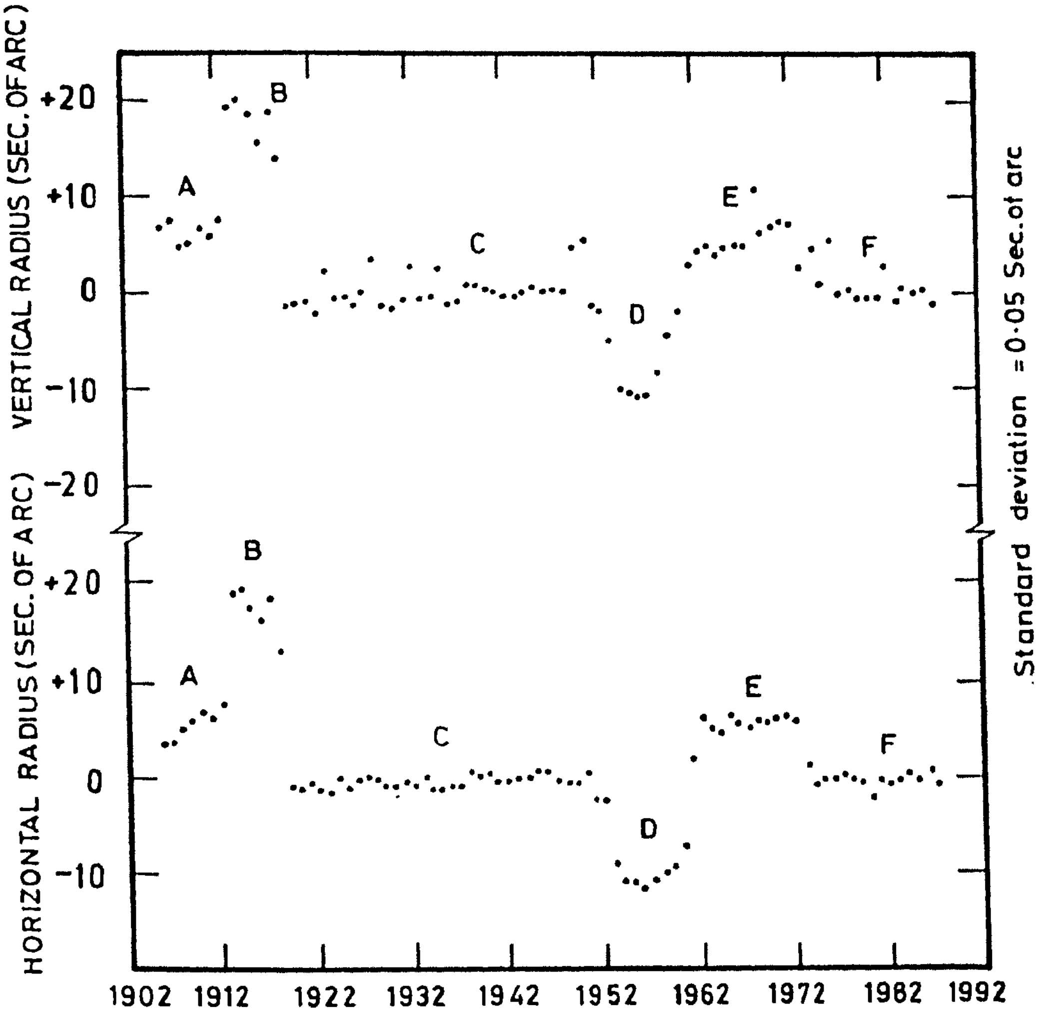


Fig.III.8 Plot of the horizontal radius and vertical radius of the sun in arc sec for the years 1906-1987 of the Kodaikanal data. The radii values in the regions covered by C and F alone have been used for detecting solar diameter changes. The zero in the radius axis corresponds to a value of 960 arc sec.

iii) temperature and pressure at the time of observations which change from season to season over the year.

iv) scattered light in the telescope.

From the Figure III.8, it can be seen that the solar radii (both horizontal radius, R_H and the vertical radius , R_V)values for the years 1919 to 1952 (designated as C) and 1973 to 1987 (designated as F) are centered around the value 960 sec. of arc. During the periods 1906 - 1919 and 1953 - 1972 the values of the radii from our measurements show deviations as much as 5 to 18 arc sec. from the central value of 960 arc sec. The deviations seen during the years 1906 - 1919 are presumably due to the changes made in the telescope optics which have been narrated in chapter II. The reasons for the deviation seen in the period 1952 - 1972 are not clear. We have examined the possibility and the extent to which these deviations in the radius values would be propagated into the rotation rate determinations. We have examined the annual values of the constants A and B of the rotation rate equation in the years 1919 - 1952. The maximum deviations in A and B from the mean values for this period are 0.023 and -0.007 respectively. Similarly the maximum deviations in A and B during the years 1953 - 1972 from the same mean is 0.008 and -0.050 respectively. Thus, eventhough the deviations in radii appear large (which is due to the enlarged scale used for the Y - axis) in Figure III.8 they are not of such magnitude as to cause errors in the rotation rates to any significant level. The stability in the values of A and B over these years bears testimony to this conclusion.

It should be mentioned at this stage that solar diameter determinations employ different parameters like, optical depth or limb intensity and using different techniques. Although these different techniques aim to determine the solar diameter changes, the results from one method are not directly comparable to another. A comparison between solar diameter values derived by using the same technique and by employing any one parameter on the sun for measurements above would be justifiable in seeking for secular changes in solar diameter. To do this, we need a longer period data. With this in view, photoheliograms data spanning over solar cycles taken with a single telescope, have been used for the solar diameter study. This is the first time that such an attempt is made to use Kodaikanal photoheliograms data (1906 - 1987). From the Figure III.8 we find that the solar diameter does not show any variation.

CHAPTER IV

VARIATIONS IN SIDEREAL ROTATION WITH THE SUNSPOT CYCLE

4.1 Sidereal Rotation in Maximum and Minimum epochs

4.1.1 Method of Analysis

The annual mean sidereal rotation rates in degrees/day were calculated for the three classes of spot sizes and for every 5° latitude bins from 0° to 30° N and 0° to 30° S for the years 1906 to 1987. These annual rotation rates are the most suitable indices for the study of the variation in solar rotation in each latitude belt with the sunspot cycle. The long data base line would provide the necessary reliability and the three area classes would help to look for any size dependence which have not been attempted by anyone so far.

The dependence of the rotation rates on the size of spots has been illustrated well in Figure III.3 as well as from the investigations of Howard, Gilman and Gilman (1984). With this in mind, it is logical to ask the question whether a similar behaviour is present even in the solar cycle variations in rotation rates; or in other words, whether the solar cycle variations in rotation rates are different for the three size classes? If there are real differences, it should be possible to detect them from these data.

In this analysis north and south hemisphere results have been maintained separately throughout with the intention of bringing out the north - south asymmetry also at all stages. These raw rotation rate vs years plots are however very noisy and also have a number of gaps in them. These plots are shown in Appendix I. These gaps are mainly in the minimum years when no big spots appear and in some years even the small spots are absent.

The number of sunspots of areas < 5 millionths are always far higher than the other two sizes (5-15 and > 15 millionths) in all latitudes. Spots of all the three sizes occur in sufficient numbers in latitudes between 5° - 25° so that there would be fewer gaps if these are spatially averaged. This is understandable as these are the most crowded regions in the sunspot butterfly diagram. Generally during solar minimum, there are no big spots at all and the small numbers that appear in any latitude are the small spots. The numbers of the larger sizes (5-15 to > 15 millionths) become appreciable only when the maximum picks up and also during the peak of activity. Again two or three years after the maximum, i.e., in the descending phase of the cycle, the small size spots are seen in far greater numbers than the large spots. Thus during the minimum years, the count of the large sunspots were zero and in some of the minimum years like 1914 and 1954, even the small spots were absent in the core of the minimum and this led to the gaps in the rotation vs years plots. These gaps became more numerous in the plots for large spots.

The noise and the gaps in the plots of the raw rotation rate vs years (Shown in Appendix I) can be reduced by substantial spatial averaging (averaging over the latitudes) and also by combining the spots of the 3 size groups into one group. Once reasonably stable plots are obtained free from noise, one can look for the real peaks (indicating enhanced rotation rates) in the rotation vs years plots. Accordingly the residual rotation rates were spatially averaged for the three area classes and separately for the northern and southern hemispheres. For each latitude belt the residuals in rotation rates were derived. These residual rotation rates are the differences between the annual values and the mean rotation rate for the respective latitude. This mean rotation rate is the one corresponding to the mean latitude value for each belt. For example, for the latitude belt 0 – 5° the mean rotation rate is the one for 2.5° ; for the 5 – 10° , the rotation rate is for 7.5° and so on. These mean rotation rates for each 5° belts were calculated using

$$\omega = A + B \; Sin^2 \; \phi$$

$$for \; \phi = \; 2.5^\circ, \; 7.5^\circ, \; 12.5^\circ, \; 17.5^\circ, \; 22.5^\circ \; and \; 27.5^\circ$$

Finally the rotation rate at 30° latitude was taken as the mean rotation rate for the latitude belt > 30°.

The values of A and B used and the mean rotation rates corresponding to each of the above ϕ values are given in Table IV.1. We have then combined the annual average residuals for the 7 latitude bins and derived the grand mean residuals averaged over all the 7 latitude bins for the three area classes separately for the northern and southern hemispheres. This gave three plots for

Table IV.1

Coefficients A and B for the three size groups (< 5, 5 - 15, and > 15 millionths) in north and south and the rotation rates at ϕ values of 2.5°, 7.5°, 12.5°, 17.5°, 22.5°, 27.5° and 30° in north and south

Hemisphere	Spot size (in millionths)	Coefficients	Latitude Zone	Rotation rates (Degree/Day)						
				0 - 5	5 - 10	10 - 15	15 - 20	20 - 25	25 - 30	>30
			Mean Latitude	2.5	7.5	12.5	17.5	22.5	27.5	30
North	< 5	A	14.623 (± 0.005)							
		B	– 3.046 (±0.050)	14.62	14.57	14.48	14.35	14.18	13.97	13.86
	5 - 15	A	14.490(± 0.008)							
		B	– 2.610(±0.070)	14.48	14.44	14.37	14.25	14.11	13.93	13.84
	> 15	A	14.378(± 0.009)							
		B	– 2.928(± 0.088)	14.37	14.33	14.24	14.11	13.95	13.75	13.65
South	< 5	A	14.641 (± 0.006)							
		B	– 2.691(± 0.058)	14.63	14.59	14.51	14.40	14.25	14.07	13.97
	5 - 15	A	14.536 (± 0.008)							
		B	– 2.662(± 0.076)	14.53	14.49	14.41	14.30	14.15	13.97	13.87
	> 15	A	14.376 (± 0.011)							
		B	– 2.582 (± 0.098)	14.37	14.33	14.25	14.14	14.00	13.83	13.73

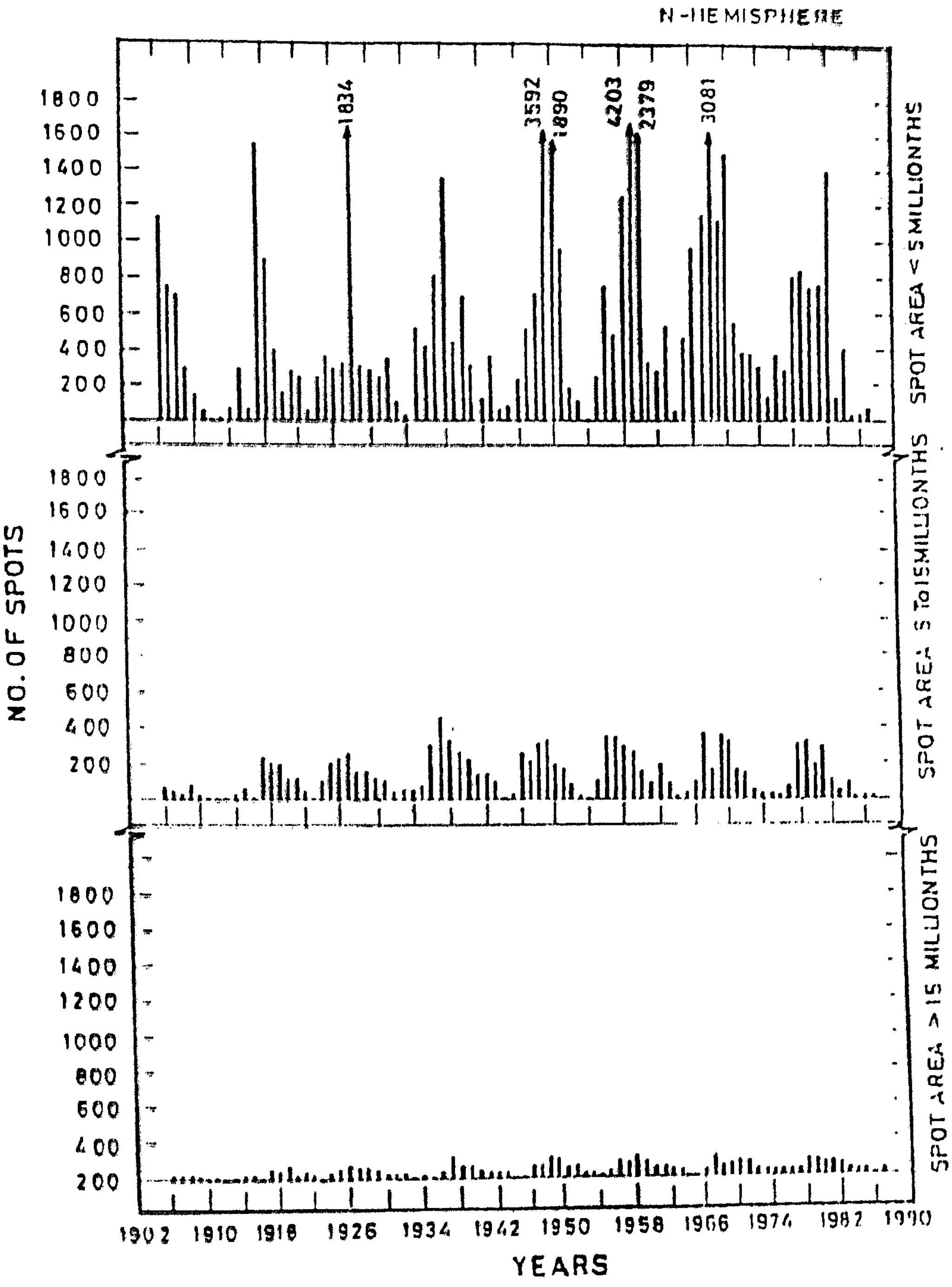
the north hemisphere corresponding to the three area sizes ($<$ 5 , 5 - 15 and $>$ 15 millionths) and three similar plots for the south hemisphere. These plots (Figures IV.1 and IV.2) have substantially less noise than the raw rotation plots and hence show the relative contribution by the three area sizes better.

4.1.2 Results

The plot for spots of areas $<$ 5 millionths, in the north hemisphere shows strong positive peaks in all the minimum years 1914, 1923, 1944, 1954, 1964 and 1976 except in 1933. These peaks are marked by A, B, D, E, F and G in Figure IV.1. In order to maintain the continuity the position where a peak would be expected in 1933 is marked as C although there is no peak there. In the southern hemisphere (Figure IV.2) the peaks are better defined, the only exception being in the year 1976. Here also the peaks for the same minimum years have been marked by A, B, C, D, E, F and G. Also in both these plots there are secondary peaks 2 - 3 years after the maximum years which illustrate the fact that small spots appear in large numbers at this phase of the spot cycle who show higher rotation rates. These secondary peaks are marked as a, b, c, d, e, f and g.

The plot for the spots of area 5 - 15 millionths for the north hemisphere has peaks in the minimum years as well as 2 - 3 years following the maximum years, but with smallar ampilitudes than in the plot for $<$ 5 millionths. In the plot (Figure IV.2) for the south hemisphere the peaks in the minimum years as well as in the post maximum years are better defined than in the northern hemisphere (Figure IV.1). Here too the peaks in the minimum years are marked B, C, D, E, F and G and the peaks in the post maximum years are marked a, b, c, d, e, f, and g. There is no peak corresponding to A.

The rotation plots from the $>$ 15 millionths spots for the north and south hemispheres (Figures IV.1 and IV.2) do not show the positive peaks during the minimum years nor in the post maximum years. But their expected locations are marked as A, B, C, D, E, F and G and a, b, c, d, e, f and g with the help of the peaks in the plots of the smaller spots. Thus, it is clear that the small spots of area $<$ 5 millionths and to a certain extent spots of area 5 - 15 millionths act as tracers during the minimum years for determining the rotation, while the large spots and also the smaller ones are the tracers during the post maximum


N-HEMISPHERE
1834
3592
1890
4203
2379
3081
1800
1600
1400
1200
1000
800
600
400
200
SPOT AREA < 5 MILLIONTHS
SPOT AREA 5 To 15 MILLIONTHS
SPOT AREA > 15 MILLIONTHS
NO. OF SPOTS
1902
1910
1918
1926
1934
1942
1950
1958
1966
1974
1982
1990
YEARS

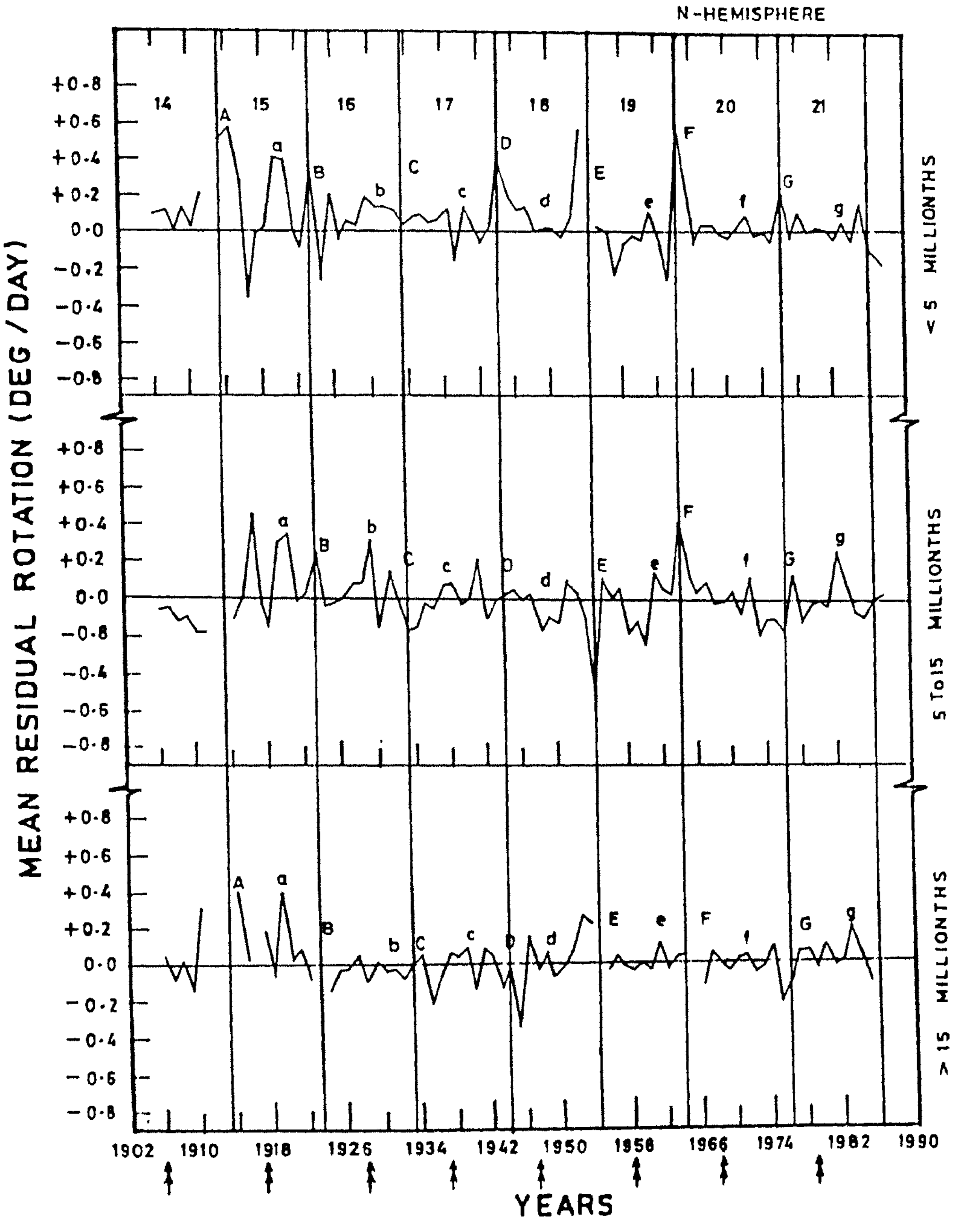
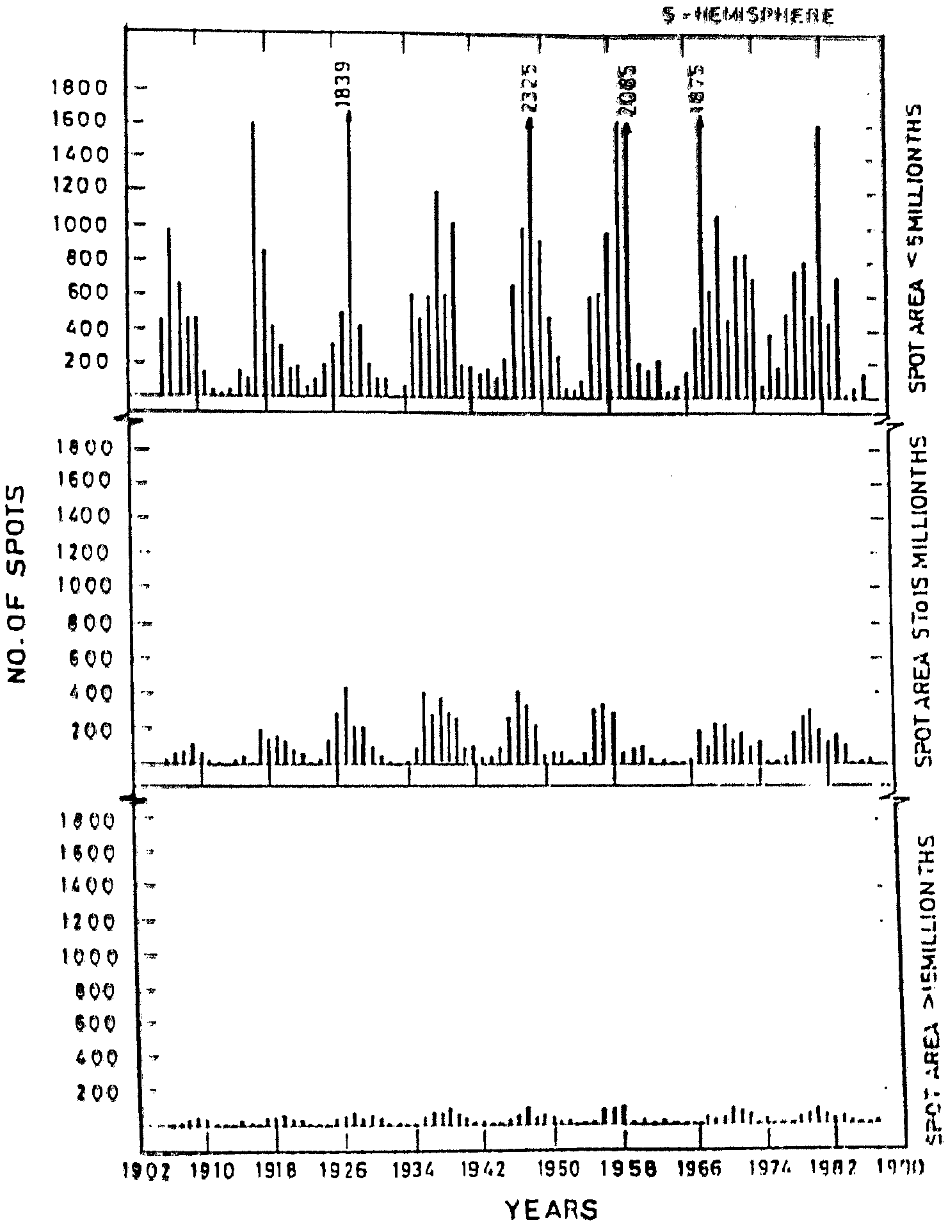


Fig.IV.1 Residual rotation rates (annual means) averaged over the seven, 5° latitudes bins (0° - 30° and > 30°) in the northern hemisphere for the spots of the 3 size groups. Vertical lines denote the years of sunspot minimum and the numbers in the vertical boxes (14 through 21) stand for the number of the spot cycle. The double arrow indicators shown along the horizontal axis mark the epochs of solar maximum.

The number of spots of the 3 size groups in each year is shown in a transparency. This transparency can be placed over Fig.IV.1 and the number of spots in any year can be related to the amplitude of the residual rotation rates.


S-HEMISPHERE
NO. OF SPOTS
1800
1600
1400
1200
1000
800
600
400
200
1839
2325
2085
1875
SPOT AREA < 5 MILLIONTHS
SPOT AREA 5 To 15 MILLIONTHS
SPOT AREA > 15 MILLIONTHS
1902
1910
1918
1926
1934
1942
1950
1958
1966
1974
1982
1990
YEARS

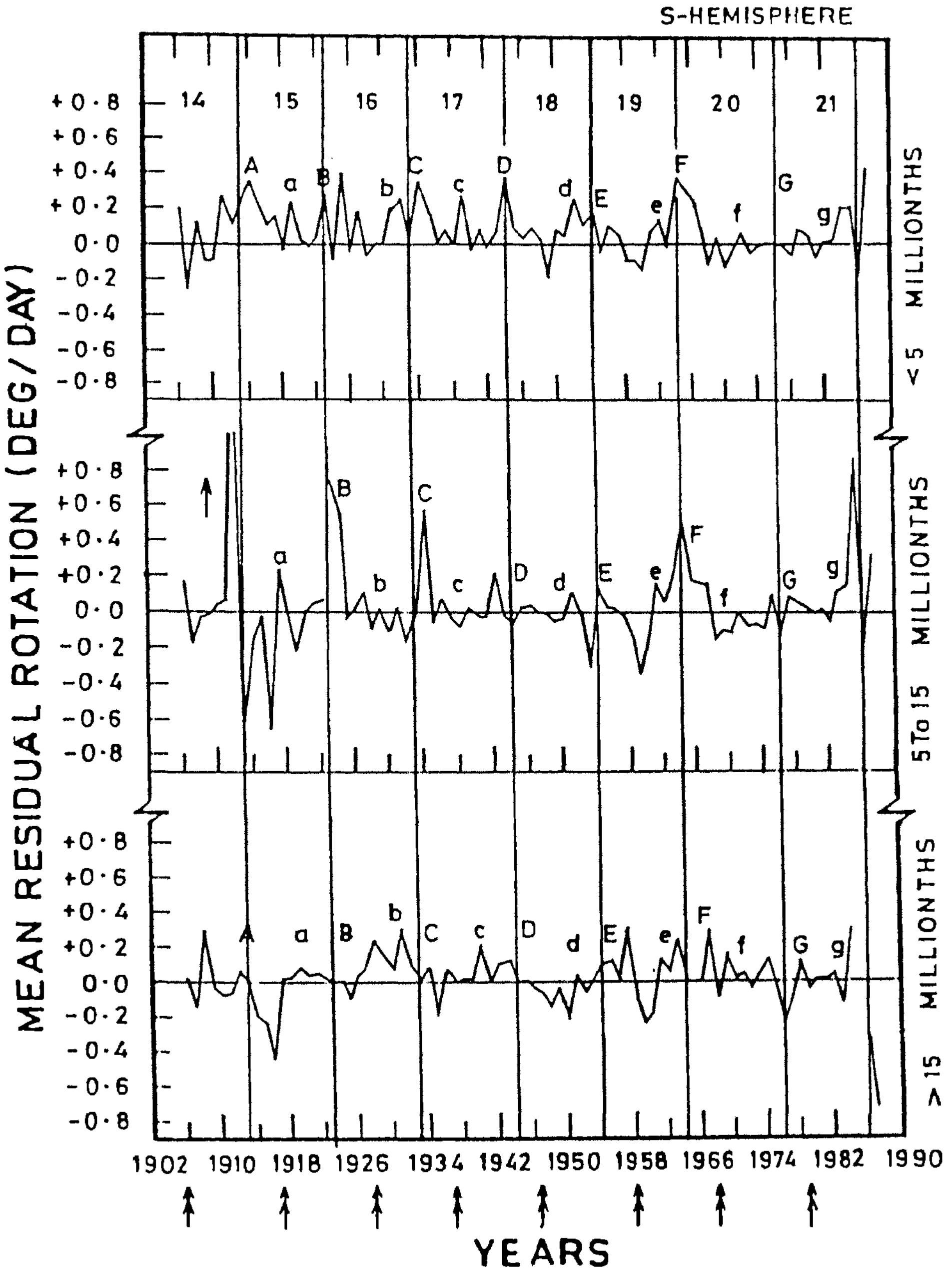


Fig.IV.2 Residual rotation rates (annual means) averaged over the seven, 5° latitudes bins (0° - 30° and > 30°) in the southern hemisphere for the spots of the 3 size groups. Vertical lines denote the years of sunspot minimum and the numbers in the vertical boxes (14 through 21) stand for the number of the spot cycle. The double arrow indicators shown along the horizontal axis mark the epochs of solar maximum.

The number of spots of the 3 size groups in each year is shown in a tranparency. This transparency can be placed over Fig.IV.2 and the number of spots in any year can be related to the amplitude of the residual rotation rates.

years. Between the north and south, the southern hemisphere data are less noisy and hence show the peaks better. The numbers of spots in a year counted for calculating the rotation rates in each size group are shown in the transparent sheets. These transparent sheets can be placed over Figures IV.1 and IV.2 and the numbers of spots for any year can be read off.

In the next step, the mean of the three plots for the north hemisphere was derived, thus averaging the behaviour of all the 3 area sizes and similarly for the south hemispshere. Figure IV.3.(i) is the plot which is the average of the 3 sizes for the north hemisphere and Figure IV.3.(ii) is a similar mean plot for the south hemisphere. Those peaks for the minimum which were very tall in the < 5 millionths plot are still conspicous, whereas others have been averaged out by the troughs in the plot of the spots > 15 millionths. Thus it is clear that in these plots as well as in the plot for the whole sun (Figure IV.3.iii) whatever peaks are seen in the minimum years are those contributed mostly by the spots < 5 millionths and to a lesser extent by the spots 5 - 15 millionths in area.

Finally, one grand mean plot of the residual of rotation velocities vs years was derived combining the north and south hemisphere plots. These plots contain minimum number of gaps as all the spots have been taken into account while averaging. There is a substantial decrease in the noise and these plots can be used as a sort of guide to identity the real peaks in the plots of Figures IV.1 and IV.2. There is very good agreement between the plots averaged over all latitudes and the individual plots for the 6 latitude bins. The peaks or increase in rotation within ± 1 year of the minimum is striking; the decrease in rotation rate around the maximum is also striking, but not the secondary maximum, in the post maximum years as reported by Gilman and Howard (1984). The grand mean residual plot (Figure IV.3.iii) which is the average over all the 3 area sizes and latitude belts and north and south combined was derived with the sole purpose of comparing it with a similar plot of Gilman and Howard (1984). Whereas our conclusions are based on the Figures IV.1, IV.2; and Figure IV.3 boxes (i) and (ii). A comparison of the peaks from the two studies has been given in Table IV.2. There is good agreement between the two studies except for the disagreement at two places; there is strong negative peak in their curve in 1923 (minimum) and a strong negative peak in 1954 (minimum) in the present study. These should have been both positive peaks.

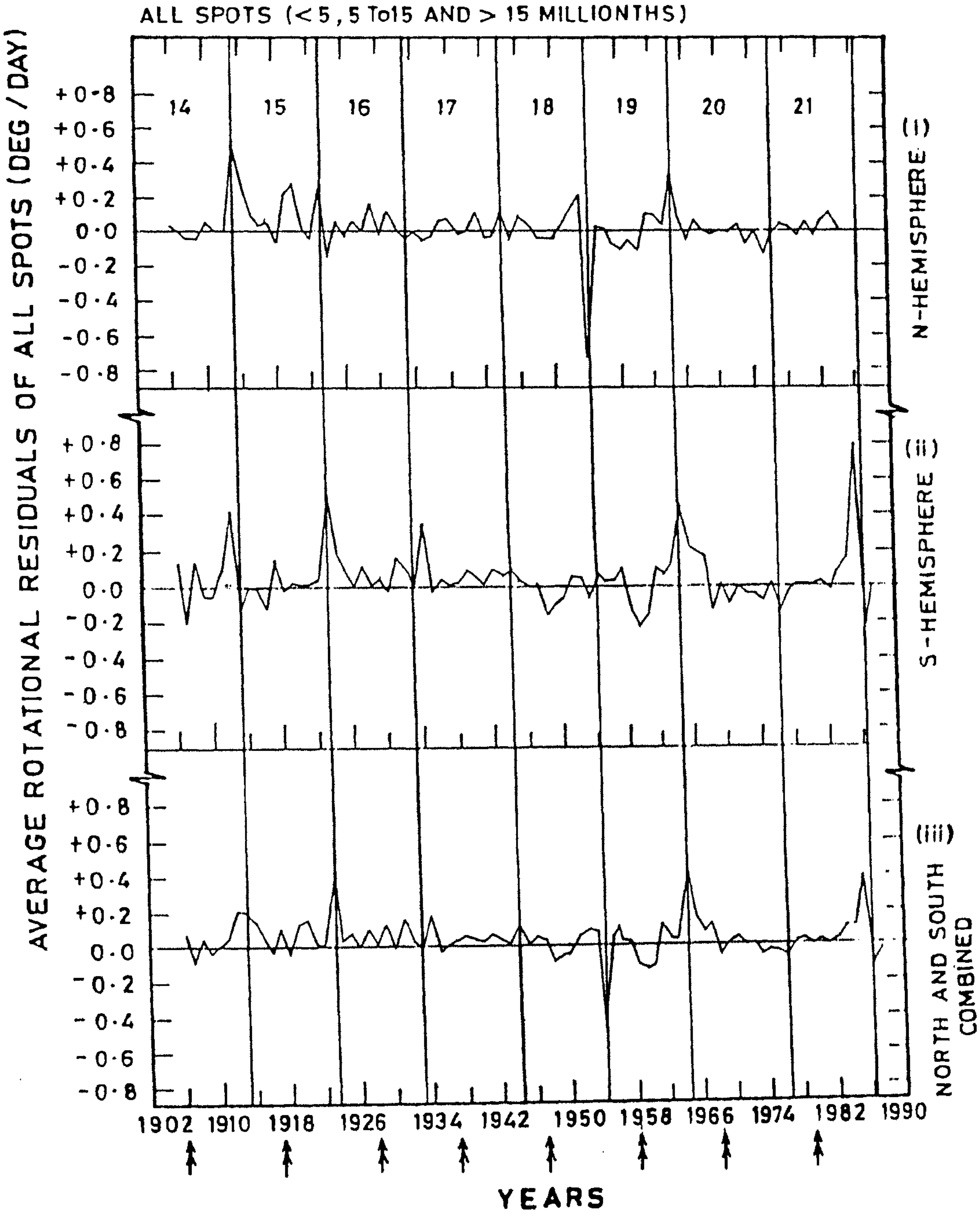


Fig.IV.3

Box (i) Residual rotation rates (annual means) averaged over the seven 5° latitude bins (0° - 30°) and for the 3 spot size groups put together for the northern hemisphere.

Box(ii) Same as above for the southern hemisphere.

Box(iii) Average of north and south hemispheres.

Table IV.2

Positions of the positive and negative peaks
in the residual rotation rates from Kodaikanal
data and Mt. Wilson data

Data source	Positive peak years			Negative peak years
Present study Kodaikanal data (1906-1987)	1913 1923	1934 1944 1964	1985	1954
Gilman, Howard, 1984 (Figure 5 of their paper) (1921-1982)		1933 1944 1954 1964 1976		1923

4.1.3 Relative roles of large and small size spots in the solar cycle variation of the rotation

The variation in spot rotation during the solar cycle has already been illustrated from Figures IV.1, IV.2, and IV.3. Such systematic variation in sidereal rotation rate of the sun was seen earlier by Gilman and Howard (1984). Hathaway and Wilson (1990) using the same Mt. Wilson data as Gilman and Howard, found variations over the phase of the spot cycle with rapid rotation occurring at minimum. Similar result was also noticed by several other workers: Balthasar and Wöhl (1980), Arevalo et al (1982); and Lustig (1983). Also Hathaway and Wilson (1990) found systematic variations from cycle to cycle: the most rapid rotation taking place during the weakest cycles. They hypothesise that, a possible cause of these variations is due to spot magnetic fields arising from more slowly rotating layers below the convection zone. The spot magnetic fields would then slow the rotation rate in the convection zone while increasing the rotation rate of the magnetic fields to match the convection zone rate. These cycle variations may also imply periodic angular momentum exchange between the photosphere and the deeper layers of the convection zone.

We offer an alternative explanation for the variation in rotation rate with the spot cycle in the following. Although this may not be the sole cause of the variation, but certainly would provide, a significant fraction of the solar cycle variation noticed. In Figure III.3 (of Chapter III) it has been shown that the sidereal rotation rates derived from spots of areas < 5 millionths are faster than those derived from spots of areas 5 - 15 millionths. Similarly the rotation rates from spots of areas 5 - 15 millionths are faster than those using spots of areas > 15 millionths. If the same trend continues even within anyone area group one should expect a monotonic increase in rotation rate as one goes from the bigger to smaller and smaller sizes i.e. from 15 to 5 and from 5 to 1 millionths. The only impediment in verifying this trend would be that the number of spots in each 1 millionths area bin available for such a study would be so small, that statistically stable values for the rotation rates may not be possible. Now, the commonly accepted explanation for the dependence of the rotation rates on the spot size is the one offered by Gilman and Foukal (1979). According to them these differences are due to the different rotation rates at different depths within the sun at which the spots of different sizes are anchored. Since small spots are anchored to shallower depths and bigger spots are anchored to deeper layers within the sun, the rotations derived from small and bigger spots reflect the rotation rates of the respective layers to which these spots are anchored.

Now, the gaps in the rotation plots during the minimum years are due to the absence of spots. The number of spots of the 3 categories of sizes are plotted year wise on the transparent sheets. The numbers of spots in any year can be read off by over laying the transparent sheet over Figures IV.1 and IV.2. It is clear from these plots that during minimum, bigger spots never appear and if at all only the small ones and even among the two categories (5 - 15 and < 5 millionths) spots of areas < 5 millionths are more common than the bigger ones; while during solar maximum bigger spots appear in larger numbers. Thus the rotation rates during the minimum years are those derived from the smaller spots, while during the maximum years both the large as well as the smaller spots participate in the rotation measurements. Since these measurements are area weighted, during the maximum the spots with large area will have higher weightage and so the rotation rates appropriate to them will predominate during these epochs of the cycle.

Two to three years after the maximum, it is known that spots of smaller sizes 0 - 5 millionths and 5 - 15 millionths occur in larger numbers than the big spots. Thus in this epoch of the cycle, the rotation derived will show the substantial contribution by these small size spots i.e., a higher rotation rate than that appropriate to the large size spots. Thus the conclusion is that the samples available for rotation determination during minimum and maximum epochs are not the same; during minimum the rotation rates determinations use the spots of small areas (spots of size < 5 millionths) while the rotation rate determinations during maximum use both big and small spots with higher weightage for the former. Since the big spots are anchored to greater depths within the sun than the smaller spots, we are thus looking at the rotation rates at different depths within the sun during the maximum and minimum epochs. The steep positive peaks seen in some of the minimum years in the rotation plot for any single size spots (Figures IV.1 and IV.2) are most probably because at these times, the rotation derived are from the sample that contains more of the still smaller spots like 1 , 2 millionths and so on. Such peaks during minimum years are not prominent in the size group 5 - 15 millionths in any of the latitude belts, whereas the systematic variation in the rotation with cycle becomes obvious only in the average plot i.e., in the plot where all the sizes and latitudes are averaged.

To illustrate the reality of the —— positive peaks in the solar minimum year, we divided the data into 7 blocks, each block beginning 4 years before the minimum, through the minimum and 4 years after the minimum. The seven blocks are 1909-1917, 1919-1927, 1929-1937, 1940-1948, 1950-1958, 1961-1969 and 1972-1980. We proceeded as follows: taking the sunspot minimum years as the central reference epoch, the annual residuals of rotation rates for four years before and four years after the minimum were superposed cycle by cycle, for the three spot size groups. These residuals were the averaged over the 7 latitude bins in the northern hemisphere and in the southern hemisphere. The means of the plots for the three size groups were derived for the north and south separately (Figures IV.4 and IV.5). At the next stage the three spot sizes were combined to give two plots - one for the northern and the other for the southern hemisphere (Figure IV.6 boxes (i) and (ii)). Finally these were also averaged to get one plot for the whole sun for all the three size groups put together.

It can be seen from Figure IV.4 that the plot for the size group ($<$ 5 millionths) shows a tall peak at the minimum year which extends to 2 years before and 2 years after the minimum. The plots for the other two sizes (5 - 15 and $<$ 15 millionths) have negative residuals at the minimum years, but the secondary peaks 2 - 3 years in the post maximum and pre maximum periods are seen. In the southern hemisphere too similar features are seen (Figure IV.5). Figure IV.6 shows the mean residuals from the three size groups for the north and south hemispheres and illustrates well the pattern of rotation rates around the minimum year.

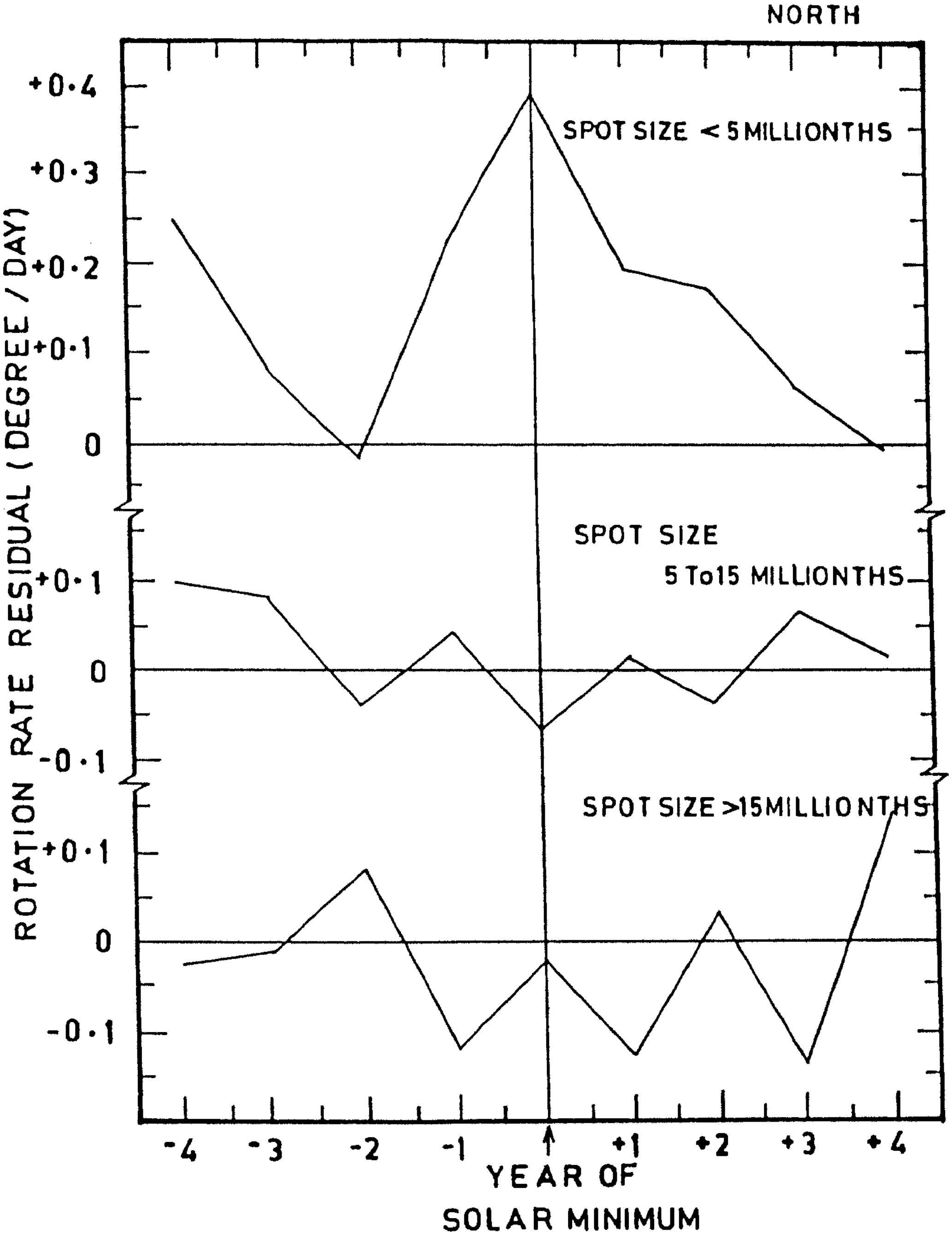


Fig.IV.4 Superposition of the annual mean residual rotation rates of seven blocks, (1909-1917; 1919-1927; 1929-1937; 1940-1948; 1950-1958; 1961-1969 and 1972-1980) for the three size groups. Each block has 9 annual means of residual rotation rates with the minimum year at the centre and with 4 years on either side – northern hemisphere.

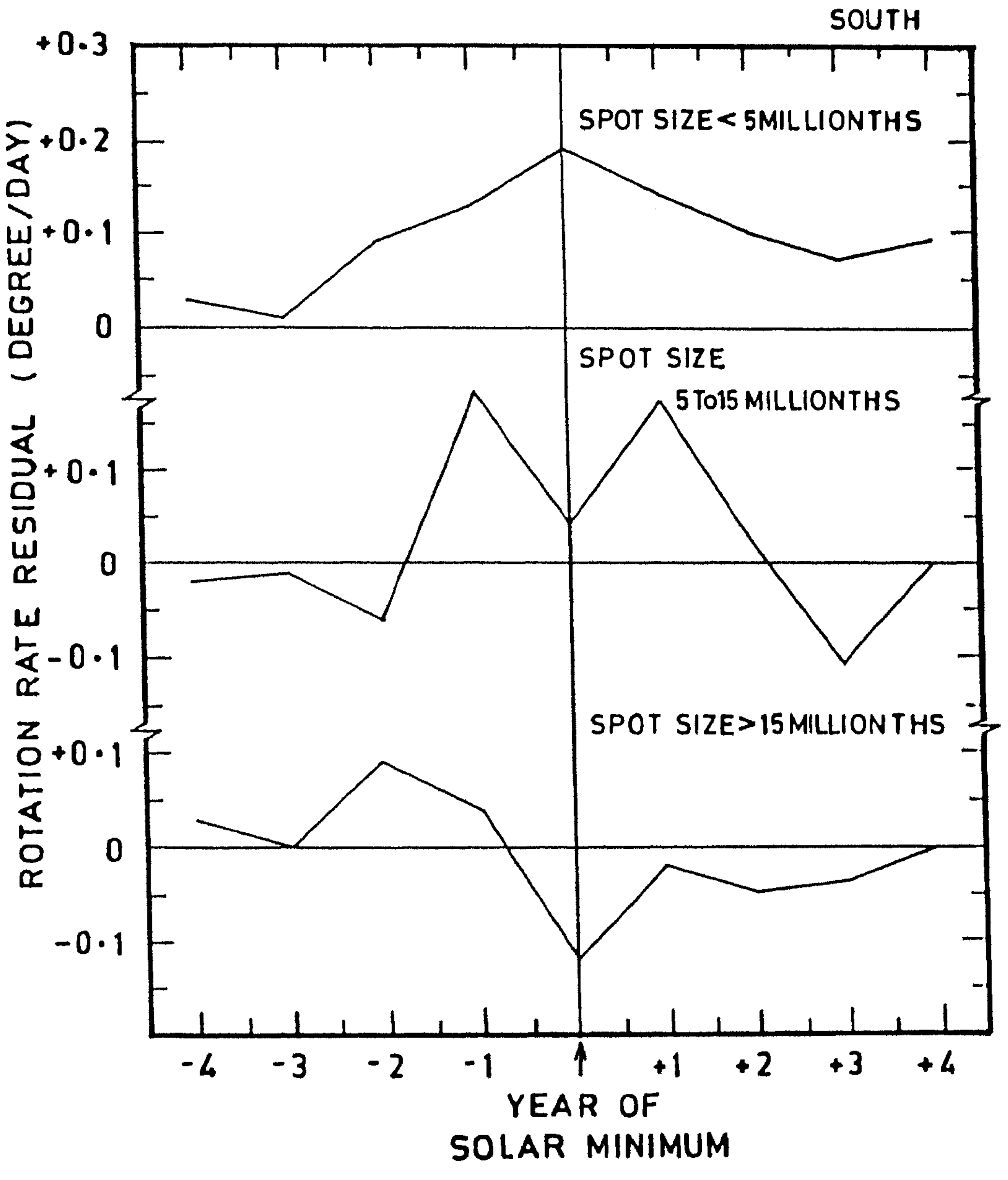


Fig.IV.5 Superposition of the annual mean residual rotation rates of seven blocks, (1909-1917; 1919-1927; 1929-1937; 1940-1948; 1950-1958; 1961-1969 and 1972-1980) for the three size groups. Each block has 9 annual means of residual rotation rates with the minimum year at the centre and with 4 years on either side – southern hemisphere.

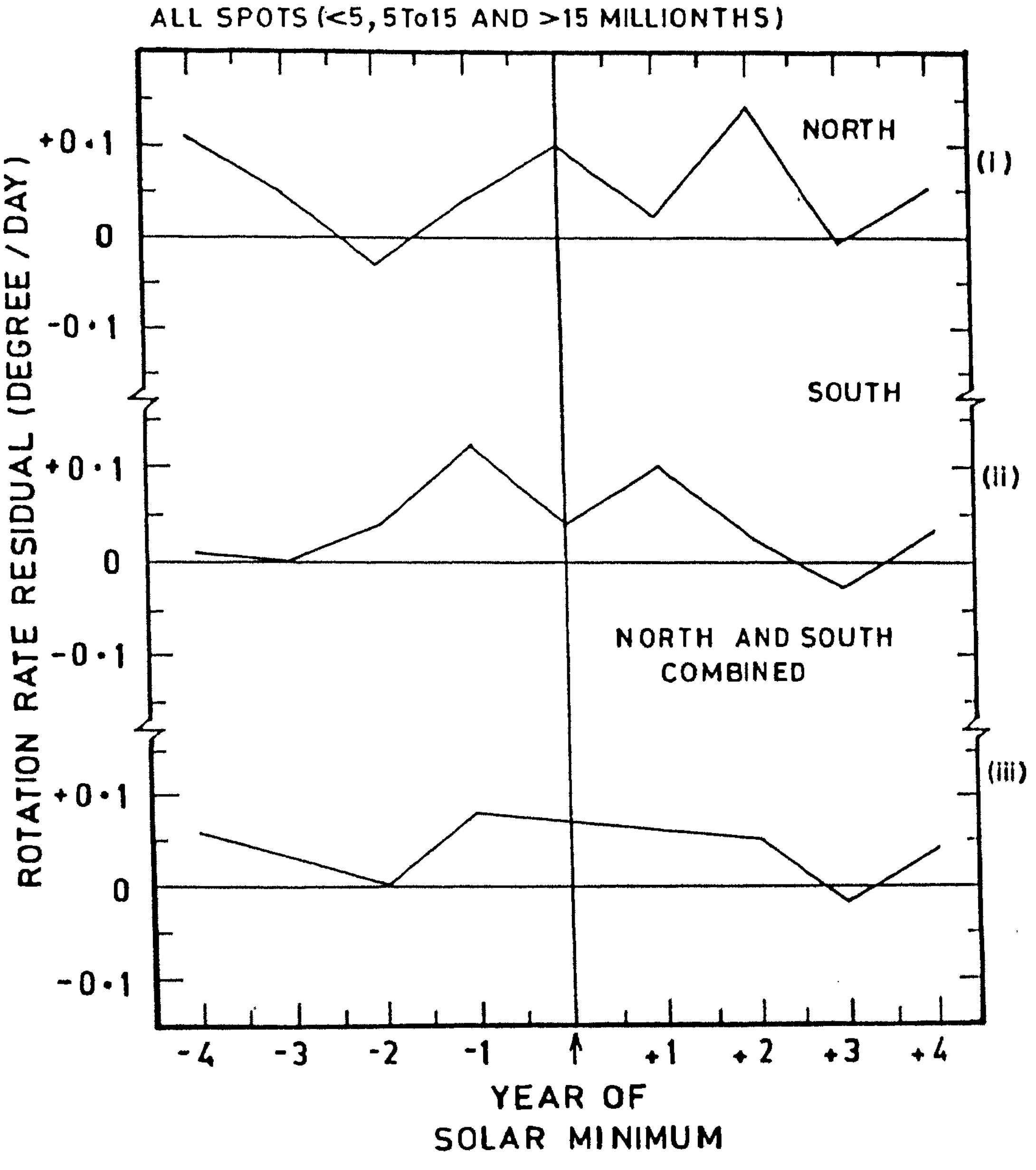


Fig.IV.6

(i) Same as for Fig.IV.4 with the rotation rates for the 3 size groups averaged for the northern hemisphere.

(ii) Same as for Fig.IV.5 with the rotation rates for the 3 size groups averaged for the southern hemisphere.

(iii) Mean of the northern and southern hemispheres.

CHAPTER V

SUMMARY OF THE RESULTS AND CONCLUSIONS

In this thesis we have presented the results of the analysis of 82 years' photoheliograms of the Kodaikanal observatory covering the period 1906-1987. Although the programme of obtaining photoheliograms continues even to-day at Kodaikanal, the target set for the present work was for the years 1904-1987 and of these the data for the years 1904 and 1905 had to be abandoned as there were many gaps in the data. The entire work involving measurement of the positions and areas of 332,620 spots was done by me over a period of 3 years and I was very eager to see the final results rather than taking up the measurements of the data subsequent to 1987. This is the longest data so far analysed for rotation studies with the accuracies described in Chapter II.

The summary of results of the analysis of these measurements are given below:

The siderial rotation rates of the sun and the variation of this rate over the sunspot zone latitudes have been derived, using the sunspot as tracers. In this processes, we have derived solar rotation rates for spot groups, individual spots of three size groups and all spots where the three size groups are combined or size averaged. The rotation rates from spot groups and all spots were derived with the intention of comparing our results with the results already available in the literature from the analysis of the classical Greenwich observatory sunspsot records.

The agreement shows the stability of our measurements. But our measurements done with higher accuracy could not bring out any differences as these are smaller compared to uncertainities in fixing group affiliations in the case of spot groups. In the case of sunspots these differences get averaged out over the years. But when the individual spots are broken into groups according to the sizes (< 5, 5 - 15 and > 15 millionths) many results come to light.

(1) The rotation rate from spots of area < 5 millionths is faster by 2% than the rate derived from the bigger spots (> 15 millionths).

(2) The rotation rates from the three size groups of the Kodaikanal data are higher than the corresponding rates from Mt. Wilson data by 0.09 deg./day or ~ 13 metres/sec. This is not an insignificant difference, considering the statistical stability of the data. This increase is reflected not only at the equator but also at higher latitudes.

(3) Combining the rotation rates from our measurements with the published rotation curve for the solar interior (from helioseismological data) we have derived the possible depths to which the spots of the 2 groups (5 - 15 and > 15 millionths) are anchored. If this is valid, then the rotation rates we have determined reflect the rotation of the subsurface layers at these depths.

(4) The meridional motions determined from the proper motions of the sunspots for the 3 size categories show that the motion is poleward in the northern hemisphere at all latitudes and is equatorward in the southern hemisphere in the mid latitudes and away from the equator at higher latitudes.

(5) We find no evidence for any secular changes in solar diameter.

(6) The variation in sidereal rotation with the sunspot cycle is brought out clearly from our analysis. During the sunspot minimum of the solar cycle, the rotation rates are higher than in the sunspot maximum years. The average amplitude of enhancement in the minimum is of the same order as the increase in rotation rate exhibited by the small spots (< 5 millionths) over that from the bigger spots (> 15 millionths). We have demonstrated that during the minimum periods, the rotation ratrs are those derived from the small size spots which alone appear on the sun whereas during the solar maximum the rotation rates are those derived from a sample that has more of the larger spots than the small spots. Thus the samples available for rotation determination during solar maximum and minimum epochs are not the same. This difference would at least partly explain the variation of the rotaion rate with the sunspot cycle.

Future prospects

There are other observatories like Greenwich, Mt. Wilson, Meudon, where one has similar long data base of photoheliograms over 6 to 7 cycles. In all these places generally single observation is done on each day of the year, weather permitting. To continue the important results presented here, it would be worthwhile to combine the Kodaikanal data set, with those from Mt. Wilson, to increase the precision in the results. As the Mt. Wilson observations are done 12 hours later to Kodaikanal observations, it would help to study the contributions from small spots better as some of the small spots which have lifetimes of less than a day can still be captured with the combined data series which would have been missing in the data from any one station. Also the weather patterns at the two sites (Kodaikanal and Mt.Wilson) are complementary: during the monsoon months at Kodaikanal, Mt.Wilson has the best observing season and the bad winter months of Mt.Wilson coincide with the best season at Kodaikanal. Thus combining the data would considerably increase the overall accuracy of the final results. Efforts in this direction have already been started.

Another aspect to which the future study should be directed is to examine whether the rotation rates would monotonically increase if the present small spots class (< 5 millionths) is subdivided into smaller sizes. This can be done only with the combined data from the two stations as such small spots last only a fraction of a day. Also, it would be worthwhile to study the rotation rates from the smaller spots at close latitude intervals, say 0 to 2, 2 to 5 and so on. This should reveal the subtle differences in the differential rotation and meridional motions better.

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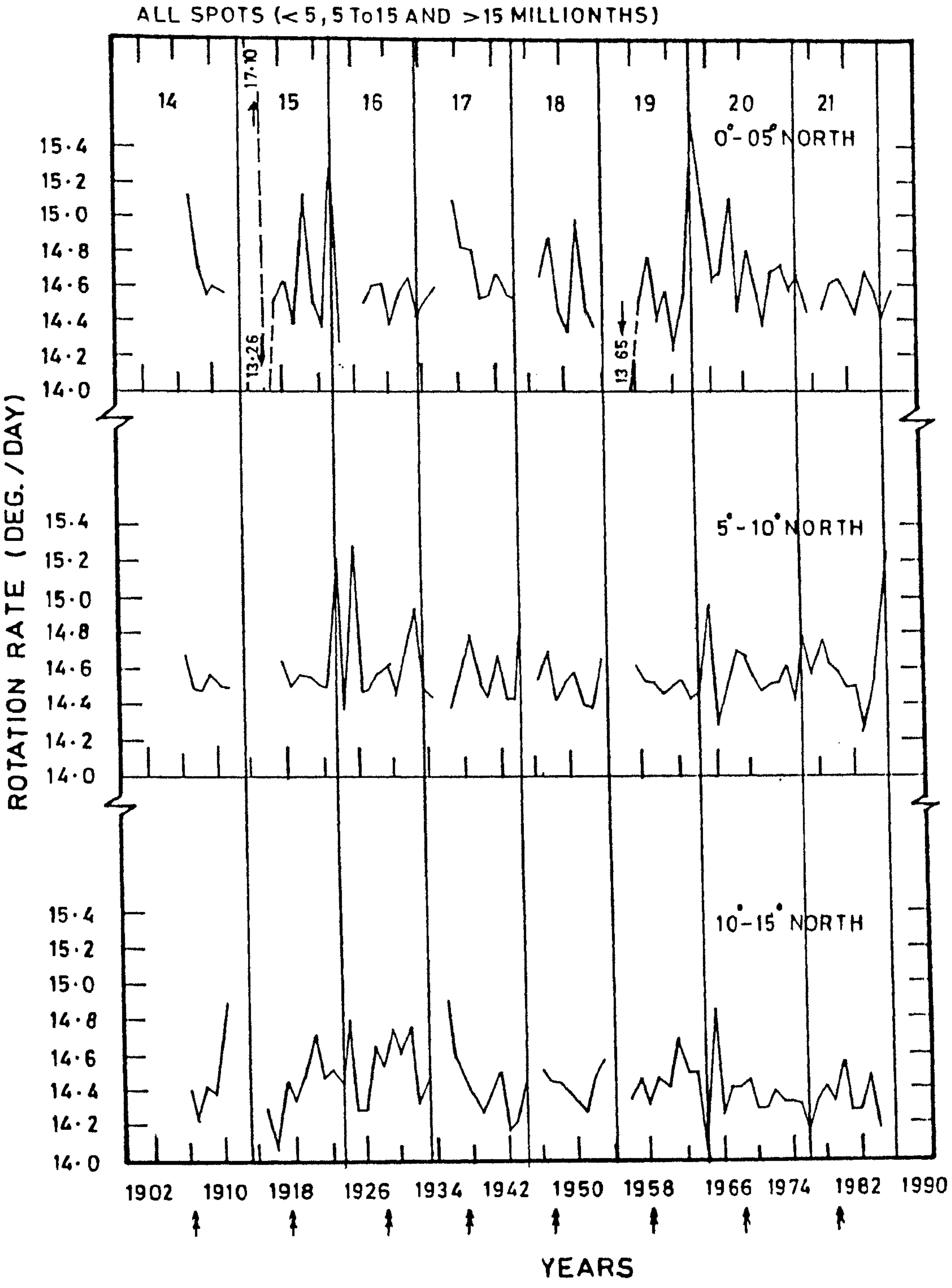
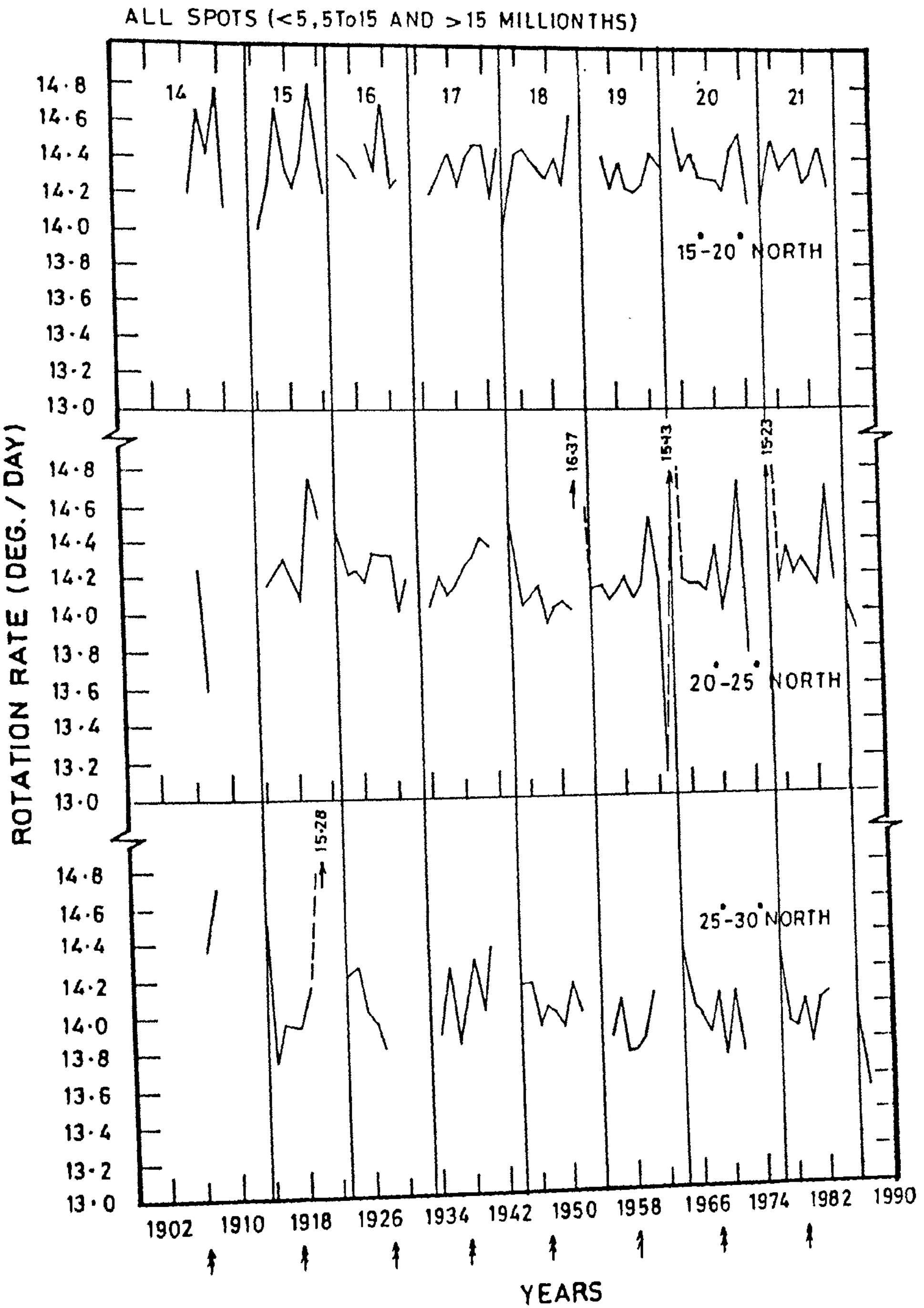
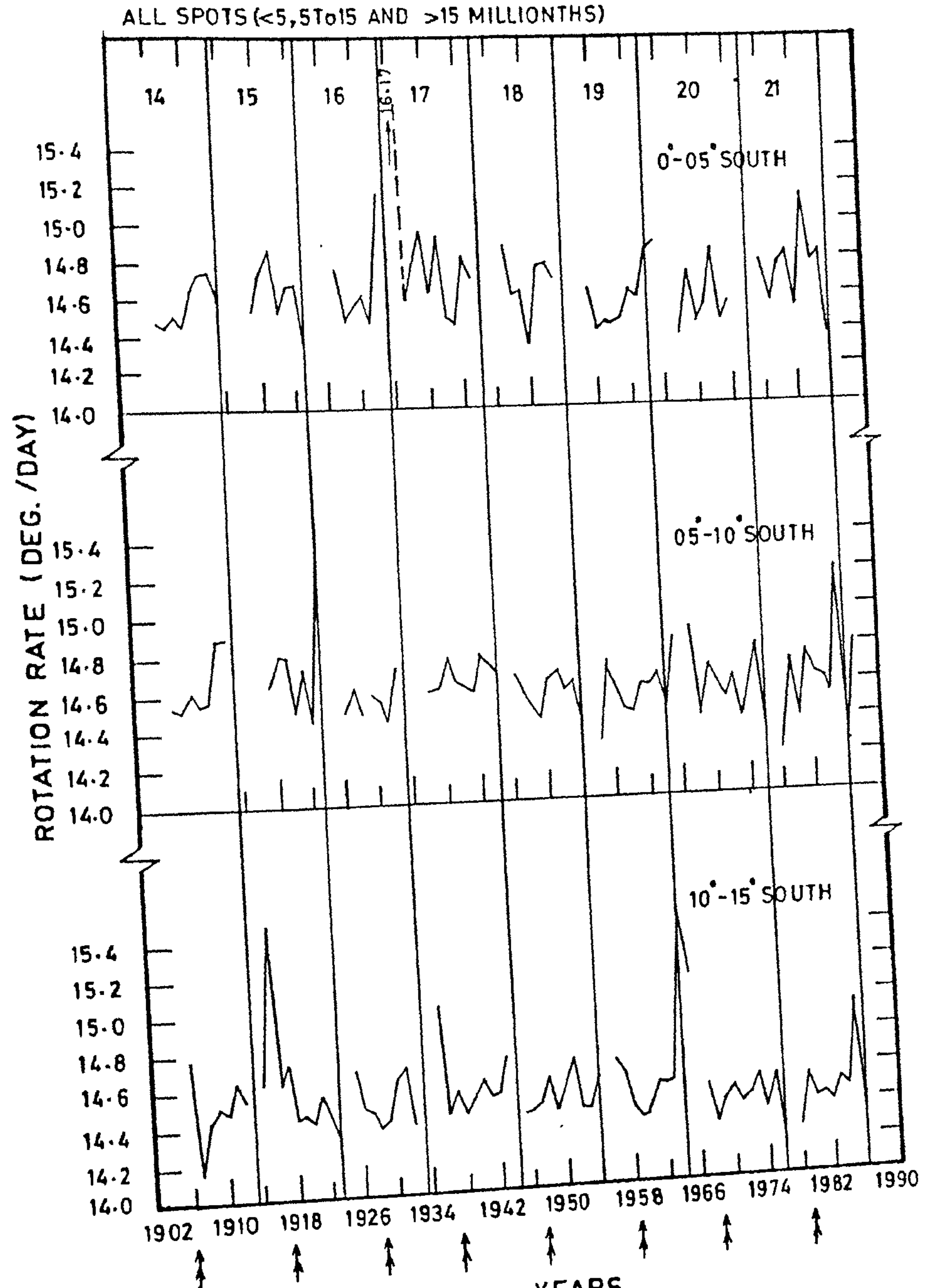
Appendix I



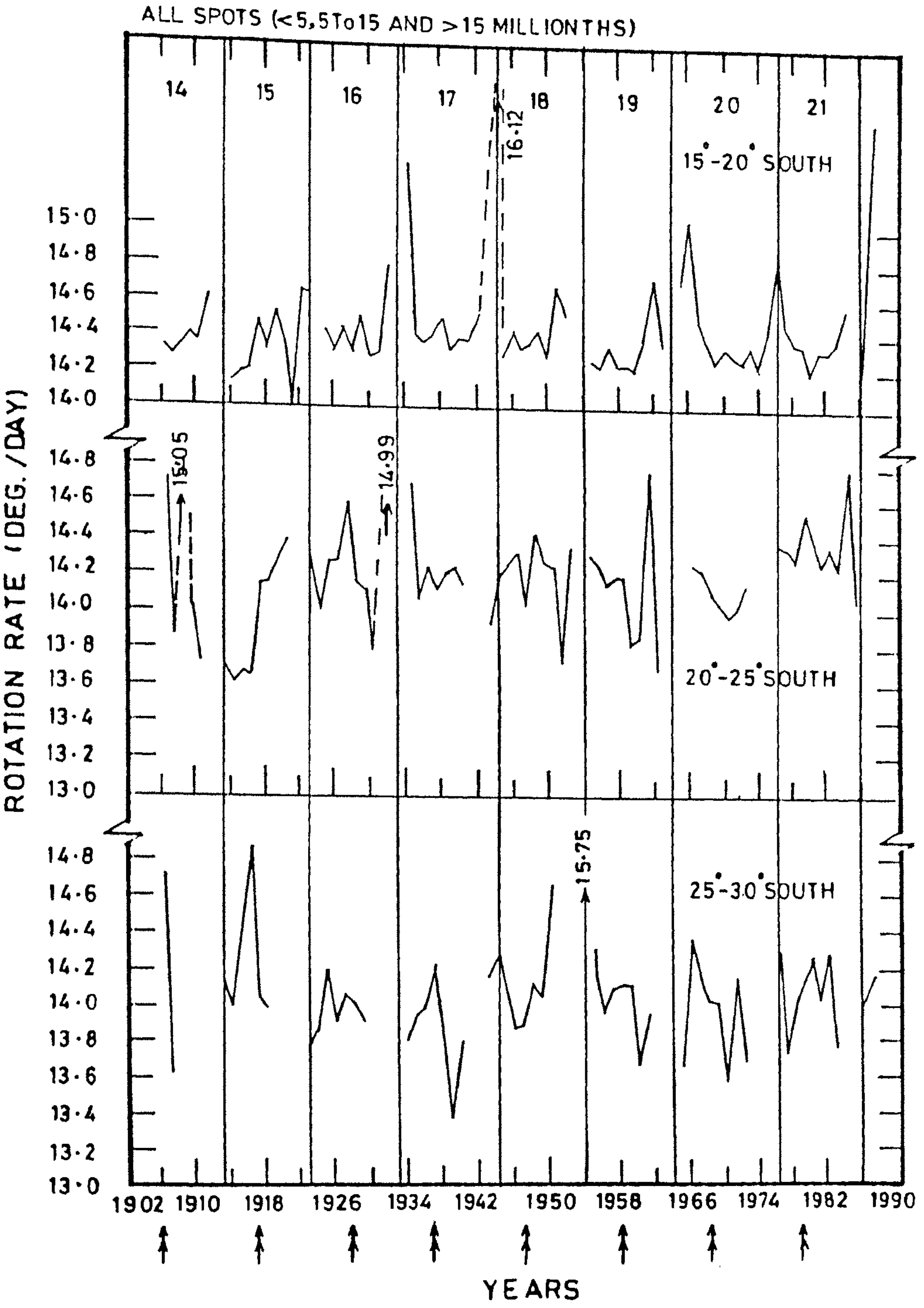
Appendix I Raw rotation rate vs year plots for the three size groups separately (<5, 5 - 15 and > 15 millionths of the solar hemisphere) for all the latitudes (0° - 30° and > 30° north and 0° - 30° and > 30 south) in 5° latitude bins. Notice the gaps in the data due to the absence of spots in the minimum years. The plots are noisy and need spatial and temporal averaging. These averaged rotation rates are presented in Figures IV.1, IV.2 and IV.3.



Appendix I Raw rotation rate vs year plots for the three size groups separately (<5, 5 - 15 and > 15 millionths of the solar hemisphere) for all the latitudes (0° - 30° and > 30° north and 0° - 30° and > 30 south) in 5° latitude bins. Notice the gaps in the data due to the absence of spots in the minimum years. The plots are noisy and need spatial and temporal averaging. These averaged rotation rates are presented in Figures IV.1, IV.2 and IV.3.



Appendix I Raw rotation rate vs year plots for the three size groups separately (<5, 5 - 15 and > 15 millionths of the solar hemisphere) for all the latitudes (0° - 30° and > 30° north and 0° - 30° and > 30 south) in 5° latitude bins. Notice the gaps in the data due to the absence of spots in the minimum years. The plots are noisy and need spatial and temporal averaging. These averaged rotation rates are presented in Figures IV.1, IV.2 and IV.3.



Appendix I Raw rotation rate vs year plots for the three size groups separately (<5, 5 - 15 and > 15 millionths of the solar hemisphere) for all the latitudes (0° - 30° and > 30° north and 0° - 30° and > 30 south) in 5° latitude bins. Notice the gaps in the data due to the absence of spots in the minimum years. The plots are noisy and need spatial and temporal averaging. These averaged rotation rates are presented in Figures IV.1, IV.2 and IV.3.

Appendix II

SUNSPOT MOTIONS FROM A STUDY OF KODAIKANAL AND MOUNT WILSON OBSERVATIONS

ROBERT F. HOWARD
National Solar Observatory
National Optical Astronomy Observatories †
Tucson, Arizona 85726, USA

K. R. SIVARAMAN
S. S. GUPTA
Indian Institute of Astrophysics
Bangalore 560 034, India

PAMELA I. GILMAN
Mount Wilson Observatory
And Department of Astronomy
University of California, Los Angeles
Los Angeles, California 90024, USA

ABSTRACT. A study of the daily motions of individual sunspots and of sunspot groups has been undertaken using Kodaikanal and Mount Wilson white-light observations. The Mount Wilson measurements were completed several years ago, and the Kodaikanal measurements have recently been started, using a technique identical to that used for the Mount Wilson data. Both data sets started early in this century, and both have continued to the present time. Preliminary results of a comparison of the two data sets are presented which show a good agreement between them in spot areas and motions. Analysis of the combined data set for the years that are available so far indicates that many more spots are available in such a reduction because the 12-hour time difference between the two sites allows for a more certain identification of individual spots from one observation to the next than in the case when data from only one site are used. This greatly increases the number of sunspots available for motion studies, particularly the smaller sunspots. Preliminary rotation and latitude drift reduction from the combined data set confirm earlier results from the Mount Wilson data alone.

1. Introduction

The important processes that produce the solar activity cycle take place beneath the solar surface. One method to advance our knowledge of the cycle mechanism is to study the motions at the surface of tracers that are rooted below. Considerable effort has gone into this activity in recent years (*e.g.* Schröter, 1985).

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8A

Several years ago the Mount Wilson white-light photoheliogram data set was measured for positions and areas of individual sunspots. This program resulted in a number of studies of properties of spots and spot groups (Howard, Gilman, and Gilman, 1984; Gilman and Howard, 1984a; 1984b; Gilman and Howard, 1985; 1986: Howard and Gilman, 1986; Bogdan, et al., 1988). A similar data set has been accumulated at the Kodaikanal site of the Indian Institute of Astrophysics. This set of plates extends back to 1905 and is in excellent condition.

Recently a digitizing instrument similar to that used for the Mount Wilson data was installed at Kodaikanal, and measurements of sunspots were begun. It is planned to measure every one of the plates at Kodaikanal from 1905 to the present. The technique being used is as nearly identical as possible to that used for the Mount Wilson plates and described in the first of the series of papers that came from this work (Howard, Gilman, and Gilman, 1984).

The measurement of the Kodaikanal white-light plates will soon give us a valuable data set with which to study the motions of sunspots at the solar surface and the radius of the sun during this century. This will be the largest collection of individual sunspot position measures in existence. The Greenwich data set, which has been much used in studies of sunspot motions, is an invaluable set of excellent data covering spot group areas and positions, but it does not have a complete set of individual spot area results. It has, however, been used to study the motions of single-spot groups (*e.g.* Wöhl, 1988).

The work is now in progress and so far several years of data have been measured. In this paper we discuss the measurements and present some of the first results from the Kodaikanal measures and from the combined Kodaikanal-Mount Wilson data set.

2. Observations, Measurements, and Reduction

Daily white-light photographs of the full-disk sun have been taken at Kodaikanal since 1905. These observations have been made with the same instrument and using the same observing procedures during the entire interval. The image size is approximately 20 cm, which is slightly larger than the Mount Wilson images.

The measuring technique for the Kodaikanal plates is quite similar to that used for the Mount Wilson plates. This is done in order to provide a uniform data set for comparison of the Kodaikanal results with the Mount Wilson results and for ease of combining the two data sets. A digitizing pad which is operated by a cursor containing a crosshair is used, as has been described for the Mount Wilson measurements (Howard, Gilman and Gilman, 1984). The accuracy of the digitizer is several times better than that of the best seeing on any of the solar images, and is therefore not a factor in the reduction process.

The measurement of a plate for a day consists of making 8 evenly spaced determinations of the limb position and then two position measures for each sunspot on the disk within 60 deg of the central meridian.

Each year is reduced separately as a whole. Accurate corrections to the positions of the limb points and the spot measures are made for atmospheric refraction using the time and date of the observation. The radius and central position of the solar disk are derived in digitizing board coordinates. Then solar coordinates are derived for each spot measurement point. Areas and positions of the spots are derived. Areas are corrected for projection effects.

With the data for each day tabulated, a search for consecutive day's observations is made. In the case of the combined data set from both observatories, the next available observation is used. The majority of the time differences in the combined data set cluster around 1/2 day. In practice, an upper limit of 1.5 days is used for the determination of 'daily' motions.

For each consecutive pair of observations, differences in latitude and CMD are calculated for each spot group and for each spot. In the case of groups the positions are area-weighted. The technique for determining the identification of individual sunspots is described (Howard, Gilman, and Gilman, 1984).

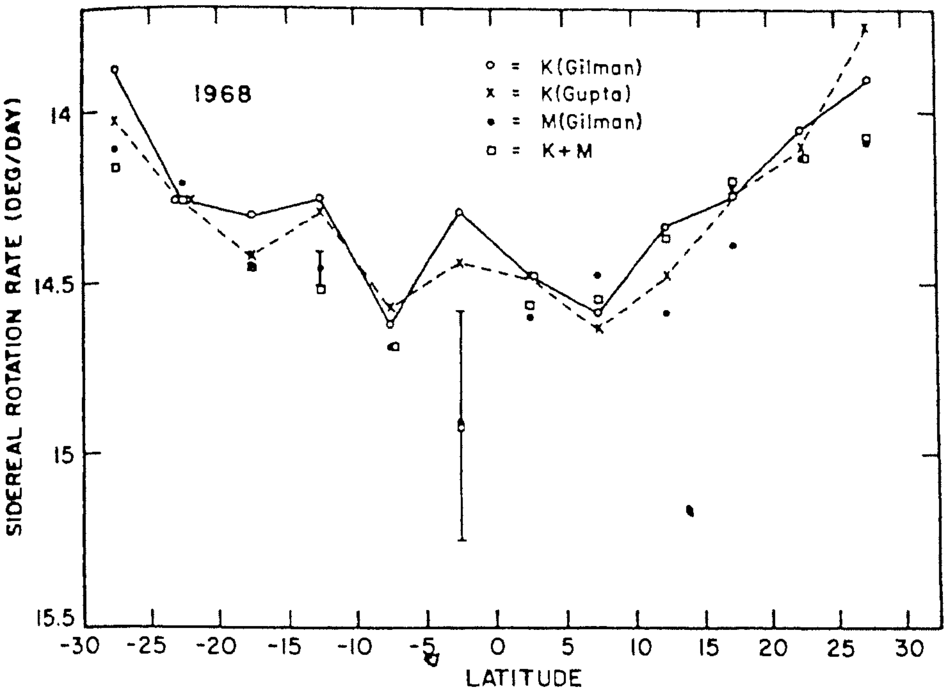

Figure 1. Sidereal rotation rate as a function of latitude for sunspots for the first 9 months of 1968. Circles are Kodaikanal data measured by Gilman; crosses are Kodaikanal data measured by Gupta; solid circles are Mount Wilson data; and squares are combined Kodaikanal-Mount Wilson data (Kodaikanal data measured by Gupta). Typical error bars are shown for low and intermediate latitudes. The errors at high latitudes are similar in magnitude to those at low latitudes.

3. Results

Figure 1 shows a comparison of solar rotation results for various data and various measurements for the first 9 months of 1968. Only part of the year was used because only that part of the year was available for the measurements of Kodaikanal data by PG. The

Kodaikanal data are represented by open circles and crosses. The agreement between the measurements made by the two measurers is seen to be well within the limits of the errors. Error bars (± one standard deviation) are shown for the Mount Wilson measures for low latitudes at 0 to −5 deg (typical also for high latitudes) and for intermediate latitudes at −10 to −15 deg, where there are more spots. The low latitude error bars refer to an average over 11 sunspots, and the intermediate latitude error bars refer to an average over 264 sunspots. These errors are of similar magnitude for Kodaikanal measures and slightly lower for the combined data set.

The differential rotation curve in Figure 1 representing the combined data set is indicated by squares. For the combined data set we used the Kodaikanal measures of SSG. The combined results for any latitude do not always fall between the results from the two separate measurements because it is not a simple average of the two sets. With many 1/2-day determinations in the combined result, this means that there are many individual determinations in this result that are not in the individual observatory results.

We determined the value of A, the sidereal equatorial rotation velocity in the representation $\omega = A + B\sin^2\phi$, where ϕ is the solar latitude. For the full disk data averaged over the 9-month interval, this value is 14.39 deg/day for the Mount Wilson data, 14.52 for the Kodaikanal data measured by PG, 14.62 for the Kodaikanal data measured by SSG, and 14.64 for the combined data set (Kodaikanal data measured by SSG). For the full 1921 through 1982 interval of the Mount Wilson data published earlier (Howard, Gilman, and Gilman, 1984) A is 14.522, although there were significant variations from year to year.

The results of the Kodaikanal measurements so far are encouraging, and we anticipate that we will have an opportunity soon to add significantly to our knowledge of solar dynamics.

4. Acknowlegements

This research has been made possible through a Grant from the US-India Cooperative Science Program of the Division of International Programs, US National Science Foundation. We gratefully acknowledge assistance from a number of people, including A. V. Ananth and R. Srinivasan, of the Indian Institute of Astrophysics, John E. Boyden of the University of California, Los Angeles, and Stephen A. Colley of the National Solar Observatory.

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DISCUSSION

PRIEST: (i) What do you believe is the cause for small spots to rotate faster than large ones? Is it because the small ones are affected more by the photospheric plasma?
(ii) Are there implications for the validity or otherwise of Piddington's tree-trunk model of the subphotospheric structure?

HOWARD: (i) I believe that the important factor is probably the age of the spot. This is not really an answer to this question, but one can imagine a weakening of the connection between the lower and surface layers as the spot ages and grows. Thus the spot would tend to rotate closer to the surface plasma rate as it aged.
(ii) It was this model that led me to assume that each spot group rotated as a whole *i.e.* showed no differential rotation. The fact that this is not the case implies either that the tree trunk model is not valid or that the connection of the spots to the subsurface layer is weak.

MEASUREMENT OF KODAIKANAL WHITE-LIGHT IMAGES

I. *A Comparison of 35 Years of Kodaikanal and Mount Wilson Sunspot Data*

K. R. SIVARAMAN, S. S. GUPTA
Indian Institute of Astrophysics, Bangalore 560034, India

and

ROBERT F. HOWARD
National Solar Observatory, National Optical Astronomy Observatories, Tucson, AZ 85726, U.S.A.*

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Abstract. A program of digitization of the daily white-light solar images from the Kodaikanal station of the Indian Institute of Astrophysics is in progress. A similar set of white-light data from the Mount Wilson Observatory was digitized some years ago. In both cases, areas and positions of individual sunspot umbrae are measured. In this preliminary report, comparisons of these measurements from the two sites are made. It is shown that both area and position measurements are in quite good agreement. The agreement is sufficiently good that it is possible to measure motions and area changes of sunspots from one site to the next, involving time differences from about 12 hours to about 36 hours. This enables us to trace the motions of many more small sunspots than could be done from one site alone. Very small systematic differences in rotation rate between the two sites of about 0.4% are found. A portion of this discrepancy is apparently due to the difference in plate scales between the two sites. Another contributing factor in the difference is the latitude visibility of sunspots. In addition it is suggested that a small, systematic difference in the measured radii at the two sites may contribute a small amount to this discrepancy, but it has not been possible to confirm this hypothesis. It is concluded that in general, when dealing with high precision rotation results of this sort, one must be extremely careful about subtle systematic effects.

1. Introduction

A program to measure the area and position of each sunspot umbra from the daily photoheliograms of the Kodaikanal station of the Indian Institute of Astrophysics was initiated recently. These photographic data extend back to 1904 in an observational series that continues to the present day. The measurements follow the procedure adopted for the measurement of a similar data set in the interval 1917–1985, done at Mount Wilson several years ago. The instrument, the data set, the method of digitizing, and the first results of the Mount Wilson measurements were discussed in an earlier paper (Howard, Gilman, and Gilman, 1984; herafter HGG).

We intend to compare the results from the two data sets and also to combine the data to obtain a more comprehensive data set of sunspot areas and positions, covering most of this century. We expect several advantages in these combined data. Of course, the Kodaikanal data extend about one activity cycle further back

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than does the Mount Wilson data set. Also the 12-hour time difference between the two sites should make it possible to follow (from one observation to the next at the other site) more small spots than would be possible from any one site because of the short lifetimes of these features. Furthermore, the climate patterns of the two sites are complementary: the winter is the best observing season, in terms of number of clear days, at Kodaikanal and the summer is the worst, while at Mount Wilson the summer is the best season, and the winter is the worst. This factor should significantly increase the total coverage when the data from both sites are combined.

An earlier paper (Howard *et al.*, 1989) discussed the measurements and presented some very preliminary comparisons of the two data sets. Now a substantial fraction of the Kodaikanal measurements have been completed, and in this paper we present the results of some detailed comparisons of measurements at the two sites.

2. The Telescope and the Camera

Direct photography of the Sun commenced at Kodaikanal in August 1903 when the photoheliograph Dallmeyer No. 4 was overhauled and put into operation. This used a 4-inch (10 cm) aperture, 5-foot (1.5 m) focal length objective lens (made by Dallmeyer), modified to give an 8-inch (20 cm) diameter image of the Sun similar to the photoheliographs operated from Dehra Dun (India), Mauritius, and Greenwich. Photography on a systematic, daily basis started in 1904, using this photoheliograph on Lantern plates of size 10 × 10-inches (25.4 × 25.4 cm) and continued until 31 July, 1912. In 1908 the objective lens was replaced by a new one of superior quality. Starting on 31 July, 1912 the direct photography was carried out with the Lerebour and Secretan equatorial telescope having an aperture of 6 inches (15 cm) and a focal length of 8 feet (2.44 m). This, one of the oldest of the Kodaikanal telescopes, was brought to the site in 1898 from Madras and had been used since 1901, with an enlarged 8-inch diameter image, for visual observations of the solar disk. In 1912 the original objective was replaced by John Evershed with a Cooke photo-visual objective of the same aperture, and it was installed in its present location and adapted for direct photography, using an 8-inch diameter solar image in addition to the visual observations. Photography of the Sun continued regularly with this setup until June 1915, when this telescope was dismantled and the objective and auxiliary optical components were moved to Kashmir, where John Evershed used them for solar photography during 1916. The telescope was reinstalled at Kodaikanal and regular observations as before were resumed starting in February 1917. In June 1918, the 6-inch photo-visual lens, used until that time, was replaced by a visual achromat of the same diameter and focal length, and a green filter was also added to the telescope. This gave very good quality images of the Sun, and regular photography was resumed. Since then the photoheliograms have been obtained with this telescope using the same procedures up to the present time.

The camera used to photograph the solar image has a focal plane shutter which is a metal plate in the form of a sector with a filter mount on it for mounting a broad-band filter. The shutter is activated by releasing a metal spring, and the shutter then slides across an aperture, providing an exposure with a duration of about 0.001 s. In 1975 the availability of plates (Ilford special Lantern, 10 × 10-inch size) became irregular and these were replaced by high-contrast film of the same size.

The plates/films containing the images are stored individually in paper envelopes, and they have been carefully preserved over the years in the plate vault at Kodaikanal under good conditions for preservation of the materials. The observing logs for each day of the observations are also well preserved.

3. Data

For each site, the observational data consist of daily, white-light photographs of the full solar disk. The Kodaikanal images are about 20 cm in diameter, as described above, whereas the Mount Wilson images have a diameter of about 16.5 cm, so the image size (area) of the Kodaikanal observations is larger than that of the Mount Wilson images by nearly 50%. Exposure times are comparable and other features of the observations, notably the average photographic densities, appear to be about the same. Observations at both sites were generally made in the early morning, although in recent decades at Mount Wilson other observations have interfered, and the result has been that many of the daily plates were taken later in the morning, or even in the afternoon. An effect of this change in observing time will be discussed below.

The Kodaikanal data set was started early in 1904, and continues to this date. The Mount Wilson data set started early in 1917, and measurements have been made through the data of 1985, although the observations continue.

The program for measuring the Kodaikanal data is similar to that carried out for the Mount Wilson data: measurements are made in full-year intervals, and the order of the years is random – a different random selection of years than that used for the Mount Wilson data. In a later table in this paper we listed the 35 years for which the Kodaikanal measurements have been completed so far (as of September 1992) and thus for which (after 1916 and before 1986) there are data to compare from the two sites. There are actually a total of 46 years that have been measured to date from this data set, and of these 35 overlap with available Mount Wilson data.

Altogether there are 16568 days of data and 13838 consecutive day pairs in the full (69-year) Mount Wilson data set, and 9712 days of Kodaikanal data with 7924 consecutive day pairs in the full (46-year) data set measured so far. In the overlapping 35 years there were 8057 days and 6689 consecutive day pairs in the Mount Wilson data set and 8046 days with 6635 consecutive day pairs in the Kodaikanal data. In the combined data set for the 35 years of overlapping data there are 16178 'days' of observation and 15563 'consecutive day pairs

4. Measurement Technique

The technique of measurement of the umbral areas used in this study was discussed in detail in the earlier paper (HGG). An identical technique is used at Kodaikanal. Each plate is oriented with the axis of rotation along the Y-axis of a digitizing pad. Positions are recorded in the two coordinates of the pad by use of a hand-held cursor. The Mount Wilson plates have pole markers on them, which were exposed using a small mask at the time the image was exposed. The Kodaikanal images have a straight line exposed across them, near disk center, denoting the east–west direction in the sky. This line is formed by a thin wire stretched across the focal plane of the telescope. The Kodaikanal plates are adjusted in orientation angle at the time of the measurement, using the ephemeris P-angle to orient the rotation pole along the Y-axis.

The measurement of each day's plate begins with the digitization of 8 limb points, equally spaced apart. This is done to determine the precise position of the solar disk in the coordinates of the digitizing pad. Then each umbra on the plate is measured with two positions, as described in the earlier paper (HGG). The positional accuracy of the measurements at each site is limited by the seeing, which is generally 1–2 arc sec. The accuracy of the umbral areas varies with the size and shape of the spot, of course. In the earlier work this accuracy for individual spot measurements was estimated to lead to individual random errors of about 30% for small spots, and it is likely that for the more recent measurements approximately the same estimate would apply. The larger plate scale for the Kodaikanal plates should lead to smaller random measurement errors. Possible systematic errors are discussed below.

5. Reduction Technique

The reduction technique is described in the earlier work (HGG). Corrections are made for atmospheric refraction in both the limb solution and the individual umbral area and position determinations. All areas are corrected to disk center.

In practice, the same computer programs are used for the data of both sites for all phases of the reduction, with only some necessary differences between the sites included (such as the site latitude, which is needed in the determination of the altitude of each measurement, which in turn is needed for the calculation of the atmospheric refraction correction).

The result of this first phase of the analysis is a list of umbral areas, positions, dates, and times of observation. The second step is to combine data from adjacent days. In the case of observations from one site only, we can simply use the date to determine whether an observation is from the next day or not. When we combine the data from two sites, we use the date and time, and define the 'next day' to

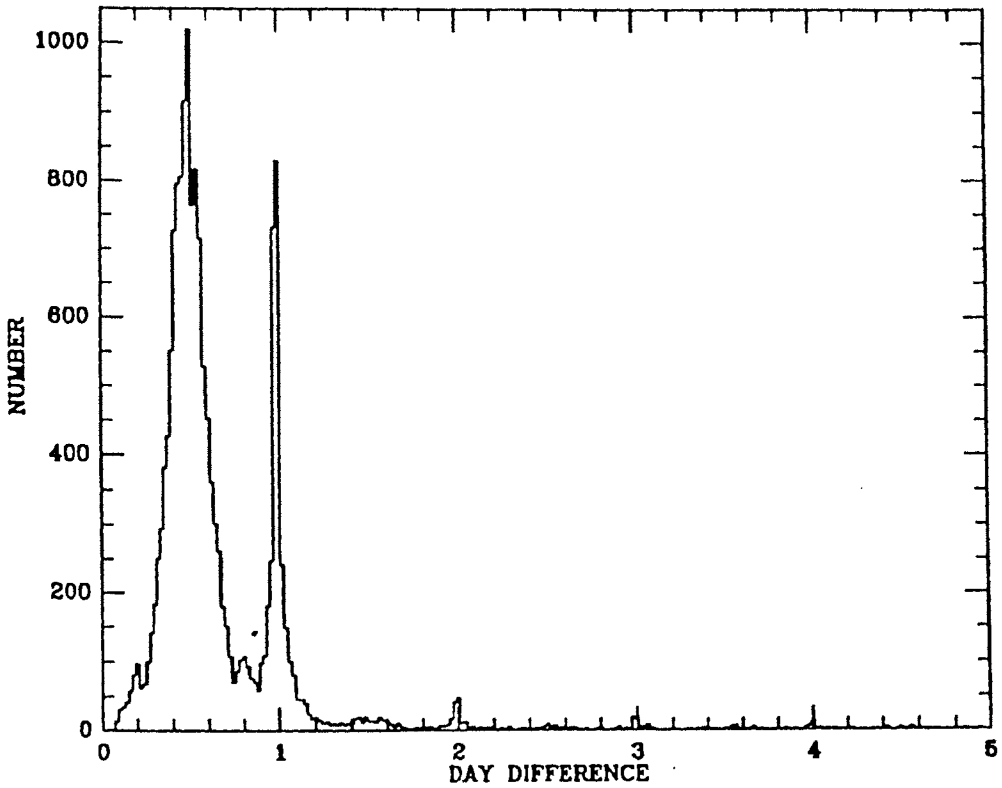

Fig. 1. Distribution of time differences (in days) between the data at the two sites. The 35-year, two-site data set was used to compute these differences.

be the next observation (from either site) which falls within some time interval. This interval has been chosen arbitrarily to be 1.8 days (43.2 hours) from the first observation. This interval was chosen to include all the 1 $\frac{1}{2}$-day differences, judging from Figure 1. If there is no observation at either site that falls within this time interval, then, by definition, we do not have two 'consecutive days'.

Figure 1 shows a plot of time differences between 'consecutive days' for the combined data set from the two sites (35 years of data). This is shown to illustrate the relatively large number of 12-hour 'returns' that are found in the combined data set. Clearly the biggest peak falls near 12 hours. The second biggest peak is centered on 24 hours, and this represents pairs of observations from the same site. Much smaller peaks may be seen at 12-hour intervals for several days. The rather broad width of the 12- and 36-hour peaks is due to the fact that there is a wide range of times of observations at the two sites. This is particularly true at Mount Wilson, where, as mentioned above, recently other observations have interfered with the cadence of the white-light photographs. If one plots individual years in the same way as Figure 1 is plotted, one sees for the early years a rather narrow peak at 12 and even 36 hours. In later years, in plots similar to Figure 1, there is a tendency to see a double peak centered near these two times, one corresponding to Mount Wilson observations made early in the day and one corresponding to Mount Wilson observations made late in the day.

Rotation rates are determined, as in the earlier work (HGG), by dividing the longitude difference of the group or of each individual sunspot within the group by the time difference between the two observations. One would expect therefore that random measurement errors would lead to somewhat larger random errors in rotation determinations for the combined data set than for the data set from either site, because the time differences are shorter on average for the combined set. Note, however, that errors in longitude measurement of this sort do not result in random, independent errors in the average rotation rates. This is because an error in one longitude in a string of such measurements will increase the derived rotation rate on one side and decrease it on the other side, so that the average will be relatively unaffected (Howard, 1992). This means that we may expect that to some extent the errors derived for the rotation rates from these data will be overestimated. In addition, of course, for the combined data set, the larger number of days of observation will lower the errors below those derived from the individual sites.

6. Area Comparisons

Table I gives daily, full-disk sunspot umbral areas, in μhemisphere, averaged over $\frac{1}{12}$ of a year and full years for both sites for the 35 years for which there is overlapping data. One would not expect perfect agreement between these results from the two sites because the coverage is rarely exactly the same, i.e., rarely in a month (and never in a year) is exactly the same set of days covered in both data sets. Nevertheless, the agreement is encouragingly good. Generally active months are seen as such at both sites. (Note that these results for Mount Wilson are not identical to a similar table given in the earlier paper (HGG). The reason for this is that over the years a number of small improvements have been made to the software, which has resulted in identifying generally a somewhat greater number of sunspots from these observations than was done before.)

It can be seen in Table I, however, that generally, although not for all months or years, the Kodaikanal daily spot areas are larger than those from the Mount Wilson plates. This result will be discussed in more detail in what follows.

In the full (69-year) Mount Wilson data set there were a total of 366680 sunspots measured on 13838 day pairs, and of these 111070 were identified as the same spot (a 'return') on the next day. Using just the overlapping 35 years, the Mount Wilson spot measurements on 6689 consecutive day pairs numbered 180667, and of this total, 55357 spots were measured as returns. In the same set of Kodaikanal years 188408 spots were measured on 6635 consecutive day pairs, and of this total, 70178 spots returned. For the full 46 years measured so far at Kodaikanal, the numbers are 77527 returns out of 209006 spots. In the 35-year combined data set, there were a total of 421678 spots, out of which 74372 spots returned in 15563 'day pairs'. (Note here that 'spots' means spots measured on a single day. One spot may be counted more than once as it rotates across the solar disk.)

From these numbers, several conclusions may be drawn. To begin with, there are about 5% more spots measured per day on the Kodaikanal plates than on the Mount Wilson plates. This, we believe, is due to the larger image scale for the Kodaikanal observations. We are seeing a greater number of smaller spots in this data set. It is possible also that systematic seeing differences between the two sites may play a role in this comparison, but it is impossible to quantify this effect. It is known that both sites are quite good seeing sites, but beyond that we cannot determine quantitatively any differences without experiments that are beyond the scope of this study.

Another conclusion from the results cited above is that there is a greater fraction of returns seen on the Kodaikanal plates (≈ 0.37) than on the Mount Wilson plates (≈ 0.31). This is most likely due firstly to the fact that, as mentioned above, more small spots are seen at Kodaikanal, and thus it is more likely that spots can be followed longer as they decay to a smaller size, and, secondly, that more spots measured means that better group positions are determined, and thus, since individual spot returns are determined from the relative positions of spots within groups on two consecutive days (HGG), better identifications of individual spot returns can be made. Note that the fraction of returns for the combined data set (≈ 0.41) is greater than that for either site reduced separately. This undoubtedly results from the fact that with a shorter time base more short-lived spots can be traced from one observation to the next.

As one means of testing this explanation for the presence of more spots at Kodaikanal than at Mount Wilson, we have examined the distribution of spot sizes at the two sites. Table II gives the distributions and relative distributions of sunspot counts in various size categories at the two sites.

These data come from the 35-year overlapping data set for the two sites, and they represent the total number of spots measured, not the number that were identified as returns or even the number seen on consecutive days. Thus the number of spots is larger than that discussed elsewhere in this paper. It may be seen that the largest differences and the largest percentage differences are seen at the smallest spot sizes, which is what one would expect because of the difference in image scales. It should be remembered that at both sites, for the smallest spots, comparable to the size of the cross-hair, the measurer did not attempt to measure the size of the spot in the usual manner, but instead placed the cross-hair centered on the spot and entered two identical positions into the computer. Such small spots were arbitrarily assigned an area of 0.05 μhemisphere (HGG).

We consider that Table II represents a very satisfactory agreement between the area measurements at the two sites, considering the image-scale differences mentioned above. Except for the smallest spots, the distributions agree within a few tenths of a percent. These area distributions compare well with a recent detailed

TABLE I

Average daily sunspot areas for the two sites (μ hemisphere)

Year	1	2	3	4	5	6	7	8	9	10	11	12	Full year
1917													
Kodai	258.7	80.1	92.4	89.3	110.9	104.0	113.2	169.8	142.6	57.7	93.1	167.8	111.9
Mt. W.	52.4	92.8	81.8	76.3	105.2	89.3	103.6	214.4	136.0	63.4	96.9	147.6	107.4
1918													
Kodai	108.7	40.7	60.8	68.5	50.1	73.1	93.2	97.0	60.7	69.8	54.3	59.0	67.9
Mt. W.	91.9	48.3	62.1	70.0	65.5	41.1	89.5	106.7	62.9	67.1	48.6	40.2	67.9
1922													
Kodai	7.0	53.6	101.8	17.7	10.4	5.4	18.0	13.4	3.1	7.0	6.1	74.6	35.7
Mt. W.	3.3	42.7	56.3	13.3	8.5	4.3	8.7	14.8	6.2	7.2	1.7	40.5	21.5
1923													
Kodai	21.0	1.5	5.3	3.0	2.7	5.3	0.3	2.4	3.9	2.9	6.5	3.9	5.0
Mt. W.	16.8	0.1	6.7	1.5	2.6	7.3	1.6	4.4	2.5	3.1	5.1	0.5	4.1
1924													
Kodai	2.8	32.2	8.3	42.1	17.6	30.3	31.7	27.1	24.1	28.6	68.7	22.3	30.5
Mt. W.	0.7	17.5	2.4	24.9	14.7	23.3	20.1	14.9	16.6	16.2	35.0	10.8	18.6
1928													
Kodai	117.8	65.6	92.4	76.8	68.3	86.5	111.7	64.8	114.1	63.4	28.1	60.9	81.3
Mt. W.	100.0	67.1	93.9	70.9	55.9	67.4	103.1	65.0	117.7	47.4	29.0	36.2	74.3
1932													
Kodai	9.1	16.7	6.4	15.3	18.0	17.7	5.9	8.7	0.2	5.2	9.7	17.7	12.4
Mt. W.	8.3	21.8	5.8	12.9	12.9	17.4	4.8	3.6	0.2	5.2	11.5	11.8	10.7
1933													
Kodai	11.8	70.2	10.0	3.6	1.4	2.3	1.8	0.0	2.8	13.3	0.0	0.0	19.0
Mt. W.	17.5	68.5	15.6	0.7	2.4	5.3	0.0	0.0	9.5	13.8	0.0	3.3	20.1
1935													
Kodai	14.6	14.2	18.6	11.7	33.1	42.5	32.8	22.8	34.3	58.8	102.9	147.5	40.0
Mt. W.	23.0	7.1	15.5	10.9	34.4	39.3	29.4	19.5	30.2	51.6	85.4	83.2	37.6

Table I (continued)

Year	1	2	3	4	5	6	7	8	9	10	11	12	Full year
1938													
Kodai	169.6	123.3	112.4	171.2	172.6	106.3	216.6	116.9	79.5	89.8	163.0	128.7	138.0
Mt. W.	112.8	88.2	81.9	124.7	135.7	101.9	176.3	113.0	79.6	87.9	137.8	75.0	115.0
1940													
Kodai	78.9	67.3	107.7	42.9	56.0	96.7	64.2	133.2	76.0	52.5	41.2	67.7	73.9
Mt. W.	49.8	49.4	77.4	36.4	49.6	78.1	61.2	103.6	58.4	52.5	37.3	46.8	58.3
1943													
Kodai	7.8	48.2	46.4	49.5	4.9	4.0	17.2	8.0	16.3	21.4	11.6	29.7	26.0
Mt. W.	4.6	40.6	36.3	46.3	15.0	3.6	23.7	5.3	22.1	30.7	13.7	18.9	22.4
1944													
Kodai	2.2	0.0	22.8	0.0	1.0	0.0	18.1	10.4	26.6	12.2	15.2	52.8	19.1
Mt. W.	1.6	0.0	0.0	6.7	1.4	2.7	5.4	9.7	21.8	11.0	9.2	45.2	15.9
1947													
Kodai	199.4	156.8	237.5	229.9	230.9	137.1	161.5	187.8	125.0	134.0	82.1	137.4	166.4
Mt. W.	139.3	141.8	206.5	192.6	183.3	104.0	164.0	210.8	180.0	163.7	110.3	139.3	161.5
1948													
Kodai	105.5	93.5	127.1	275.4	229.6	189.1	170.5	178.3	121.8	112.5	76.4	162.0	153.4
Mt. W.	76.9	56.8	103.8	234.0	193.3	156.9	139.8	130.6	113.7	92.8	48.4	121.4	102.2
1949													
Kodai	144.5	287.3	259.2	150.0	93.1	107.5	157.0	139.9	143.2	122.0	160.8	100.7	158.1
Mt. W.	86.8	198.3	184.5	108.3	98.4	105.3	111.1	128.8	122.2	159.3	129.9	82.3	125.5
1950													
Kodai	88.9	120.3	104.9	153.3	74.0	65.5	52.0	61.0	36.7	30.0	43.1	34.1	76.0
Mt. W.	69.4	122.3	104.6	163.7	99.9	70.1	83.8	70.1	28.0	34.5	44.8	46.5	82.5
1951													
Kodai	62.4	36.1	59.0	107.7	246.7	133.2	37.4	48.3	77.0	43.1	38.4	44.9	80.3
Mt. W.	39.7	25.1	55.2	105.7	176.2	126.3	49.3	54.7	61.6	38.0	33.1	30.7	70.2

Table I (continued)

Year	1	2	3	4	5	6	7	8	9	10	11	12	Full year
1952													
Kodai	33.4	48.6	25.0	29.9	28.5	37.7	33.0	49.6	27.6	19.8	32.8	44.0	33.7
Mt. W.	13.1	14.8	17.3	13.3	14.6	33.9	38.7	45.5	22.2	20.6	29.0	26.0	25.2
1953													
Kodai	23.1	0.0	22.7	83.6	6.7	17.6	15.0	33.1	9.1	6.9	0.6	4.4	25.7
Mt. W.	13.1	1.1	7.2	19.5	7.0	10.9	12.5	24.0	11.1	4.7	0.0	3.3	12.0
1957													
Kodai	174.7	116.7	129.6	115.3	223.0	345.2	224.9	179.6	367.3	398.9	237.1	291.3	223.0
Mt. W.	160.2	99.8	100.0	125.2	154.2	304.3	213.6	182.6	338.5	309.4	194.3	298.3	208.1
1959													
Kodai	301.8	141.8	248.5	125.6	148.0	147.7	155.5	152.3	97.4	65.3	129.7	129.0	160.8
Mt. W.	266.3	118.1	231.0	133.4	157.2	184.8	159.8	186.0	109.8	92.3	110.0	125.9	185.5
1960													
Kodai	140.3	100.7	67.4	107.4	98.4	79.6	79.1	148.7	103.8	76.2	49.6	59.7	93.7
Mt. W.	121.5	86.3	72.7	115.5	103.6	85.8	101.3	110.5	82.9	58.3	85.3	43.0	88.1
1963													
Kodai	19.8	11.7	7.5	33.9	29.0	19.3	11.5	20.2	53.5	33.8	11.8	3.0	22.7
Mt. W.	19.0	22.7	0.0	0.0	0.0	0.0	12.2	16.4	36.8	27.2	6.7	4.3	18.3
1965													
Kodai	10.7	2.8	7.0	4.8	22.4	12.5	9.5	5.1	9.0	8.7	16.9	12.8	10.3
Mt. W.	9.1	3.5	4.5	3.6	26.0	11.3	8.6	3.5	12.3	9.9	11.8	9.1	9.2
1966													
Kodai	17.7	16.2	58.4	59.4	31.3	23.3	50.2	31.4	79.5	44.5	51.9	77.6	45.9
Mt. W.	20.6	23.2	55.3	57.5	39.2	29.2	64.5	48.4	71.5	40.7	54.8	115.6	50.5
1968													
Kodai	123.3	97.6	68.6	67.7	127.3	113.8	78.3	102.5	75.0	86.9	66.1	104.2	92.9
Mt. W.	103.6	93.0	70.3	53.5	105.1	95.2	80.6	77.0	71.9	81.6	58.4	84.5	80.3

Table 1 (continued)

Year	1	2	3	4	5	6	7	8	9	10	11	12	Full year
1973													
Kodai	40.0	23.4	55.3	45.3	25.5	23.3	7.0	22.9	75.7	29.1	19.7	53.3	36.2
Mt. W.	24.1	32.0	52.3	57.2	33.5	29.9	26.0	29.5	71.9	25.2	34.9	17.6	38.3
1974													
Kodai	19.5	27.9	6.5	50.7	48.9	41.8	45.7	29.7	69.4	64.8	25.3	28.6	37.8
Mt. W.	30.6	31.3	9.1	45.5	25.5	29.5	54.3	28.6	56.1	60.2	12.4	22.7	36.2
1975													
Kodai	15.0	9.2	8.4	8.4	18.1	12.1	22.7	83.9	2.2	4.4	20.7	6.4	17.3
Mt. W.	14.1	15.7	22.7	12.2	18.1	29.5	35.2	76.8	4.4	13.8	29.1	17.4	28.6
1976													
Kodai	11.4	2.9	34.8	28.2	3.8	10.1	1.0	16.8	9.0	13.1	1.5	13.6	15.5
Mt. W.	18.5	3.2	33.2	12.7	3.9	8.2	1.1	17.5	9.1	15.0	6.1	11.4	14.2
1977													
Kodai	14.4	19.4	2.6	10.5	16.1	54.0	15.0	18.6	48.1	44.8	20.0	56.8	27.9
Mt. W.	16.1	19.1	5.5	8.0	13.1	31.9	12.1	20.5	37.4	32.7	15.4	47.1	23.7
1981													
Kodai	105.4	221.2	192.1	254.7	188.4	126.0	240.1	241.4	246.1	387.1	161.7	257.2	208.8
Mt. W.	59.1	123.8	106.5	153.5	109.6	79.0	159.5	230.4	156.3	143.5	53.7	143.9	136.5
1982													
Kodai	103.2	223.5	215.4	111.2	71.9	133.1	50.6	140.5	99.9	92.2	105.2	171.5	137.9
Mt. W.	124.2	161.4	162.7	106.5	72.2	156.3	104.6	137.3	88.8	54.6	51.5	135.2	112.9
1984													
Kodai	91.3	138.3	80.9	146.6	98.5	33.4	17.6	11.6	20.2	6.6	15.8	7.6	70.6
Mt. W.	90.7	89.4	56.3	79.3	69.7	32.2	19.4	12.5	13.9	3.6	9.9	5.0	43.5

TABLE II

Sunspot area distributions

Area	Mount Wilson		Kodaikanal	
μhemisphere	Number	%	Number	%
0–1	106146	51.0	108287	49.1
1–2	28503	13.7	32044	14.5
2–3	16842	8.1	19692	8.9
3–4	11508	5.5	13533	6.1
4–5	8296	4.0	9463	4.3
5–6	6268	3.0	6914	3.1
6–7	4888	2.4	5083	2.3
7–8	3995	1.9	3952	1.8
8–9	3054	1.5	3085	1.4
9–10	2588	1.2	2364	1.1
10–11	2023	1.0	1908	0.9
11–12	1688	0.8	1557	0.7
12–13	1416	0.7	1352	0.6
13–14	1210	0.6	1082	0.5
14–15	988	0.5	956	0.4
15–16	882	0.4	869	0.4
16–17	804	0.4	745	0.3
17–18	653	0.3	594	0.3
18–19	599	0.3	538	0.2
19–20	506	0.2	500	0.2
20–21	465	0.2	461	0.2
21–22	423	0.2	389	0.2
22–23	311	0.2	349	0.2
23–24	325	0.2	312	0.1
24–25	274	0.1	323	0.2
>25	3287	1.6	3994	1.8
Total	207942	100.0	220346	100.0

study, using the Mount Wilson data (Bogdan *et al.*, 1988). Figure 2 shows the distribution of the areas for the data from each site. Note that in these comparisons, we are not in all cases comparing images from the same days. Seasonal differences in coverage undoubtedly affect the spot size differences, both in this figure and in the tables in this paper, especially for the largest spots, which are few in number, and perhaps also to some extent for the rotation rate comparisons discussed below.

In order to test this proposed explanation (sampling differences) for the spot size differences, we chose seven years near five different activity maxima and examined in detail the numbers of spots with areas greater than 40 μhemisphere.

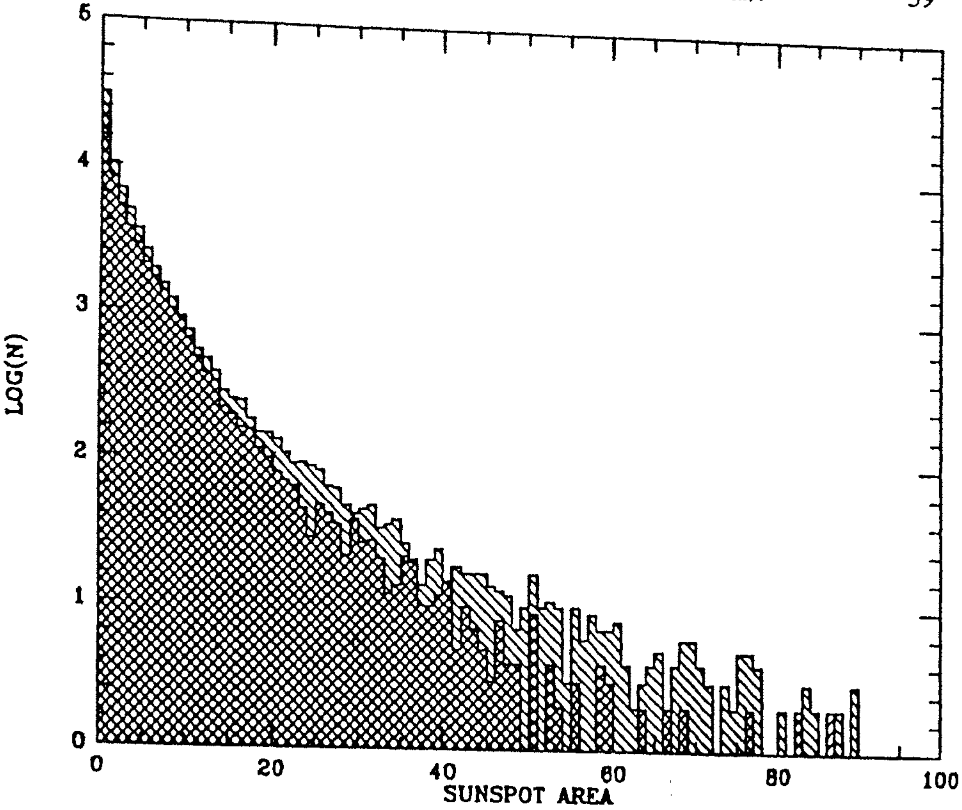

Fig. 2. Histograms of the distribution of sunspots by area (in μhemisphere). The two identical 35-year data sets are used for this plot. The lower, hatched curve represents the Mount Wilson data, and the upper curve is the Kodaikanal data.

These seven years represent a substantial fraction of the large spots seen in this data set. In these years there were 744 such spots measured on the Kodaikanal plates and 463 such spots measured on the Mount Wilson plates. Altogether 188 of these large spots were observed at both sites on the same days. Five-hundred forty of the spots were measured at Kodaikanal on days when there were no observations at Mount Wilson, and 259 of the spots were measured on the Mount Wilson plates on days when there were no observations at Kodaikanal. Only a few percent of the large spots were missed at either site in the measurement process, judging from a comparison with the sunspot drawings in *Solar Geophysical Data*, and many of these are likely to be spots which developed between when the photographs were taken and when the drawings were made. Thus statistical fluctuations account for the differences in the numbers of large sunspots measured at the two sites.

TABLE III

Sunspot sidereal rotation rates by latitude zone, data set, and site

Latitude	<-30	-30-25	-25-20	-20-15	-15-10	-10-5	-05-00	+00+05	+05+10	+10+15	+15+20	+20+25	+25+30	>+30
35 years														
Kodai rate	13.73	14.03	14.19	14.37	14.47	14.56	14.59	14.62	14.54	14.41	14.30	14.13	14.01	13.74
St. dev.	0.034	0.020	0.012	0.009	0.007	0.009	0.016	0.014	0.007	0.006	0.008	0.010	0.016	0.029
No. of spots	627	1551	3503	6831	9600	6907	1908	2691	8615	11543	7806	5615	2363	617
Mt W. rate	13.68	13.92	14.12	14.29	14.38	14.53	14.60	14.53	14.46	14.37	14.25	14.13	14.00	13.75
St. dev.	0.043	0.027	0.017	0.012	0.010	0.011	0.020	0.020	0.011	0.009	0.011	0.014	0.024	0.044
No. of spots	477	1140	2940	5785	7276	6429	1892	1822	6330	9212	6088	3858	1567	538
Both sites	13.90	14.10	14.25	14.44	14.49	14.62	14.67	14.59	14.55	14.47	14.35	14.24	14.18	13.92
St. dev.	0.058	0.031	0.021	0.015	0.011	0.013	0.025	0.022	0.012	0.010	0.013	0.016	0.028	0.054
No. of spots	507	1390	3233	6369	8741	6782	2004	2277	8193	11807	7960	5162	1989	515
46 years														
Kodai rate	13.74	14.04	14.20	14.36	14.46	14.56	14.58	14.63	14.54	14.41	14.30	14.13	14.02	13.73
St. dev.	0.033	0.020	0.012	0.008	0.007	0.008	0.015	0.013	0.007	0.006	0.008	0.009	0.015	0.029
No. of spots	663	1675	3974	7763	10605	7710	2237	2962	9722	12615	8471	6042	2460	627

7. Rotation Comparisons

Table III shows average rotation rates in deg day^{-1} sidereal for individual sunspots in 5-deg latitude zones for the overlapping 35-year data sets for Kodaikanal, Mount Wilson, and the combined data set. For each latitude zone the standard deviation of the mean is listed from the determination of the mean rate in that zone, as well as the number of spots that were measured in each zone. Also given here is the same set of results for the full 46-year data set that is available for the Kodaikanal data. We have also determined the coefficients A and B in the expression $\omega = A + B \sin^2 \phi$ deg day^{-1}, where ϕ is the latitude. These solutions were determined for all the individual spots, not using the average latitude zones. For the 35-year Kodaikanal data, $A = 14.591 \pm 0.004$ and $B = -2.92 \pm 0.042$. For the same years, the Mount Wilson result is $A = 14.533 \pm 0.006$ and $B = -2.87 \pm 0.071$. For the combined data set, $A = 14.610 \pm 0.007$ and $B = -2.448 \pm 0.070$. For the 46-year data set, the Kodaikanal results are: $A = 14.589 \pm 0.004$ and $B = -2.90 \pm 0.04$.

It is clear that the average sunspot rotation rate is greater by about 0.4% ($\approx$ 8 m s^{-1} at the equator) for the Kodaikanal data than for the Mount Wilson data. This is obtained from the equatorial rates listed above. This is a small difference, of course, nevertheless it is a significant difference, as may be judged from the errors – which as noted above are likely to be overestimated – and from Figure 3, which compares the latitude dependence of the rotation rate at the two sites, using the 35-year data sets. This difference could be due to the fact that, as discussed above, more smaller spots are measured in the Kodaikanal data set, and, as has been determined previously, smaller spots rotate faster than larger spots (HGG).

In order to investigate this possibility further, and also in order to examine the possibility that there are small systematic differences in the measuring technique between the two measurers that might lead to systematic differences in the measured areas of some or all sunspots at the two sites, we have examined the rotation rate of spots as a function of spot size from the two data sets. Figure 4 shows a plot of average rotation rates for sunspots averaged over area bins of 1 μhemisphere. It can be seen in Figure 4 that the Kodaikanal rotation rates are significantly faster than the Mount Wilson rates in the interval 2–8 μhemisphere by roughly 0.05 deg day^{-1}, or about 0.3%. Curiously, the rotation rates of the smallest (1 μhemisphere) spots are quite close.

Judging from the relative area distributions of Table II and the rotation rate distributions of Figure 4, one can estimate that the rate differences seen in Figure 4 will affect the average rotation rates at the two sites by enough to account for a rotation rate difference of 0.18% between the two sites. This was done by weighting the rotation rates at each area bin by the number of spots in that bin at the other site and deriving an average rate for each site from these weighted quantities. This may still leave about half the rotation rate difference between the two sites unaccounted for. This appears to be well within the errors of measurement, although, as discussed above, these errors are overestimates.

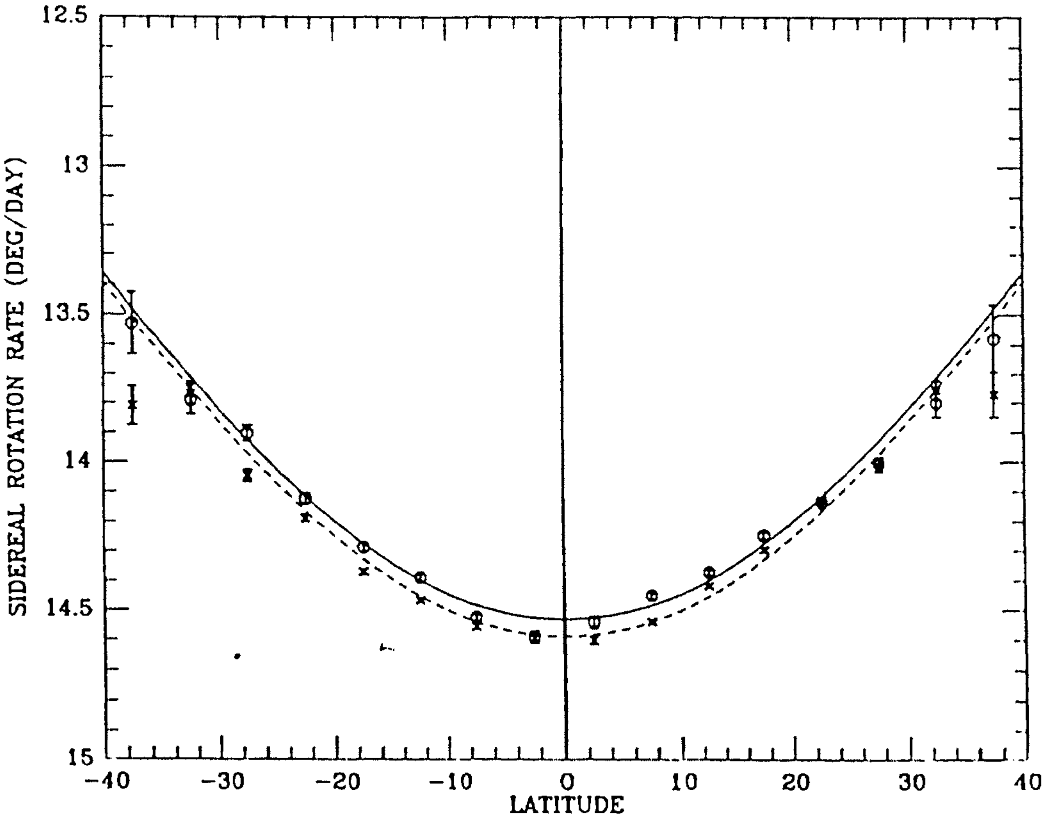

Fig. 3. Rotation rate in deg day^{-1} averaged over 5-deg bins of latitude for Mount Wilson (solid lines) and Kodaikanal (dashed lines) data. The identical 35-year data sets are used for this plot. Full error bars for this and the remaining figures are two standard errors. (For many of the points here the error bars are smaller than the points.)

Another possible cause for systematic rotation rate differences between the two sites is a systematic difference in the radius determinations. This results from the projection from the plane of the photographic plate to coordinates on the solar surface. Such an error in the measured radius could be caused by slightly different techniques used by the two measurers, for example. But this seems a bit unlikely. In order to account for the full measured difference in rotation rate, the systematic difference in radius would have to be 2 arc sec. This is larger than the average seeing effect at either site, and corresponds to about 0.2 mm on the plates. It is possible, however, that systematic differences in this quantity do exist and affect the rotation results at some level.

It is possible to estimate the effect of radius error on the rotation rate by examining the rotation rate derived at various central meridian distances (CMDs). Because of projection effects, the effects of radius errors on measured rotation rates will be greater at greater distances from the central meridian. Figure 5 shows average rotation rates for various CMD values. This was done for umbral areas

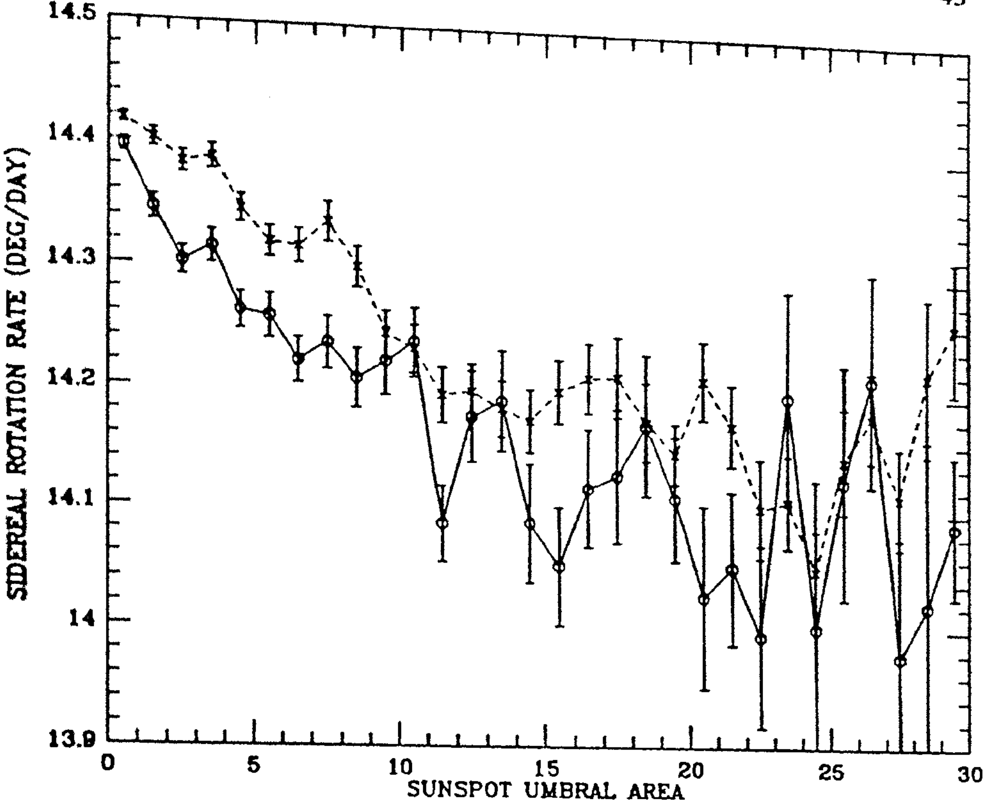

Fig. 4. Average rotation rates for spots in area bins of 1 μhemisphere. Circles and solid lines represent the 35-year Mount Wilson data set, and the ×'s and dashed lines represent the same years in the Kodaikanal data.

<1 μhemisphere only in order to avoid the effects of varying visibility of sunspots at different CMD values combined with the faster rotation shown by smaller sunspots. Clearly there is little or no significant effect here. Linear least-squares solutions for the points that go into Figure 5 give identical slopes for the two data sets (+0.0003), but the errors in each case are nearly the same as the slopes. Shown for reference are two modelled results (from a simple geometrical model of the projection of coordinates on a sphere) for a 0.1% and 0.2% error in radius – a radius measured to be too large by those amounts. The results for either site are probably within that possible error, although the noise in this determination is somewhat higher than the difference we are looking for. The very low value for the Mount Wilson data nearest the limb also has a relatively large error bar. It should be noted that the model results indicate that the error in the rotation rate should not change very rapidly with CMD, nor does it depend significantly on latitude within the sunspot belt nor on B_0. Furthermore, the percentage error in the derived rotation rate is nearly equal to that in the radius for small values of CMD, so, for example, in the 0.1% modelled result shown in Figure 5, the 'true' rotation rate is 14.436 deg day^{-1},

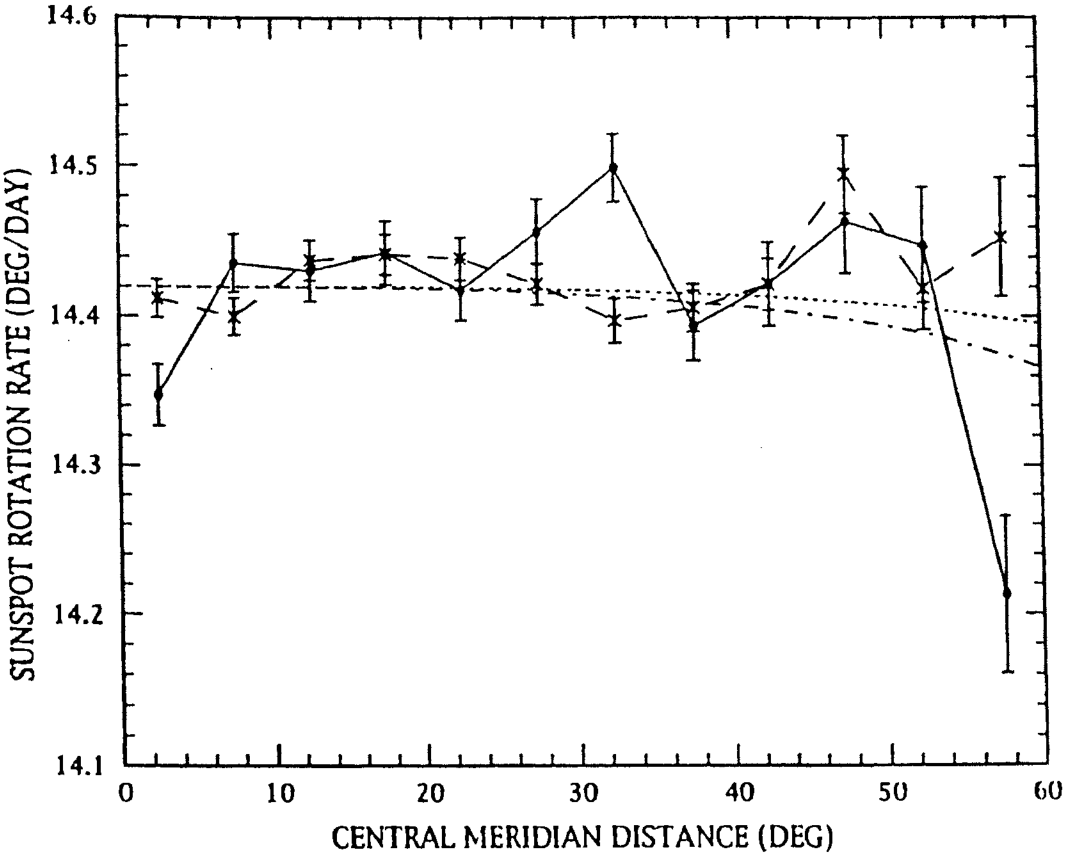

Fig. 5. Average rotation rates averaged over 5-deg intervals of CMD. Circles and solid lines represent the 35-year Mount Wilson data set, and the ×'s and dashed lines represent the same years in the Kodaikanal data. Also shown are the results of model calculations of rotation rates for the cases where the measured radius is too large by 0.1% (upper dot-dashed curve) and by 0.2% (lower dot-dashed curve). Only spots with areas less than 1 μhemisphere and CMDs greater than −45 deg (to avoid rotating beyond the 60-deg limit for measurements) were used here. There are 29557 Kodaikanal spots and 18919 Mount Wilson spots included in this calculation.

although the low-CMD value is 14.2 – and averaging all values would lead to a lower average than that.

Yet another potential source of error in rotation determinations is the image distortion that results from the projection from the celestial sphere to the plane of the photographic plate in a telescope system with a finite focal length (Smart, 1977). The difference in these effects between two systems with different focal lengths can, in principle, lead to systematic positional errors. However, the magnitude of these errors for the focal lengths of the instruments used for these observations (≈ 0.0003%) lies more than an order of magnitude lower than the small differences we find.

We should point out that a factor that will tend to make the rotation rate from the combined data set faster is the fact that the combined data set will include more small spots than will the data set from either site, because the smaller spots have shorter lifetimes, and with only a 12-hour time difference between observations, more of these small spots will be observed to 'return'.

Furthermore, differential (with latitude) rotation results may be expected to be affected by the rotation rate-spot size relationship because of possible differences in spot size or in the visibility of smaller spots with latitude. This may be the cause of the difference in the coefficient B between the Mount Wilson and Kodaikanal results given above. The larger (absolute) value for Kodaikanal data suggests that there is a greater falloff of visibility of small spots with latitude in these data. This hypothesis is supported by the finding that there is a much greater discrepancy in average spot areas between the two sites at high latitudes than at low latitudes.

As a test of this hypothesis, Figure 6 shows *residual* rotation velocities of spots from both sites. The residual rotational velocity is the rotational velocity of a spot minus the average rotational velocity of all spots for that latitude. This eliminates the latitude effect in the rotation analysis. In this plot, which is similar to Figure 4, there is no difference between the size dependence of the rotation rates for the two sites, except for the smallest spots.

In addition, when differential rotation for the two sites is calculated only for spots with areas greater than 5 μhemisphere, it is found that the values of B are not significantly different. They are -2.58 ± 0.04 for 10603 Mount Wilson spots and -2.65 ± 0.05 for 13079 Kodaikanal spots. The value of A derived in this experiment is significantly reduced (about 14.4). The rather low value of B found in this study for the combined data set is puzzling, and cannot be explained easily by discrepancies in spot sizes. This effect will be examined in more detail in a later study.

Altogether, this distribution of possible rotation rate errors should be a caution to those who are tempted to trust any rotation rate determinations (tracer, Doppler, or helioseismic) to one or two tenths of a percent. At this level of precision the results are sensitive to a large array of possible systematic errors.

8. Summary and Conclusions

We may draw the following conclusions from this study:

(1) The agreement between the measured sunspot positions and rotation rates derived from measurements of Kodaikanal and Mount Wilson white-light photographs is generally quite satisfactory.

(2) Combining the two data sets gives significantly more sunspot next-day 'returns' than are derived from any one site.

(3) The differences in overall relative area distributions may be explained partly by the fact that the Kodaikanal image scale is larger than that of the Mount Wilson plates and partly by random selection differences because of different weather patterns at the two sites.

(4) Small ($\approx 0.4\%$) deviations in the average measured rotation rates of

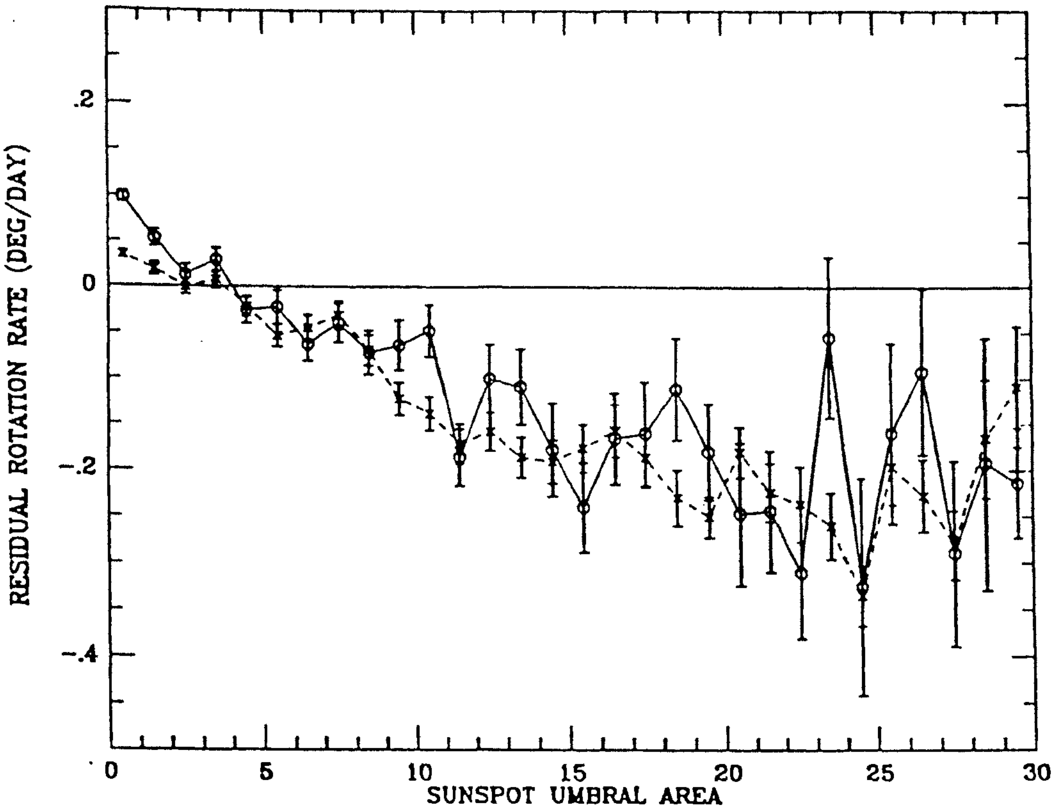

Fig. 6. Similar to Figure 4 except the ordinate is the residual rotation velocity for each site (solid = Mount Wilson; dashed = Kodaikanal). The residual rotation velocity of a spot is the rotation rate of the spot minus the rotation rate of all spots for that latitude.

sunspots from the two sites are detected and are demonstrated to be due in part to differences in the numbers of small spots measured at the two sites.

(5) Errors in the determination of the radius of the Sun can also affect the derived rotation rates, but this effect does not appear to have a large influence on these results.

(6) The latitude dependence of the derived rotation rate is shown to be sensitive to the presence of small sunspots, which rotate faster than larger spots, and whose distribution with latitude is visibility sensitive.

(7) In general, there are a number of subtle systematic differences that can affect rotation results at the level of 0.1%.

One purpose in measuring the Kodaikanal data set is to compare various parameters with previous results from the Mount Wilson data. Another purpose is to combine the two data sets to obtain a larger and more complete data set. For both these purposes it is necessary to have data from the two sites that are sufficiently similar to be comparable and compatible. This study demonstrates that we have achieved that goal for spot areas and for spot rotation rates, at least to the level of a few tenths of a percent.

The spot rotation comparisons suggest that for high precision measurements, one should be very careful about subtle, systematic effects. A similar conclusion was reached in the earlier work (HGG) resulting from the discovery of the effects of small, systematic errors in image orientation on derived meridional flow rates.

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