Bull. Astr. Soc. India (1993) **21**, 27-32

Temperature and brightness distributions in the components of binary systems

S. K. Barman
Milki H. S. School, AT/Puratuli, Malda 732 101, West Bengal

Received 1992 July 6; revised 1992 November 11; accepted 1992 December 2

Abstract. From the theoretical model of a binary system for gravity darkening (Barman 1991), the temperature and brightness distributions along the surface of the components have been calculated using the third and fourth orders of the ratio (r/R) in the tidal potentials created by the secondary components, where r is radial distance and R the distance between the centres of gravities of the binary stars. Results of both distributions have been presented in seven figures. On analysing the results, it has been suggested that if the expansion technique be used to calculate the combined potentials and related matters e.g. the temperature and brightness distributions of a binary system, one should at least consider the fourth order of the ratio (r/R).

Key words : binary system—brightness distribution—gravity darkening

1. Introduction

In the paper (Barman 1991, henceforth referred to as paper 1) a model of a close binary system for gravity darkening due to tidally and non-uniformly rotating Roche components was presented. Using formulations of the temperature and brightness distributions deduced in the model, we have shown here the usefulness of considering the fourth order of (r/R) term in the tidal potential created by the secondary component over the third or second order terms if the expansion technique be applied to them.

The binary system is in radiative and hydrostatic equilibrium. The origin of the coordinate system is the centre of gravity of the primary; the x-axis is the line joining the centres of gravities of the components; the axis of rotation of the primary component is the z-axis. The z-axis is perpendicular to the orbital plane which is also the equatorial plane of the primary component. Any point on the surface of the primary is given by the polar coordinates (r, θ, ϕ).

2. Formulation of the model

In Paper 1 the author described in detail the methods of formulations of the model. Here the same is stated briefly. The binary system has masses m_1 (primary component) and m_2 (secondary component). For a Roche model of mass m_1 rotating nonuniformly according to the law (Ireland 1966),

$$\Omega = b_1 + b_2 \varpi^2, \qquad \dots (1)$$

where ϖ is the distance r measured from the axis of rotation, the combined potential of the gravitational and rotational forces can be put in the form :

$$\psi = \frac{G m_1}{R}\left\{\frac{1}{(r/R)} + \frac{m_2}{m_1}\sum_{j=2}^{4} P_j(z)\left(\frac{r}{R}\right)^j + \sum_{j=1}^{3} t_j \sin^2\theta\left(\frac{r}{R}\right)^{2j}\right\}, \qquad \dots (2)$$

where

$P_j(z)$ are the Legendre polynomials

$z = \cos\phi \sin\theta$

$$t_j = c_j \cdot \frac{(x-1)^{j-1} f}{x^2} \cdot \frac{1}{(r_e/R)^{2j+1}}, \quad (j = 1, 2, 3)$$

$$c_1 = \frac{1}{2},\ c_2 = \frac{1}{2},\ c_3 = \frac{1}{6}, \qquad \dots (3)$$

and x is the ratio of the equatorial to the polar angular velocities, f the ratio of the centrifugal to the gravitational forces at the equator, r_e the equatorial radius of the distorted component, and θ and ϕ are the colatitude and azimuthal angles respectively.

The effective temperature T_e at any point on the tidally and rotationally distorted components is given by

$$\frac{T_e}{T_p} = \left(\frac{g}{g_p}\right)^{1/4}, \qquad \dots (4)$$

where T_p is the black body temperature at the pole.

The brightness H at any point is given by (Kopal 1959)

$$\frac{H}{H_p} = 1 + \frac{b}{4}\left(\frac{g}{g_p} - 1\right), \qquad \dots (5)$$

where H_p is that at the pole. The surface gravity, g, at any point follows from the relation $g = -\text{grad}\,\psi$ and thus the ratio g/g_p is worked out, where g_p is that at the pole (cf. Paper 1). The value of b is given by

$$b = \frac{a}{1 - e^{-a}},\quad a = \frac{hc}{\lambda k T}, \qquad \dots (6)$$

where h is the Planck's constant, c the velocity of light, k the Boltzman constant, T the temperature and λ the wavelength.

We can now calculate the temperature and brightness distributions using relations (4) and (5).

3. A brief description of the numerical methods

To calculate the above stated distributions, equations (2) to (6) are employed in the following manner : a few appropriate values for x, f, m_2/m_1 and r_e/R are selected. t_j (j = 1, 2, 3) have been calculated from equation (3). r_p/r_e has been calculated from the condition of equipotential surfaces :

$$(\psi)_{\theta=90^\circ} = (\psi)_{\theta=90^\circ}\,, \qquad \ldots (7)$$

and using

$$\left(\frac{r}{R}\right)_{\theta=90^\circ} = r_e/R \;\text{ and }\; \left(\frac{r}{R}\right)_{\theta=0^\circ} = \frac{r_p}{R} = \frac{r_p}{r_e}\cdot\frac{r_e}{R}\,. \qquad \ldots (8)$$

Now this value of r_p/r_e has been used to calculate r/R and hence r/r_p for different θ and ϕ from the relations

$$\frac{r}{R} = \frac{r}{r_p}\cdot\frac{r_p}{r_e}\cdot\frac{r_e}{R}\,, \qquad \ldots (9)$$

$$(\psi)_{\text{at}\,\theta} = (\psi)_{\theta=0}\,. \qquad \ldots (10)$$

Using these values of r/r_p, the surface gravity $\tilde{g}$ and g/g_p have been calculated from the relations :

$$\tilde{g} \equiv -\,\text{grad}\,\psi \equiv (g_r\,, g_\theta\,, g_\phi)\,,$$

$$g = \left(g_r^2 + g_\theta^2 + g_\phi^2\right)^{1/2}\,,$$

$$g_p = (g)_{\theta=0^\circ}\,. \qquad \ldots (11)$$

Substitution of g/g_p into the equations (4) and (5) enables us to calculate the temperature and brightness distributions. Calculations have been presented in seven figures for the fourth and third order of (r/R) in second term of relation (2) for the parameters m_2/m_1, f, r_e/R and T.

4. Results and discussions

Here the distortions of the Roche surface arises simply from a combination of mass-point components, centrifugal forces and tidal forces. The second term of equation (2) namely

$$\frac{Gm_2}{R}\sum_{j=2}^{4} P_j\,(x)\left(\frac{r}{R}\right)^j$$

is the gravitational potential due to the secondary component. This term represents distortions created by the presence of the secondary component. In the summation we have retained $j = 2$ to 4, whereas in general case it is $j = 2$ to ∞. For $j = 4$ the summation gives terms containing fourth order of (r/R). For any change of j, the secondary component remains a mass-point. So changes in the curves of figures 1 to 7 are due to a numerical artifact. If the second term in relation (2) is omitted, the formulations reduce to those of Ireland's paper (1967) which describes the case of a single star. If the terms $(r/R)^3$ and $(r/R)^4$ of the second term in relation (2) are ignored, the formulations reduce to those of Peraiah's papers (1969, 1970) which extended the analysis of Ireland (1967) to the case of close binary system. The retention of $(r/R)^4$ in the tide generating potential remains within the case of mass-point secondary component to which the second

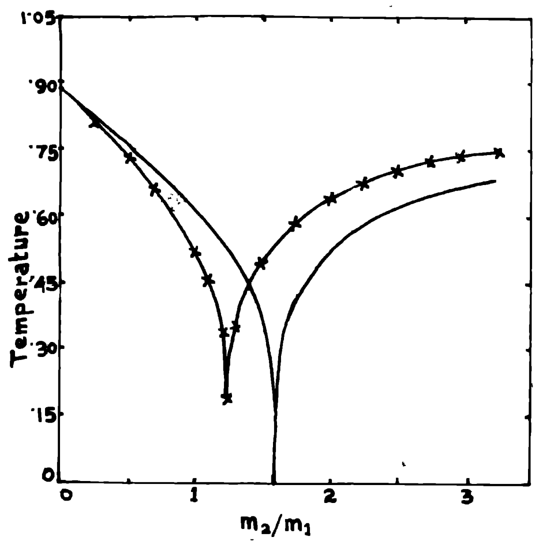

Figure 1. Distributions of temperature T_e/T_p versus m_2/m_1 for $T = 6000$°K, $\lambda = 6500$ Å, $x = 5$, $f = 0.3$, $r_e/R = 0.5$.

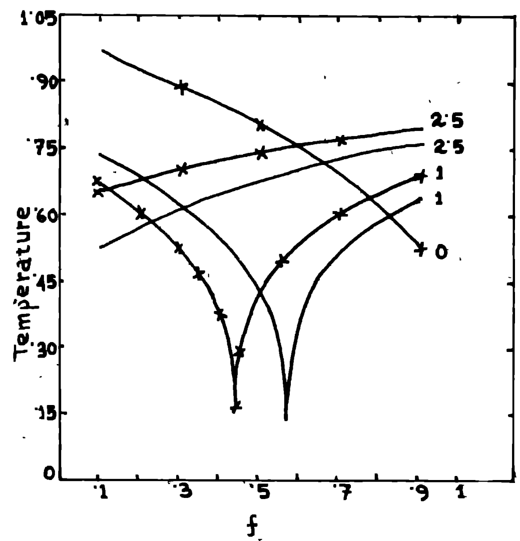

Figure 2. Distributions of temperature T_e/T_p versus f for $m_2/m_1 = 0$, 1, 2.5 and $T = 6000$°K, $\lambda = 6500$ Å, $x = 5$, $r_e/R = 0.5$.

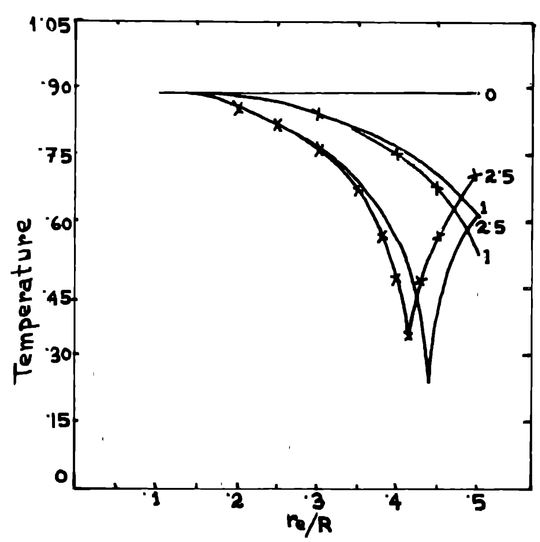

Figure 3. Distributions of temperature T_e/T_p versus r_e/R for $m_2/m_1 = 0$, 1, 2.5 and $T = 6000$°K, $\lambda = 6500$ Å, $x = 5$, $f = 0.3$.

and higher order terms of tidal surface harmonics governing their distortions (Kopal 1959) are ignorable.

Figures 1-3 present the temperature distributions for the parameters m_2/m_1, f, r_e/R and figures 4-7 present the brightness distributions for the above parameters and

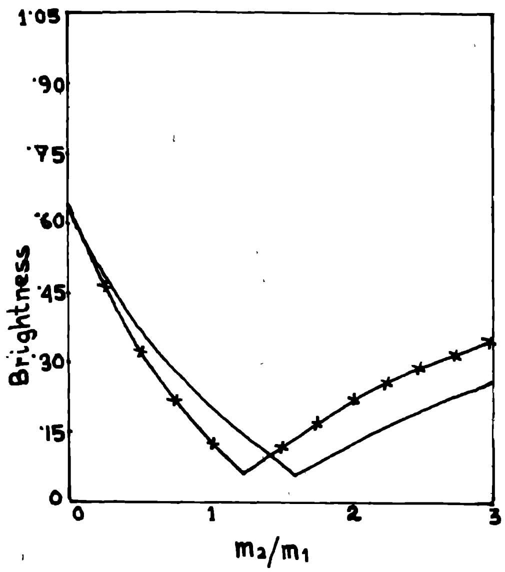

Figure 4. Distributions of brightness H/H_p versus m_2/m_1. Others are same as in figure 1.

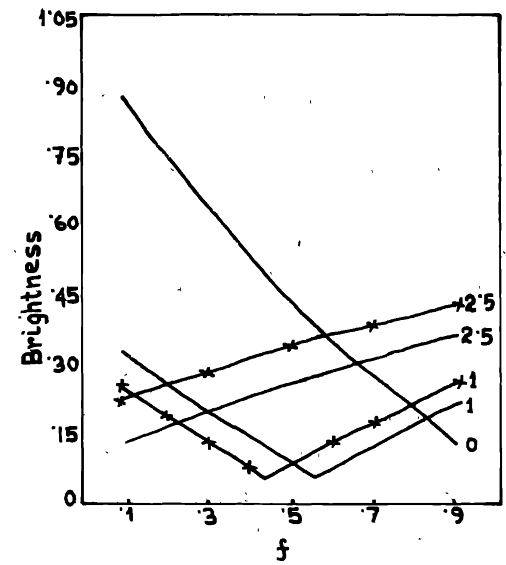

Figure 5. Distributions of brightness H/H_p versus f for m_2/m_1 = 0, 1, 2.5. Others are same as in figure 2.

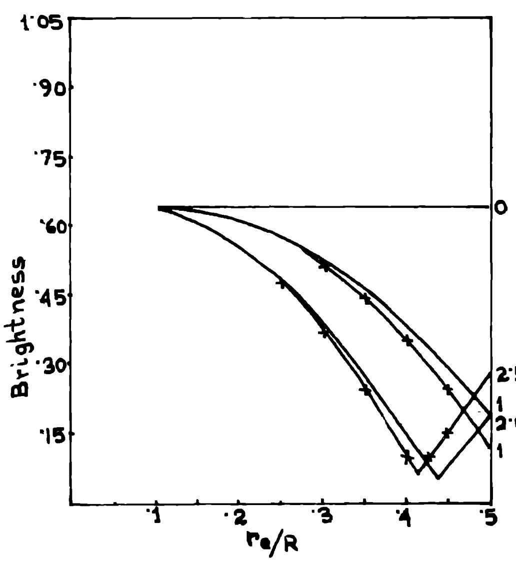

Figure 6. Distributions of brightness H/H_p versus r_e/R for m_2/m_1 = 0, 1, 2.5. Others are same as in figure 3.

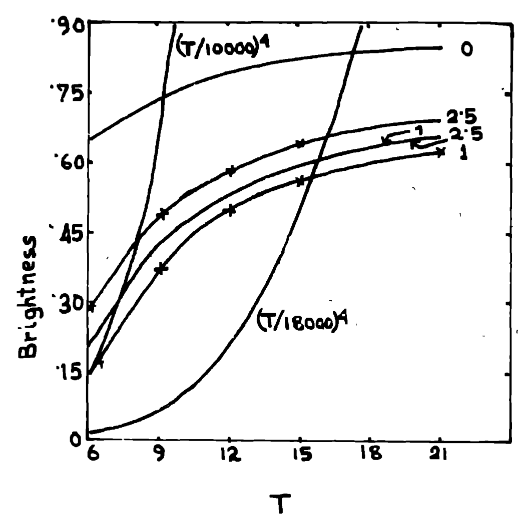

Figure 7. Distributions of brightness H/H_p versus T for m_2/m_1 = 0, 1, 2.5 and λ = 6500Å, x = 5, f = 0.3, r_e/R = 0.5.

N.B. – All the results have been presented for the equator only. In all the figures, the curves with crosses and without crosses represent fourth and third orders of (r/R) respectively. The values of the mass-ratio have been levelled in the curves.

effective temperature T for the tidal third and fourth orders in (r/R). Here only equatorial results have been presented, since these show prominent differences between the third and fourth order values. Specifically, it is seen from figure 1 that the third order value of T_e/T_p is zero for the mass-ratio $m_2/m_1 = 1.6$ whereas the fourth order values show minimum equal to 0.1888 for the mass-ratio $m_2/m_1 = 1.24$, other parameters remaining the same for both the orders. In all the other figures, the mass-ratio $m_2/m_1 = 0$ gives results for a single star. In figure 7 the black body radiation curves $(T/T_0)^4$ for polar temperature T_0 = 18,000°C and 10,000°C have been presented to compare with the brightness distributions H/H_0 versus T curves. It will be, however, difficult to adjust the polar temperature T_0 for comparison between the $(T/T_0)^4$ and H/H_0 curves, since in our model the polar value of H has been calculated by taking $\theta = 0°$. Even so it can be said that the H/H_0 curve rises steadily say up to 15,000°C but thereafter it moves up slowly towards unity contrary to the black body radiation where the curve moves up steadily towards unity.

5. Conclusions

From all these results and discussions, it can be concluded that if an expansion technique as in the text be applied to calculate combined potentials and related matters e.g. temperature distributions, brightness distributions of a close binary system, one should include at least the fourth order of (r/R) in the tidal potential caused by the secondary component. If this model be applied to the theory of contact binary system where the ratio r/R increases, then even higher order of r/R may be necessary.

Acknowledgement

Author wishes to thank the referee for his valuable comments.

References

Barman S. K., 1991, BASI, 19, 59.
Ireland J. G., 1966, publ. obs. Edinburgh, 5, 63.
Ireland J. G., 1967, Z. Ap., 65, 123.
Kopal Z., 1959, Close Binary System, John Wiley, New York.
Peraiah A., 1969, A & A, 3, 163.
Peraiah A., 1970, A & A, 7, 473.