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On the electric circuit approach of coronal heating : a review

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Abstract. It is now widely believed that the photosphere of the Sun and its outer atmosphere (chromosphere and corona) are electrodynamically coupled through interconnecting magnetic fields. Similar situation may exist in other stars. The electric circuit approach provides a formalism through which the coupling of inner and outer regions of stars can be adequately described, and many seemingly diverse heating mechanisms can be unified. The physics of stellar atmospheres can be better understood through electric circuit analogues because the theory of electric circuits is well understood.

Key words : Sun, stars—corona, heating—electric circuit approach

1. Introduction

Low density, X-ray emitting, plasmas (temperature $T > 10^6$K) are found in the Sun, early and late-type stars (Vaiana *et al.* 1981), accretion disks (Ionson & Kuperus 1984; Kuperus & Ionson 1985), interstellar clouds (Rosner & Hartquist 1979), galaxies (Sturrock & Stern 1980) etc. All such systems possess regions in which matter moves with differential velocities.

The solar corona is the only astrophysical system whose X-ray structure has been to some degree spatially resolved. The observations show that the solar corona is highly structured and it contains a variety of closed, looplike regions of enhanced radiative emission. These regions have been found to be spatially coincident with coronal magnetic loops (Withbroe & Noyes 1977; Withbroe 1988). The solar observations may be modeled to study other as yet spatially unresolved X-ray emitting astrophysical systems.

It is now widely believed that the photosphere and the corona of the Sun are electrodynamically coupled through interconnecting magnetic field characterized by $\vec{B_0}$ because extra-polations of the observed photospheric magnetic energy into the corona exceeds the thermodynamic energy present there (Ionson 1982). It is quite convenient to describe the behaviour of aforesaid regions through plasma parameter, β, defined by

$$\beta \equiv \frac{v_s^2}{v_A^2} = \frac{4\pi p}{B_0^2} . \qquad \dots (1)$$

Here $v_s = (p/\rho)^{0.5}$ is the isothermal sound speed and v_A is the Alfvén speed defined by

$$v_A \equiv B_0/(4\pi\rho)^{0.5} \qquad \ldots (2)$$

where ρ is the matter density and p is the thermal pressure of the medium. As a matter of fact $\beta \geq 1$ for photosphere and $\beta < 1$ for the solar corona.

Golub *et al.* (1980) have pointed out that in $\beta \gtrsim 1$ region of the Sun there are two types of velocity fields. The "surface layer" velocity field is characterized by relatively small convective time and size scales compared to those of deeper 'subsurface layer'. Thus, the mechanical motion of the smaller gaseous regions at the surface of the star is faster than that of the larger regions lying deeper. According to Ionson (1982) the loop structure (potential magnetic field configuration) is established by large scale subsurface velocity fields. In contrast, the relatively smaller scale surface convection provides energy to the confined plasma which results in its heating.

The transient inductance (L)-resistance (R) circuit discharge analogues have been used to explain solar flare problems (Alfvén 1977 and references therein). A detailed review of electric circuit approach, applicable to solar flare problems, may be found in Spicer (1982).

Ionson (1982) has been the first to apply a series LCR circuit analogue to the steady electrodynamic heating of coronal loops. His physical system consists of a single $\beta < 1$ magnetic loop of a loop-cluster and an underlying region of a $\beta \gtrsim 1$ velocity fields which electrodynamically couple to and drive electrodynamic activity within the $\beta < 1$ loop via interconnecting magnetic induction $\vec{B}_0$. The hot X-ray emitting plasma is confined within a shell of volume $\pi l_\perp l_\parallel \Delta r$, where $l_\perp$ is the loop diameter in coronal part of the loop. $l_\parallel$ is the field-aligned length of the loop from one foot point to the other and Δr is the X-ray shell thickness (figure 1). The central core of this structure contains relatively cooler plasma.

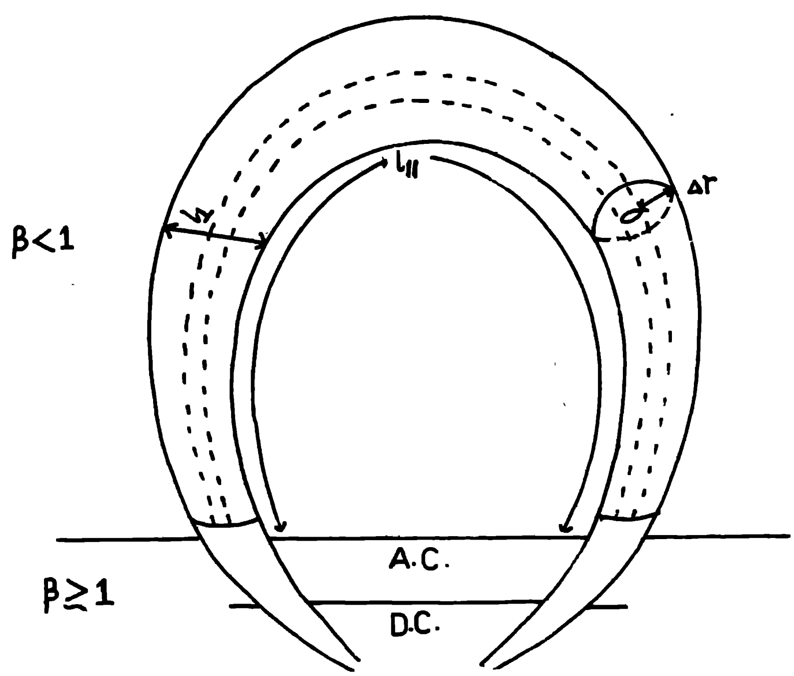

Figure 1. Schematic diagram of a coronal magnetic loop. Surface velocity fields generate A.C. and subsurface (deeper) velocity fields generate potential field (D.C.).

Ionson (1985b) has reviewed the work done by him on steady heating of coronal loops using a series LCR circuit approach. He has focussed his attention on a few problems which can be studied using circuit approach and has emphasized the need to study dissipation more deeply. Since then a number of papers using circuit approach have become available (Zappala & Zuccarello 1989; Narain & Ulmschneider 1990 and references therein). Although Narain & Ulmschneider (1990) accounted for some of the new works using circuit approach they could not pay as much attention as this powerful approach needed. In this review article considerable effort has been made to focus attention on this approach, its achievements and future prospects.

In section 2 required basic circuit relations are presented. Section 3 surveys the literature. The last section contains our conclusions. Both, C.G.S. and S.I., systems of units are used.

2. Basic circuit relations

In general, an electric circuit contains three basic elements, namely, a self-inductance, L, a capacitance, C, and resistance, R. When a source of e.m.f. (A.C. or D.C.) is put in the circuit a current, I, flows in it. The source of e.m.f. may have its own internal resistance, Rs. The magnitude of current depends on the values of L, C, R and, of course of time t.

If the circuit elements are in series with an A.C. source then we have a series LCR resonant circuit which is represented by the following equation :

$$L\frac{d^2I}{dt^2}+R\frac{dI}{dt}+\frac{I}{C}=\frac{d\varepsilon(t)}{dt}, \qquad \ldots (3)$$

where $\varepsilon(t)$ is the time-dependent e.m.f. of A.C. source. The impedance Z of such a circuit is given by

$$Z=\left[R^2+\left(\omega L-\frac{1}{\omega C}\right)^2\right]^{0.5} \qquad \ldots (4)$$

Here ω is the angular frequency of the applied A.C. source. ωL is called inductive reactance and $1/\omega C$ is called capacitive reactance of the circuit. At resonance

$$\omega L = 1/\omega C,\ Z = R,$$

which gives

$$\omega = \omega_0 = 1/(LC)^{0.5}. \qquad \ldots (5)$$

ω_0 is called the resonant angular frequency.

An important quantity for a circuit is the quality factor, Q, which is defined by

$$Q \equiv (L/C)^{0.5}/R = \omega_0 L/R = 1/\omega_0 CR. \qquad \ldots (6)$$

This relation is true for a series LCR circuit only.

Using equations (4), (5) and (6) the impedance, Z, may be rewritten as

$$Z = R\ [1 + \{(\omega/\omega_0) - (\omega_0/\omega)\}^2 Q^2]^{0.5}. \qquad \ldots (7)$$

The width of the resonance peak depends on the quality factor in the following manner

$$\frac{\Delta\omega}{\omega_0} = \frac{\Delta\nu}{\nu_0} \approx \frac{1}{Q}, \quad \dots (8)$$

where $\nu_0 = \omega_0/2\pi$ is the linear resonant frequency. The magnetic energy stored in the circuit is given by

$$W_m = {}^1/_2\, LI^2. \quad \dots (9)$$

The electric energy stored in the capacitor is

$$W_E = {}^1/_2\, CV^2, \quad \dots (10)$$

where V is the potential difference between the plates of the capacitor.

The power dissipated in the circuit, through Joule effect, is given by

$$P = I^2R. \quad \dots (11)$$

The ohmic dissipation time, t_{diss} in a L-R circuit may be written as

$$t_{diss} = L/R. \quad \dots (12)$$

This is also known as damping time.

3. Literature survey

In order to facilitate the presentation this section has been divided into subsections.

3.1. *Equivalent LCR circuit description*

The state of a magnetoplasma is described by the magnetohydrodynamic equations of momentum, mass conservation, induction, Maxwell-Ampere equation and the equation of state. In order to have knowledge about induced currents the magnetic induction can be eliminated from the set of the MHD equations. Ionson (1982, 1984), by linearization and adopting an averaging procedure which converts current density into total current I, has obtained a second order differential equation having the same form as the series LCR circuit equation (3). The equivalent inductance and capacitance of such a circuit are given by

$$L = 4l_{\parallel}/\pi c^2. \quad \dots (13)$$

and

$$C = l_{\parallel}c^2/4\pi V_A^2 m^2. \quad \dots (14)$$

These expressions show that the equivalent reactances are directly proportional to the field-aligned length $l_{\parallel}$ of the loop in the corona. V_A is the average Alfvén speed in coronal part of the loop and m is a positive integer such that $m = 1$ corresponds to fundamental harmonic, $m = 2$ to the first harmonic, etc. of resonant frequency ω_0. c is the speed of light in vacuum. The self inductance L represents the ability of loop plasma to store magnetic energy and capacitance C represents its ability to store (kinetic) electrical energy.

The equivalent resistance of the loop may be expressed as a sum of two terms as follows :

$$R = R_p + R_l, \quad \dots (15)$$

where R_p is contributed by the photospheric part and R_l by the remaining part of the magnetic loop. Thus R_p corresponds to the internal resistance of the photospheric source of e.m.f. Following Ionson (1982), the photospheric resistance R_p may be expressed as

$$R_p = \eta_p l_p/(\Delta r)_p^2. \qquad \text{... (16)}$$

Here η_p is the ohmic resistivity of matter in photospheric part of the loop, l_p is the length of the loop in the photosphere and $(\Delta r)_p$ is the the thickness of shell in the photosphere. The resistance R_l consists of contributions from chromospheric and coronal part of the loop. Both regions have ohmic (Joule) and viscous contributions. Thus R_l may be written as

$$R_l = \sum_{\substack{\text{Chromosphere}\\ + \\ \text{Corona}}} (R_{\text{Joule}} + R_{\text{vis}}), \qquad \text{... (17)}$$

with $$R_{\text{Joule}} = \eta_{\text{Joule}}\; l/(\Delta r)^2. \qquad \text{... (18)}$$

and $$R_{\text{vis}} = P^{\perp}_{r\,\text{mag}}\; R_{\text{Joule}}. \qquad \text{... (19)}$$

Here $P^{\perp}_{r\,\text{mag}}$ is the cross-field magnetic Prandtl number. Physically, it is approximately equal to the ratio of viscous-ion heating to Joule-electron (ohmic) heating. Thus the equivalent resistance R represents various dissipation mechanisms.

The equivalent e.m.f. of the source is given by

$$\varepsilon(t) = (l_\perp\; V_\phi\; B/c)_p. \qquad \text{... (20)}$$

The subscript p denotes quantities in the photosphere (i.e. $\beta \gtrsim 1$ region). $l_\perp$ is the diameter of loop measured across the potential magnetic field lines and V_ϕ is the surface velocity field in the $\beta \gtrsim 1$ region. Thus the A.C. source of e.m.f. (voltage generator) responsible for the flow of current, I, comes from the surface layer $\beta \gtrsim 1$ convective velocities.

Like an LCR circuit, a coronal loop is a resonant system. Such systems, in general, may posses more than one resonance frequency. The linear resonance frequencies, ν_{res}, for coronal loops correspond to harmonics of the inverse Alfvén transit time, t_A, i.e.

$$\nu_{\text{res}} = m/t_A,\; m = 1, 2, 3\; ... \qquad \text{... (21)}$$

where $$t_A = 2l_{\parallel}/V_A. \qquad \text{... (22)}$$

It is to be noted that Alfvén transit time, t_A, is the time taken by Alfvén waves to travel from one foot point to the other and back to the same footpoint. Ionson (1982, 1984) has studied the heating by the fundamental ($m = 1$) harmonic only.

The thickness of dissipation shell (X-ray emitting region) depends on the type of dissipation mechanism under consideration. For example, in case of spatial resonance absorption between Alfvénic surface waves (which travel along the boundary of the loop) and kinetic Alfvén waves (KAW) propagating inside the loop the width Δr, is given by (Ionson 1982) :

$$\frac{\Delta r}{a} = \pi^{1/3}\left(1 + P^{\perp}_{r\,\mathrm{mag}}\right)^{1/3} R^{-1/3}_{e\,\mathrm{mag}}, \qquad \ldots (23)$$

where the magnetic Reynolds number, $R_{e\,\mathrm{mag}}$, is defined by

$$R_{e\,\mathrm{mag}} \equiv 4\pi v_A a^2/\eta c^2 l_{\parallel}. \qquad \ldots (24)$$

The quantity a is called the scale length of gradient of Alfvén speed across the loop and is defined by

$$a \equiv [d(\ln V_A)/dr]^{-1}. \qquad \ldots (25)$$

By using equation (11) Ionson (1982, 1984) arrives at a heating flux (cf. equation (30)) which depends upon the power available at the resonant frequency in the $\beta \gtrsim 1$ region. Since the resonant frequency depends on the loop length ($l_{\parallel}$) hence different loops are heated differently depending upon the power available at and near that resonant frequency. Some of Ionson's data and results are exhibited in table 1 to have a feeling for this approach. It is obvious from the table that the resistivity of the corona is lower by five orders of magnitude than that of photosphere. Further the value of cross field Prandtl number $P^{\perp}_{r\,\mathrm{mag}}$ suggests that coronal heating via ion-viscosity is about 4 times more effective than Joule (ohmic) dissipation. Table 1 also shows that Ionson (1982) assumed that the cross section of loop in photosphere, chromosphere and corona is the same which is not quite true.

Ionson (1983) has extended above work by taking different values of cross section in the $\beta \gtrsim 1$ and $\beta < 1$ regions. In addition to this compressional viscosity in the momentum equation has been included. He has considered two cases, viz. isolated and diffuse heating. In isolated case the heated region is at a higher temperature than the remaining part of the

Table 1.

S No.	Quantity	Photosphere	Chromosphere	Corona
1.	$l_{\parallel}$ (cm)	5×10^7	5×10^7	$\sim 10^{10}$
2.	β	$\gtrsim 1$	$\lesssim 1$	~ 0.1
3.	V_A (cm/s)	$\sim 10^6$	$\sim 3 \times 10^6$	$\sim 6 \times 10^7$
4.	η (s)	$\sim 6 \times 10^{-12}$	$\sim 2 \times 10^{-13}$	$\sim 5 \times 10^{-17}$
5	$R_{e\,\mathrm{mag}}$	$\sim 4 \times 10^7$	$\sim 4 \times 10^9$	$\sim 2 \times 10^{12}$
6.	$Pr^{\perp}_{\mathrm{mag}}$	–	~ 43	~ 4.3
7.	ν_{res} (1/s)	–	–	$\sim 3 \times 10^{-3}$
8.	a (cm)	–	–	$\sim 10^9$
9.	$l_{\perp} \sim 0.1\, l_{\parallel}$ (cm)	$\sim 10^9$	$\sim 10^9$	$\sim 10^9$
10.	$\Delta r \sim 0.1 l_{\parallel}/Q$ (cm)	–	–	2×10^8
11.	V_{Φ} (cm/s)	$\sim 5 \times 10^4$	–	$\sim 10^2$

loop. In the diffuse case, through some unspecified transport mechanism, the heat gets distributed over the whole loop.

The physical mechanisms responsible for isolated and diffuse heating are Joule (ohmic) dissipation, shear-viscous dissipation and compressional-viscous dissipation. Corresponding to these three dissipation mechanisms the expressions for the size of heated region (dissipation shell), quality factor, induced current, electrodynamic heating function, the energy exchange rate, pressure and coronal velocity etc. are derived. These expressions do not depend on magnetic induction explicitly hence they may be tested without requiring accurate determination of magnetic fields.

3.2. *Unification scheme*

A number of mechanisms have been put forward to explain coronal heating problem (cf. Narain & Ulmschneider 1990; Ulmschneider, Priest & Rosner 1991 and references therein). Of these Alfvénic surface wave heating (Ionson 1978), resonant electrodynamic heating (Ionson 1982), stochastic magnetic pumping (Sturrock & Uchida 1981), dynamical dissipation (Parker 1983), complex magnetic reconnection (Heyvaerts & Priest 1984) and related ones may be considered as special cases of a more generalized mechanism (Ionson 1984).

Ionson (1984) starts with the linearized MHD equations for magnetic perturbation, B', and associated coronal velocity perturbation, V'. The Joule and viscous dissipation are taken care of through diffusion time scales $t_{\rm Joule}$ and $t_{\rm vis}$ which are related to the electrical resistivity and viscosity as well as current and velocity field localization within coronal filaments that are aligned with the ambient magnetic field (Ionson 1985b). The functional form depends upon the mechanism responsible for the filamentation. The two times may be combined to get total dissipation time $t_{\rm diss}$ as follows :

$$t_{\rm diss} = 1/(1/t_{\rm Joule} + 1/t_{\rm vis}). \qquad \ldots (26)$$

There exists a gradient in the Alfvén speed across the loop's ambient field so that each magnetic field line has its own natural frequency of oscillation. When footpoints of the loop are excited by photospheric motions, the field lines are subjected to coherent excitation, initially. Because of viscosity of medium and the inhomogeneity in the Alfvén speed the relative phase difference between field lines changes with time and the photospheric excitation no longer remains cohernt. The time in which initial coherent excitation becomes incoherent is called 'phase mixing time' and is denoted by $t_{\rm ph}$. As a result of this the amplitude of fluctuation reduces (Heyvaerts & Priest 1983).

Ionson (1982) assumed that a coronal loop is an electrodynamically closed system but Hollweg (1983, 1984) points out that there may be leakage of incoming magnetic stresses from the loop. In order to account for this Ionson (1984) introduces a magnetic stress leakage time $t_{\rm leak}$ and utilizes the same averaging procedure as earlier to arrive at the series LCR circuit equation (3). The equivalent resistance now has the following form

$$R = R_{\rm diss} + R_{\rm ph} + R_{\rm leak} \equiv L(1/t_{\rm diss} + 1/t_{\rm ph} + 1/t_{\rm leak}) \equiv L/t_{\rm loss}, \qquad \ldots (27)$$

where

$$t_{\rm loss} = 1/(1/t_{\rm diss} + 1/t_{\rm ph} + 1/t_{\rm leak}). \qquad \ldots (28)$$

It is to be noted that only R_{diss} ($\equiv L/t_{diss}$) results in energy dissipation (resistive heating) whereas R_{ph} ($\equiv L/t_{ph}$) and R_{leak} ($\equiv L/t_{leak}$) participate only in reducing the amplitude of the coronal loop's total current I.

Ionson (1984) focussed his attention on fundamental ($m = 1$) harmonic only. Ionson (1985a, b) has generalized the approach to include $m = 0$ (the zeroth harmonic) and $m = 1, 2, 3$ .... i.e., higher harmonics. He expresses the time-averaged coronal heating flux,

$$F_H = \langle I^2 R_{diss} \rangle / (\pi l_\perp^2/4), \qquad \ldots (29)$$

in the following simple form

$$F_H = F_0 \varepsilon, \qquad \ldots (30)$$

with

$$F_0 = 4\,(B_c/B_p)\left(4V_A^p/V_A^c\right)V_A^p\left(1/2\,\rho V_t^2\right)_p, \qquad \ldots (31)$$

and

$$\varepsilon = \sum_m \int_{-\infty}^{\infty} \langle V_\perp^2 \rangle / V_t^2 \left\{ Q_{loss}^2 / Q_{diss} \left[1 + \left(\nu t_A - \frac{m^2}{\nu t_A} \right) Q_{loss}^2 \right] \right\} d\nu$$

$$= \varepsilon_0 + \sum_{m>0} \varepsilon_m \; (m = 1, 2, 3, \ldots). \qquad \ldots (32)$$

Here

$$Q_{loss} = (L/C)^{0.5}/R_{loss} = 2\pi t_{loss}/t_A. \qquad \ldots (33)$$

and

$$Q_{diss} = (L/C)^{0.5}/R_{diss} = 2\pi t_{diss}/t_A. \qquad \ldots (34)$$

The subscript or superscript 'c' in equation (31) specifies $\beta < 1$ region (i.e. corona). The factor F_0 in equation (30) represents the maximum flux of energy that could enter the coronal portion of the loop and ε is the electrodynamic coupling efficiency which gives the fraction dissipating within the corona. It is this efficiency factor which facilitates unification of various heating mechanisms.

On the right-hand side of equation (31) the factor 4 accounts for two footpoints per loop and two excitation polarizations, viz. twist and shake; the factor (B_c/B_p) results from magnetic field expansion into the corona; the factor $(4\,V_A^p/V_A^c)$ is the transmission coefficient and the factor $V_A^p(1/2\rho V_{-t}^2)_p$ represents the maximum available flux of electrodynamic energy within $\beta \gtrsim 1$ region. In general, F_H contains a geometrical factor of the form $[l_\parallel/(l_\parallel + l_v)]^2$ but it goes to unity under the reasonable assumption that the scale length l_V, of convective velocity cells in $\beta > 1$ region is small in comparison with the loop length $l_\parallel$ in the $\beta < 1$ region (Heyvaerts & Priest 1984; Ionson 1985a).

A close look at equation (32) shows that it contains contributions from $m = 0$ as well as from $m = 1, 2, 3$, ... etc. If we put $m = 0$ in equation (21) the resonance frequency, ν_{res}, comes out to be zero which implies a non-wavelike character and corresponds to a DC Poynting flux. In this case there is non-resonant coupling between the $\beta \gtrsim 1$ velocity fields and responding coronal loop (Heyvaerts & Priest 1984). For m = 1, 2, 3, ..., equation (21) yields nonzero resonance frequencies which imply wavelike character and correspond to A.C. (resonant) heating process. Thus, equation (32) leads to two broad categories, viz. A.C. and D.C. heating mechanisms.

Assuming a typical form for $(< V_\perp^2 > / V_t^2)$, Ionson (1985a, b) arrives at following analytic expressions for the electrodynamic coupling efficiency ε :

$$\varepsilon_0 = \sum A(x^2 - x + 1) / (x^4 + x^2 + 1), \qquad \ldots (35)$$

and

$$\varepsilon_{m>0} = \sum A/x \left(1 + \left(\frac{t_c m}{t_A} - \frac{t_A}{t_c m}\right)^2\right) \qquad \ldots (36)$$

if

$$\frac{2\pi m\, t_{loss}}{t_A} > 1$$

or

$$\varepsilon_{m>0} = \sum A\left[\left(x^2+1\right) / \left\{x^4\left(1+D^8\right) + x^2\left(1+D^4\right)+1\right\} + x^7 D^{10} / \left(x^4 D^8 + x^2 D^4 + 1\right) - x / \left(x^4 + x^2 + 1\right)\right]$$

if

$$\frac{2\pi m\, t_{loss}}{t_A} < 1 \qquad \ldots (37)$$

with

$$x = 2\pi t_{loss}/t_c,\ A = (t_{loss}/t_{diss})\,(2\pi t_{loss}/t_A),\ D = m\, t_c/t_A. \qquad \ldots (38)$$

The summation in equations (35), (36), and (37) is to be taken over all the peaks if the convective velocity spectrum has more than one peak. Equation (36) represents under-damped (high quality) category whereas (37) represents over-damped (low quality) category. When $2\pi t_{loss}/t_A \sim 1$ for A.C. mechanisms ($m > 0$) the coupling coefficient does not have a convenient analytic representation. This is a critically damped category and the integral in equation (32) must be evaluated numerically.

For $t_{loss} \sim t_{diss}$, $x << 1$ we arrive at fluxes given by Parker (1983) and Heyvaerts & Priest (1984) provided their fluxes are multiplied by $4(B_c/B_p)$ when equations (30), (31), (35) and (38) are used. If we put $t_c = t_{peak}$, m = 1 and $x << 1$ in equations (36) and (37) we obtain Ionson's (1984) result. Ionson's (1982, 1983) resonant theory and Heyvaerts and Priest's (1983) phase-mixing theory belong to high quality coupling category represented by equation (36). Thus many of the proposed, seemingly diverse, mechanisms may be treated as special cases of one general mechanism represented by equations (30) through (38).

3.3. *Some stellar applications*

Mullan (1984) tries to explain enhancement of X-ray flux in early M-dwarfs and quite small X-ray flux in very late. M-dwarfs on the basis of electrodynamic coupling efficiency ε.

The efficiency of coronal heating depends sensitively on the matching of convective time scale t_c (associated with $\beta \gtrsim 1$ region) with Alfvén transit time t_A (associated with $\beta < 1$ region where the mechanical energy is eventually deposited). It is expected that t_c should decrease significantly as one goes to later and later spectral types, mainly because the gravity increases along lower main sequence. In the Sun the value of t_c is many hundreds of seconds but in the convection zone of coolest M-dwarfs it could be a few tens of seconds only.

From the analysis of Mullan (1984) it appears that t_A increases towards cooler stars. In the Sun t_c is larger than t_A whereas t_c tends to be smaller than t_A in cooler stars. Obviously a crossover of two time scales is bound to occur. For the Sun ε has an estimated value in the range 1/80 to 1/170. Hence stellar heating efficiencies at crossover should exceed the solar values by factors of 80 to 170. X-ray data from the Einstein satellite indicate that enhancement of heating efficiencies by factors more than 130 do occur among the early M-dwarfs.

For later spectral types, beyond the crossover, the coupling efficiency again falls off very rapidly as t_A/t_c increases. This could be a possible explanation for X-ray fluxes to be quite small in the very late M-dwarfs.

Kuperus & Ionson (1985) try to explain the observed large ratio of hard to soft X-ray emission and the bimodal behaviour of blackhole accreting X-ray sources such as Cygnus X-1 in terms of a magnetically confined, structured accretion disk corona which is electrodynamically coupled to the disk turbulent motions while the disk is thermodynamically coupled to the corona. They assume that all the energy that is delivered to the corona through electrodynamic coupling is used to heat the corona. A part of this energy is transferred via inverse Compton scattering of the disk soft X-ray photons into the hard X-ray photons while the remaining part is conducted back to the disk.

The electrodynamic energy flux dissipated in the corona is given by

$$L_c = F_0\, \varepsilon_{max}\, f. \qquad \text{... (39)}$$

We call L_c, the coronal luminosity, f is the fraction of the disk surface covered with coronal loops and ε_{max} is the maximum value of the heating efficiency, i.e.

$$\varepsilon_{max} = t_c/t_A. \qquad \text{... (40)}$$

Following Kuperus and Ionson (1985) the coronal luminosity may be rewritten as

$$L_c \simeq 8f\, (B_c/B_d)\, (V_A^d)^2\, \rho V_s\, \psi(\eta_{vis}), \qquad \text{... (41)}$$

where $\psi(\eta_{vis})$ is a function of viscosity η_{vis}. The subscript or superscript d stands for disk. The disk luminosity is given by

$$L_d = 9\, \rho V_s\, h^2\, GM\, \eta_{vis}\, /\, 8R^3, \qquad \text{... (42)}$$

where h is the thickness of the disk, G is the gravitational constant, M and R are the mass and radius of the disk, respectively. Equations (41) and (42) yield

$$\frac{L_c}{L_d} \sim 7 f(B_c/B_d) \left(\frac{V_A^d}{V_s^d}\right)^2 \psi \frac{(\eta_{vis})}{\eta_{vis}}. \quad \ldots (43)$$

Here $V_s^d \equiv (GMh^2/R^3)^{0.5}$ has been used. Under the approximation, $f = 1$ and disk plasma parameter $\beta_d = (V_s^d/V_A^d)^2 \sim 1$, equation (43), to first order terms, gives

$$L_c/L_d \sim 7.$$

The theoretically determined quantities L_c and L_d are related to observable quantities L_{hard}, L_{soft} as follows (Ionson & Kuperus 1984)

$$L_{hard} = (1-\delta)\, L_c, \quad \ldots (44a)$$

$$L_{soft} = L_d + \delta\, L_c, \quad \ldots (44b)$$

where δ is the feed-back parameter which describes the fraction of energy dissipated in the corona which is transferred back to the disk by thermal conductior and radiation and reprocessed in the disk into soft X-ray emissions. Equations (44a, b) yield

$$\frac{L_{hard}}{L_{soft}} \sim \frac{7(1-\delta)}{1+7\delta}. \quad \ldots (45)$$

For hard to soft X-ray emission ratio to be larger than unity δ must be smaller than 0.5. The bimodal behaviour of the source corresponds to minimum and maximum values of δ in the range $\delta < 0.5$.

3.4. *Magnetic energy in a sheared arcade*

The circuit approach has been used to find self-inductance, current, magnetic energy, shearing time (t_{sh}) of a set of loops forming an arcade (Zuccarello *et al.* 1987). The authors consider a magnetic arcade of length l, width $2b$ and initial magnetic induction $\vec{B}_0$. The feet of magnetic fieldlines move further apart because of shearing motions. It is assumed that perturbed magnetic induction $\vec{B}$ satisfies the force-free field condition.

$$\vec{\nabla} \times \vec{B} = \alpha\, \vec{B},$$

where α is the force-free field parameter. For a chosen set of components of $\vec{B}_0$ the parameter α takes the form

$$\alpha = \frac{\pi}{2b} \frac{\Delta y_b}{S}.$$

Here Δy_b is the displacement of one edge of arcade in the Y-direction (cf. figure 2), $\vec{B}_0$ being in the Z-direction. The quantity S is defined by

$$S = \left(b^2 + \Delta y_b^2\right)^{1/2} \quad \ldots (48)$$

As a result of the displacement Δy_b the self-inductance L of the arcade changes in the following way

$$L = L_0\ 2S^2/b\ (b + s). \qquad \ldots (49a)$$

The initial self-inductance L_0 has the form

$$L_0 = \frac{\mu_0}{\pi}\frac{b^2}{l}, \qquad \ldots (49b)$$

where μ_0 is permeability constant having value $4\pi \times 10^{-7}$ weber/amp – meter in S.I. units. The corresponding induced current is given by

$$I = I_0\ \Delta y_b/s, \qquad \ldots (50a)$$

where

$$I_0 = \frac{B_0}{\mu_0}\ l\ . \qquad \ldots (50b)$$

The total amount of stored magnetic energy would be

$$W_B = W_0\ (S - b)\ /\ b \qquad \ldots (51a)$$

with

$$W_0 = B_0^2\ lb^2\ /\ \mu_0\pi. \qquad \ldots (51b)$$

In order to have a feeling of the magnitudes of inductance, current and magnetic energy involved following realistic set of data may be used :

$B_0 = 1.6 \times 10^{-1}$ Tesla; $l = 1 \times 10^8$ meter; $b = 1.2 \times 10^7$ meter.

These data, together with equations (49b), (50b) and (51b), yield $L_0 = 0.58$ Henry; $I_0 = 1.3 \times 10^{13}$ Ampere; $W_0 \simeq 9 \times 10^{25}$ Joule.

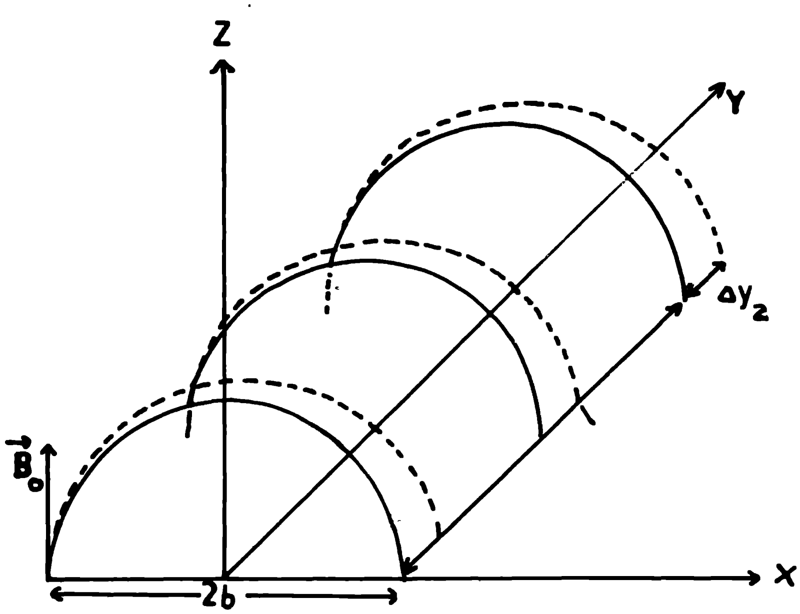

Figure 2. Schematic diagram of a magnetic arcade. Solid line refers to initial situation and dotted line to that after shearing time t_{sh}.

The time of energy build up (shearing time) may be estimated from the relation

$$t_{sh} \simeq \frac{\Delta y_b}{v_\phi} \simeq \frac{b}{v_\phi}, \qquad \text{... (52)}$$

where v_ϕ is the average velocity of shearing motions. In the Sun's photosphere $v_\phi \simeq 0.5$ km/s, which together with equation (52) gives a shearing time ~ 2.4×10^4 s (~ 6.7 hours). The ohmic dissipation time, t_{diss}, may be evaluated using equation (12) provided equivalent resistance R of the arcade is known. Zuccarello *et al.* (1987) estimate $R \sim 4.57 \times 10^{-11}\Omega$. Therefore $t_{diss} \sim L/R \simeq 1.3 \times 10^{10}$ s (~ 3.6×10^7 hours). This means that there will be more build up of magnetic energy than its ohmic dissipation. Further since shearing increases self-inductance and ohmic dissipation time is directly proportional to the self-inductance hence Joule dissipation (classical) is of no consequence. Some alternative, efficient dissipation mechanism is highly desirable. Anomalous current dissipation (Rosner *et al.* 1978) could be one of them.

Assuming that photospheric motions build up direct currents (DC) flowing parallel to the magnetic field (i.e. under force-free field approximation) Zappala and Zuccarello (1989) evaluate the range of stored energy in a magnetic arcade interconnecting two sunspot groups having different age and therefore a relative velocity. Two different values for half-width of the loops ($b = 6 \times 10^9$ and 1.6×10^{10} cm) and two different values for the length of the arcade ($l = 6 \times 10^8$ and 4×10^9 cm) are used because these values are typically observed in complex active regions (Priest 1982). They find that stored magnetic energy ranges from 2.73×10^{30} to 1.05×10^{33} erg. Besides a dependence on the length l of the arcade, the amount of energy which can be stored strongly depends on the shearing time, t_{sh}, and therefore on the relative velocity of two sunspot groups. On the contrary, it seems that the width of the arcade does not have much influence on the stored energy.

3.5. *Alfvén wave representation*

Keeping in view the transient solar flare problems Spicer (1982) represented a moving magnetoplasma (Alfvén wave) by a low loss transmission line. Ionson (1982, 1984), while considering heating of stellar coronal loops, arrives at the second order differential equation (3) which represents a series LCR circuit. Both approaches describe the same physical system (Alfvén waves) but by different equivalent circuits. Scheurwater and Kuperus (1988) make use of basic electric circuit theory and linearized MHD equations to correlate the two equivalent circuit representations.

They start with a uniform magnetic induction B_0 directed paralled to Z-direction of a cartesian coordinate system. The generator of monochromatic plane Alfvén waves of angular frequency ω is situated at $z = 0$. The boundary separating two homogenous plasma regions is situated at $z = D$. In general, Alfvén waves are partly reflected and partly transmitted at the boundary. In region 1 ($0 \leq z \leq D$) incident and reflected waves will interfere to produce standing wave pattern whereas in region 2 ($z \geq D$) only transmitted wave will be present.

For a weakly damped Alfvén waves the dispersion relation in the two regions is (Scheurwater & Kuperus 1988)

$$k_j = R_e(k_j) + i\,I_m(k_j),\; I_m(k_j) << R_e(k_j),\; j = 1, 2 \qquad \ldots (53)$$

where

$$R_e(k_j) = \omega/V_{Aj} \qquad \ldots (54a)$$

and

$$I_m(k_j) = \frac{\omega^2}{2V_{Aj}^3}\left(\frac{c^2}{4\pi\sigma_j} + \frac{\eta_j}{\rho_j}\right). \qquad \ldots (54b)$$

The subscript $j = 1$ stands for region 1 and $j = 2$ for region 2. The quantities σ_j, η_j, ρ_j are the conductivity, viscosity and matter density, respectively, in the two plasma regions. It is assumed that the Alfvén velocities (V_{Aj}) in the two regions are small in comparison with the speed of light c. The refractive index μ_j and impedance Z_j for the two regions are related to propagation number k_j as follows:

$$\mu_j = ck_j/\omega, \qquad \ldots (55a)$$

$$Z_j = 4\pi/c\mu_j. \qquad \ldots (55b)$$

The continuity of fields at the boundary $z = D$ yield reflection and transmission coefficients which enable one to define a complex impedance given by

$$Z(z) = \frac{Z_2 + iZ_1 \tan k_1(z-D)}{1 + i\frac{Z_2}{Z_1}\tan k_1(z-D)} \qquad (0 \le z \le D)$$

$$= Z_2\;(z = D). \qquad \ldots (56)$$

According to the conventions of the electric circuit theory the value of Z at $z = 0$ is called input impedance Z_I and the value of Z at $z = D$ is called the load impedance Z_l, Now equation (56) yields

$$Z_I = \frac{Z_2 - iZ_1 \tan k_1 D}{1 - i\frac{Z_2}{Z_1}\tan k_1 D} \qquad \ldots (57a)$$

and

$$Z_l = Z_2. \qquad \ldots (57b)$$

In the simplest case when both regions have the same plasma parameters, i.e. magnetoplasma extends from $z = 0$ to infinity, $Z_1 = Z_2$ and the input impedance (after dropping the subscripts 1, 2) and using equations (53)–(55) is given by

$$Z_I = \frac{4\pi}{c\mu} = \frac{4\pi\omega}{c^2 k}$$

$$= \frac{4\pi v_A}{c^2} - i\omega\frac{2\pi}{c^2 v_A}\left(\frac{c^2}{4\pi\sigma} + \frac{\eta}{\rho}\right) = R - i\omega L \qquad \ldots (58)$$

with $R \equiv 4\pi v_A/c^2,$ ... (59a)

and $$L \equiv \frac{2\pi}{c^2 v_A}\left(\frac{c^2}{4\pi\sigma} + \frac{\eta}{\rho}\right).$$ ... (59b)

It is quite obvious from equation (58) that the equivalent circuit in this case is a series L-R circuit. The minus sign in (58) is due to a phase factor $e^{-i\omega t}$ in the representation for the electric and magnetic vectors of the wave.

An extensive study by Scheurwater & Kuperus (1988) shows that whenever the plasma parameters of both regions have different values then for any arbitrary frequency the input impedance may be represented by the circuit of a terminated low loss transmission line. However, in the vicinity of frequencies corresponding to resonance (i.e. the zeros of input reactance) the proper representation is a series LCR circuit, whereas near antiresonance (i.e. the poles of the input reactance) it is a parallel LCR circuit. In all cases the character and magnitude of the circuit elements are fully determined by the plasma parameters and the functional form of the dispersion relation. The different types of circuit, though in their appearance they look completely unrelated, are in fact the limiting cases of one and the same input impedance hence transition from one type of circuit to the other is possible.

3.6. *Future prospects*

For the sake of completeness an estimation of the amount of heating by some of the proposed mechanisms is given below. Due to lack of adequate data the maximum amount of heating is difficult to evaluate in most of the cases.

A surface wave excited by 300s chromospheric oscillations heats a coronal loop at a rate

$$E_H = 2.7 \times 10^{23}\left(\frac{a\,\Delta V_A}{r_c V_A}\right)^{-1}\left(r_g^2\, l_{10}\, n_g \left|v_6^2\right| / \pi p_{300}\right)$$ ... (60)

where a is distance over which Alfvén speed changes from its value outside the loop to its value within the loop. Following Ionson (1978) $\frac{a\,\Delta V_A}{r_c V_A} \leq 0.13$, $r_g = r_c/10^9 = 0.5$, $l_{10} = l_{11}/10^{10} \sim 1$, $n_g = n_e/10^9 \sim 1$, $v_6 = v/6$ km ~ 1 and $P_{300} = P/300 \sim 1$ so that $E_H = 1.65 \times 10^{23}$ erg s^{-1} corresponding heating flux turns out to be $F_H \sim 2 \times 10^5$ erg cm^{-2} s^{-1}.

This flux is smaller by one order of magnitude for quiet regions and coronal holes and by two orders of magnitude for active regions.

Stochastic magnetic pumping of Sturrock & Uchida (1981) heats the corona at a rate

$$E_H = \frac{\Phi B_p \langle v^2 \rangle \tau_c}{4\pi\, l_{\parallel}}$$ ... (61)

where

$$\Phi = \pi r_c^2\, B_c$$ ... (62)

is the magnetic flux through the loop, τ_c is the correlation time for the photospheric velocity field, B_c and B_p are magnetic induction in corona and photosphere respectively. Following Sturrock & Uchida (1981) $B_p < v^2 > \tau_c = 5 \times 10^{16}$ which together with equation (61) gives $E_H \sim 3.1 \times 10^{24}$ erg s^{-1} where $l_{11} \sim 10^{10}$ cm and $B_c \approx 5$ Gauss have been used. The corresponding heating flux is $F_H \sim 4 \times 10^6$ erg cm^{-2} s^{-1} which is quite sufficient to heat quiet and coronal hole regions of solar corona. For active regions $B_c \sim 50$ Gauss yields heating fluxes quite sufficient to heat them.

The resonant electrodynamic heating of stellar coronal loops using LCR circuit approach leads to the following heating rate (Ionson 1982)

$$E_H = 6.23 \times 10^9 \frac{T_b}{l_{\parallel}^2}\left(\frac{R_l}{R}\right) < \rho v^2/2 >^p_{v_0} \qquad \dots (63)$$

where T_b is the black body temperature of the star and ν_0 is the linear resonant frequency of the loop and $< \rho v^2/2 >^p_{v_0}$ is the average photospheric energy density at v_0. With $T_b \sim 5770$ K (Sun), $R_1 \sim$ R and $< \rho v^2/2 >^p_{v_0} \approx 2 \times 10^3$ erg cm^{-3} Hz^{-1} (cf. figure 3, Ionson 1982). Corresponding to 300 s oscillations equation (63) yields $E_H \sim 7.2 \times 10^{-4}$ erg cm^{-3} s^{-1} which corresponds to a heating flux of $F_H \sim 7.2 \times 10^6$ erg cm^{-2} s^{-1}. This is slightly less than that required for coronal active region loops.

The dyħamical dissipation of Parker (1983) has the following expression for the heating flux :

$$F_H \sim B_c^2 V^2 t/4\pi l_{\parallel}, \qquad \dots (64)$$

where t is the time for which the strain produced by random motions accumulates. With $B_c \sim 10^2$ Gauss, $v \sim 0.4$ km s^{-1}, $l_{\parallel} \sim 10^{10}$ cm and $t \sim 1$ day (= 86400s), it is found that $F_H \sim 10^7$ erg cm^{-2} s^{-1} which is quite adequate to heat active region loops.

The complex magnetic reconnection approach of Heyvaerts & Priest (1984) may be used to obtain heating flux through the formula

$$F_H = \frac{B_c^2 v}{\mu_0}\left(\frac{l_{\parallel}}{l_v + l_{\parallel}}\right)^2 \left(\frac{\tau_{rec} v}{l_{\parallel}}\right). \qquad \dots (65)$$

With $B_c \sim 50$ Gauss, $v \sim 1$ km s^{-1}, $l_{\parallel} \sim 10^{10}$ cm, l_v (the length scale of velocity cells at the loop foot points) ~ 1000 km, $\tau_{rec} \sim 10^4$s it is found that $F_H \sim 2 \times 10^6$ erg cm^{-2} s^{-1} which is an order of magnitude smaller than that required for active regions.

In all these estimations photospheric velocity fields, correlation, reconnection and other times, coronal and photospheric magnetic fields, various length scales are not accurately known. Optimistically it may be said that many of the proposed heating mechanisms are capable of providing required heating fluxes.

The observed value of coronal velocity for the Sun is about 30 km/s but the predicted value is about 100 km/s (Ionson 1983). Since the physics of dissipation in coronal loops is closely related to coronal velocities hence extensiveattention in this direction is desirable.

It would be quite enlightening to apply circuit approach to the heating of coronal holes which have open magnetic field lines. The case of coronal holes is similar to that of open transmission lines. This problem is under our current investigation (Narain & Kumar 1992).

4. Conclusions

The matter presented in this article leads one to conclude the following :

(i) The total heating of a coronal loop at or near resonance may be calculated from basic electric circuit theory without going through the elaborate calculation of a local step by step analysis for which one needs to know dynamical viscosity, conductivity etc., at each point.

(ii) Many seemingly diverse heating theories such as Alfvénic surface wave heating, resonant electro-dynamic heating, dynamical dissipation, complex magnetic reconnection, stochastic magnetic pumping etc. can be considered as special cases of a single general mechanism.

(iii) This approach is able to explain satisfactorily the enhancement of X-ray flux in early M-dwarfs and quite small X-ray flux in very late M-dwarfs. It is also able to explain the observed large ratio of hard to soft X-ray emission and the bimodal behaviour of black hole accreting X-ray sources such as Cygnus X-1.

(iv) This approach enables one to determine self-inductance, current, stored magnetic energy, shearing time, damping (dissipation) time, quality factors etc. of loops and arcades readily. Stored magnetic energy is found to depend on shearing time strongly.

(v) In a region, having same plasma parameters throughout, the equivalent circuit representation simplifies to a series L-R circuit. When there are two regions having different plasma parameters then near resonant frequency the appropriate representation is a series LCR circuit, whereas near antiresonance it is a parallel LCR circuit. Away from these resonances and antiresonances the appropriate representation could be the circuit of a terminated low loss transmission line.

(vi) In order to bring observed and predicted solar coronal velocities into agreement the physics of dissipation needs a more detailed study.

(vii) The circuit approach should be developed further by applying it to plasmas associated with earth's atmosphere and other astrophysical systems.

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