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New approach for inclusion of the effect of gravity in magnetohydrostatic equilibrium in solar coronal arcades

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Abstract. We present a new approach for inclusion of the effect of gravity in magnetohydrostatic equilibrium in solar coronal arcades. Contrary to the findings of earlier approach, we find that the magnetic lines of force are unaffected whereas the pressure isobars are distorted owing to the inclusion of the effect of gravity. In the earlier approach, it was an essential requirement to represent plasma by an ideal gas, whereas the present approach remains unaffected whether plasma is represented by an ideal gas or not. The observations may throw light on the relative study of the two approaches.

Key words : photosphere—coronal loops—magnetohydrostatic equilibrium

1. Introduction

The observations taken by the satellites orbiting around the sun established that the solar corona is highly structured by magnetic fields. These fields are believed to originate from eruption through the photosphere (Parker 1977; Vaiana & Rosner 1978) and play an important role in solar activities (see *e.g.* Pallavicini 1989 and references therein). Some magnetised plasma structures in the solar corona appear to remain in a stable state on time scales for days, weeks or for longer time until the magnetohydrostatic equilibrium becomes unstable due to physical reasons, and it would initiate a solar activity, *e.g.* flares. Hence the solution of magnetohydrostatic (MHS) equations is of considerable importance. When the plasma pressure is negligibly small in comparison to the magnetic pressure then the magnetic field can be assumed to be force-free. In order to explain the phenomena such as solar flares, scientists solved the set of basic equations for force-free magnetic fields (*e.g.* Low & Nakagawa 1975; Low 1977; Jockers 1978). The work was further extended by Birn *et al.* (1978) as they included the plasma pressure in the set of basic equations. Birn *et al.* (1978) found two solutions for the set of basic equations under a common boundary condition at the base of the corona. However, the effect of gravity was not yet taken into account. Melville *et al.* (1983, 1984, 1986, 1987) extended the work in order to include the effect of gravity. In the approach of Melville *et al.* (1983, 1984, 1986, 1987) the plasma was essentially represented by an ideal gas, and they found that after the inclusion of the effect of gravity the pressure isobars were unaffected whereas the magnetic lines of force were found distorted.

In the present investigation we have presented a new approach for the inclusion of the effect of gravity. We found that in the present approach the condition of representing the plasma by an ideal gas is not necessary. However, the approach is unaffected whether the plasma is represented by an ideal gas or not. It is found that the present approach for the inclusion of the effect of gravity does not change the topology of the magnetic lines of force, but the pressure isobars are distorted.

2. The approach of Melville *et al.*

The basic MHS equations governing the equilibrium in solar atmosphere are

$$(\nabla \times \vec{B}) \times \vec{B}/\mu - \nabla p - \rho g \hat{e}_z = 0 \qquad \ldots(1)$$

$$\nabla \cdot \vec{B} = 0 \qquad \ldots(2)$$

where $\vec{B}$, p, ρ, and μ are the magnetic field, plasma pressure, mass density and permeability of the medium, respectively. The acceleration due to gravity is directed opposite to the unit vector $\hat{e}_z$. In the earlier work for example by Birn *et al.* (1978), Low (1977), Low & Nakagawa (1975), Priest & Milne (1980), the effect of gravity was neglected and therefore, the term $\rho g \hat{e}_z$ was not considered in the basic MHS equations. Melville *et al.* (1983, 1984, 1986, 1987) included the effect of gravity by considering the term $\rho g \hat{e}_z$ in their basic equations.

Consider that the plane $z = 0$ represents the base of the solar corona and all physical parameters vary in a two-dimensional plane (x, z). (Since the length of two-ribbon flare configuration is much larger than its width, it is reasonable to consider two-dimensional solutions.) The magnetic field can be expressed in terms of the generating function $A(x, z)$ (Low, 1977; Birn *et al.* 1978; Melville *et al.* 1983, 1984, 1986, 1987).

$$\vec{B} = -\frac{\partial A}{\partial z} \hat{e}_x + B_y(A) \hat{e}_y + \frac{\partial A}{\partial x} \hat{e}_z. \qquad \ldots(3)$$

Here $B_y(A)$ is a function of the generating function $A(x, z)$. The magnatic field (3) satisfies the condition (2). Putting (3) into (1) we get a set of equations

$$\frac{1}{\mu}\left[\nabla^2 A \frac{\partial A}{\partial x} + B_y \frac{\partial B_y}{\partial x}\right] + \frac{\partial p}{\partial x} = 0 \qquad \ldots(4)$$

$$\frac{\partial B_y}{\partial z} \cdot \frac{\partial A}{\partial x} - \frac{\partial B_y}{\partial x} \cdot \frac{\partial A}{\partial z} = 0 \qquad \ldots(5)$$

$$\frac{1}{\mu}\left[\nabla^2 A \frac{\partial A}{\partial z} + B_y \frac{\partial B_y}{\partial z}\right] + \rho g + \frac{\partial p}{\partial z} = 0. \qquad \ldots(6)$$

Since B_y is a function of A only, (5) is obviously satisfied. In order to include the effect of gravity Melville *et al.* (1983, 1984, 1986, 1987) expressed the plasma pressure p as

$$p = f(A)\, e^{-z/H} \qquad \ldots(7)$$

where H is the pressure scale height $(= RT/mg)$. Using equation (7) in (4) and (6), we get

$$\frac{1}{\mu}\left[\nabla^2 A + B_y \frac{\partial B_y}{\partial A}\right] + e^{-z/H} \frac{\partial f(A)}{\partial A} = 0 \qquad \ldots(8)$$

$$\frac{1}{\mu}\left[\nabla^2 A \frac{\partial A}{\partial z} + B_y \frac{\partial B_y}{\partial A} \cdot \frac{\partial A}{\partial z}\right] + \rho g + e^{-z/H} \frac{\partial f(A)}{\partial A} \cdot \frac{\partial A}{\partial z} - \frac{p}{H} = 0. \qquad \ldots(9)$$

In order to reduce equation (9) into equation (8) they considered the plasma to be represented by an ideal gas

$$p = \rho RT/m \qquad \ldots(10)$$

where R, T and m are the gas constant, kinetic temperature and molecular weight of plasma, respectively. They defined the dimensionless variables (by putting bars on them) as

$$\bar{x} = x/1,\ \bar{z} = z/1,\ \bar{B} = B/B_0,\ \bar{A} = A/B_0,\ \bar{f}(\bar{A}) = f(A)/p_0,$$

where B_0 and p_0 are characteristic values of field strength and pressure at the origin ($z = 0$) and 1 is the characteristic length associated with variations on the photospheric boundary. Then equation (8) becomes

$$\nabla^2 \bar{A} + \frac{1}{2} e^{-\alpha \bar{z}} \frac{\partial}{\partial \bar{A}}\{\beta f(\bar{A})\} + \bar{B}_y(\bar{A}) \frac{\partial \bar{B}_y(\bar{A})}{\partial \bar{A}} = 0 \qquad \ldots(11)$$

where

$$\nabla^2 = \frac{\partial^2}{\partial \bar{x}^2} + \frac{\partial^2}{\partial \bar{z}^2}, \qquad \beta = \frac{2\mu p_0}{B_0^2}$$

β is the ratio of plasma to magnetic pressures at the origin, and $\alpha = 1/H$ is the ratio of the width of the structure to the scale height. Melville *et al.* (1983, 1984, 1986, 1987) considered the case

$$\bar{B}_y(\bar{A}) = \text{constant} \qquad \ldots(12a)$$

and

$$\bar{f}(\bar{A}) = e^{-2\bar{A}}. \qquad \ldots(12b)$$

For the sake of convenience, the bars would be dropped from all dimensionless variables in the further discussion. Using equation (12) in (11) yields that

$$\nabla^2 A = \beta e^{-(2A + \alpha z)}. \qquad \ldots(13)$$

Equation (13) can be solved analytically by applying the transformation

$$F = A + \frac{1}{2}\alpha z, \qquad \ldots(14)$$

so that equation (13) transforms to

$$\nabla^2 F = \beta e^{-2F}. \qquad \ldots(15)$$

For the boundary condition at the photosphere

$$F(x,\ 0) = \ln(1 + x^2), \qquad \ldots(16)$$

the solution of equation (15) is given by a pair of equations

$$F_{\pm} = \ln [x^2 + z^2 + 1 \pm 2z(1 - \beta/4)^{1/2}]. \quad \ldots(17)$$

Using back transformation (14), we get

$$A_{\pm} = \ln [x^2 + z^2 + 1 \pm 2z(1 - \beta/4)^{1/2}] - \frac{1}{2}\alpha z. \quad \ldots(18)$$

Along the magnetic lines of force, the values of A remains constant. Thus there are two sets of magnetic lines of force corresponding to plus (+) and minus (−) signs. In absence of the effect of gravity the value of α is zero. Thus, the magnetic lines of force are distorted after the inclusion of the effect of gravity. The two sets of lines of force are plotted in figures 1 and 2 by solid lines.

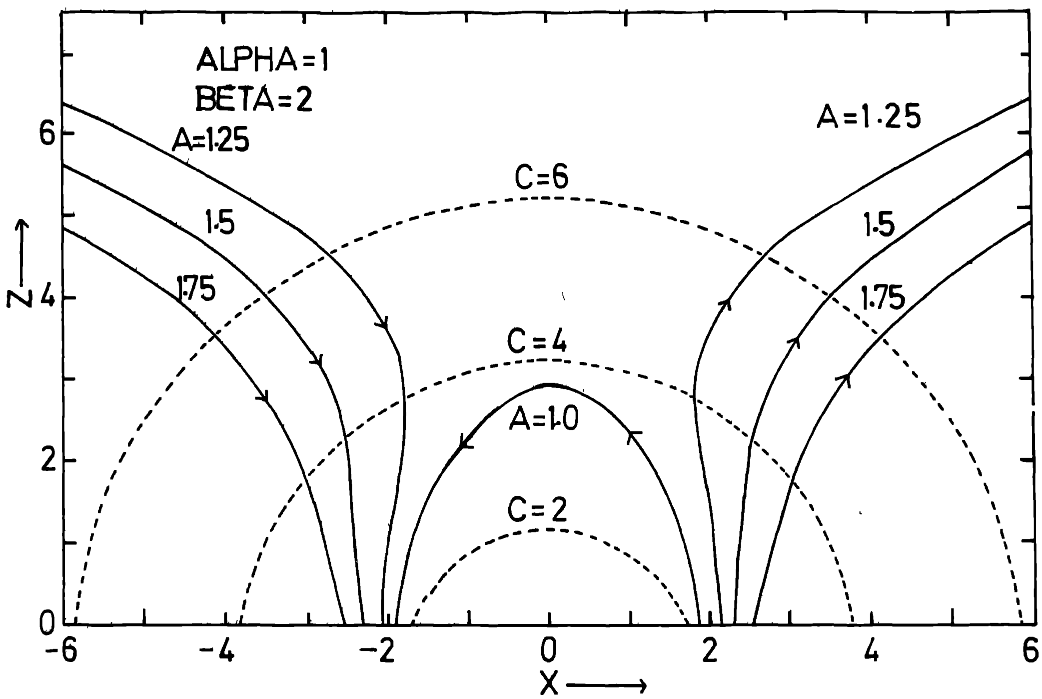

Figure 1. Figure shows the lines of force (solid lines) and pressure isobars (dotted lines) corresponding to minus (−) sign and for the values of the parameters $\alpha = 1$, $\beta = 2$ with different values of A and C. The pressure isobars are obviously circular arcs. The magnetic lines of force are distorted from the forms of the circular arcs after the inclusion of the effect of gravity.

Putting the value of $f(A)$ from equation (12b) in equation (7) and using equation (18), the pressure is given by

$$p_{\pm} = [x^2 + z^2 + 1 \pm 2z(1 - \beta/4)^{1/2}]^{-2}. \quad \ldots(19)$$

Along the pressure isobars the value of p is constant, denoted by C. The pressure isobars in figures 1 and 2 are shown by dotted lines. Equation (19) obviously does not depend on the gravity. Thus, the pressure isobars are not distorted after the inclusion of the effect of gravity.

Hence, in the approach of Melville *et al.* (1983, 1984, 1986, 1987), after the inclusion of the effect of gravity, the magnetic lines of force were distorted whereas the pressure isobars remains unaffected.

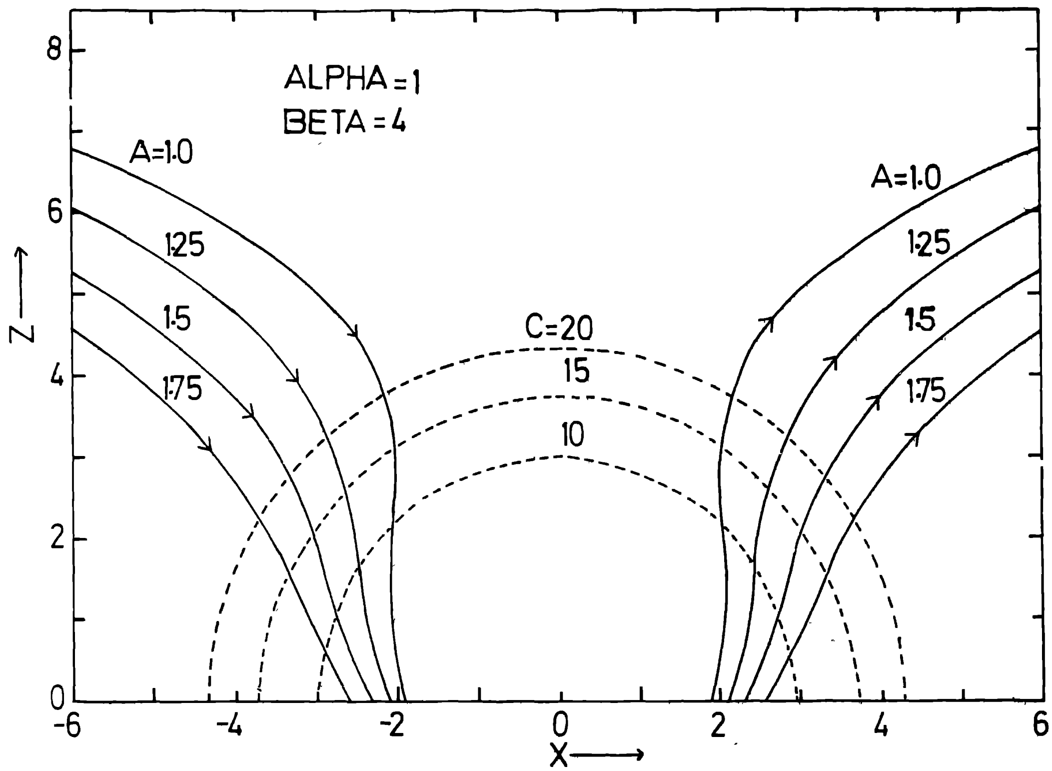

Figure 2. Figure shows the lines of force (solid lines) and pressure isobars (dotted lines) corresponding to plus (+) sign and for the values of parameters $\alpha = 1$, $\beta = 4$ with different values of A and C. The pressure isobars are circular arcs. The magnetic lines of force are distorted from the forms of the circular arcs after the inclusion of the effect of gravity.

3. Present approach

In the approach of Melville *et al.* (1983, 1984, 1986, 1987) it is obvious that equation (9) would not reduce to equation (8), if the plasma is not represented by an ideal gas [equation (10)]. We propose another approach to include the effect of gravity for which the representation of plasma by an ideal gas equation is not an essential requirement. However, we do not emphasise that the plasma cannot be represented by an ideal gas. Therefore, our approach is unaffected whether the plasma is represented by an ideal gas equation or not. We express the plasma pressure as

$$p = f(A) + G(z) \qquad \ldots(20)$$

where $f(A)$ is a function of A only and $G(z)$ is a function of z only. Using equation (20) in equations (4) and (6), we get

$$\frac{1}{\mu}\left[\nabla^2 A + B_y \frac{\partial B_y}{\partial A}\right] + \frac{\partial f}{\partial A} = 0 \qquad \ldots(21)$$

and

$$\frac{1}{\mu}\left[\nabla^2 A \cdot \frac{\partial A}{\partial z} + B_y \frac{\partial B_y}{\partial A}\frac{\partial A}{\partial z}\right] + \rho(z)\, g(z) + \frac{\partial f}{\partial A}\frac{\partial A}{\partial z} + \frac{\partial G}{\partial z} = 0. \qquad \ldots(22)$$

If we choose the function $G(z)$ as

$$G(z) = -\int_0^z \rho(z')\, g(z')\, dz' \qquad \ldots(23)$$

then equation (22) reduces into equation (21). We also changed the variables into dimensionless forms, and then dropped the bars from the dimensionless variables for the sake of convenience, so that equation (21) and equation (22) both reduce to the form

$$\nabla^2 A + \frac{1}{2}\frac{\partial}{\partial A}[B_y^2(A) + \beta f(A)] = 0. \qquad \ldots(24)$$

In the present approach also we considered

$$B_y(A) = \text{constant} \qquad \ldots(25)$$

$$f(A) = e^{-2\gamma A} \qquad \ldots(26)$$

where γ is a free parameter. Putting the values of $B_y(A)$ and $f(A)$ from equations (25) and (26) in equation (24), we get

$$\nabla^2 A + \beta\gamma e^{-2\gamma A} = 0. \qquad \ldots(27)$$

In order to transform it into a form that can be solved analytically, let us apply the transformation

$$F = \gamma A \qquad \ldots(28)$$

so that equation (27) reduces in the form

$$\nabla^2 F - \beta\gamma^2 e^{-2F} = 0. \qquad \ldots(29)$$

For the boundary condition (16) at the base of the photosphere, the analytical solution of equation (22) is given by a pair of equations

$$F_\pm = \ln[x^2 + z^2 + 1 \pm 2z(1 - \beta\gamma^2/4)^{1/2}]. \qquad \ldots(30)$$

On applying the back transformation from equation (28), we get

$$A = \frac{1}{\gamma}\ln[x^2 + z^2 + 1 \pm 2z(1 - \beta\gamma^2/4)^{1/2}]. \qquad \ldots(31)$$

Along the magnetic lines of force the value of the generating function A is constant. Thus for the given values of β and γ, there are two sets of magnetic lines of force corresponding to plus (+) and minus (−) signs. The magnetic lines of force are shown in figures 3 and 4 by solid lines. The lines of force are circular arcs. In the present approach equation (31) does not have any factor depending on the gravity. Thus, after the inclusion of the effect of gravity, the magnetic lines of force are unaffected. Using equations (23) and (26) in equation (20), the plasma pressure is given by

$$p_\pm = [x^2 + z^2 + 1 \pm 2z(1 - \beta\gamma^2/4)^{1/2}]^{-2} - \int_0^z \rho(z')\, g(z')\, dz'. \qquad \ldots(32)$$

Along the pressure isobars, the value of the plasma pressure p is constant. Therefore, the pressure isobars are given by

$$[x^2 + z^2 + 1 \pm 2z(1 - \beta\gamma^2/4)^{1/2}]^{-2} - \int_0^z \rho(z')\, g(z')\, dz' = \text{constant} = C. \qquad \ldots(33)$$

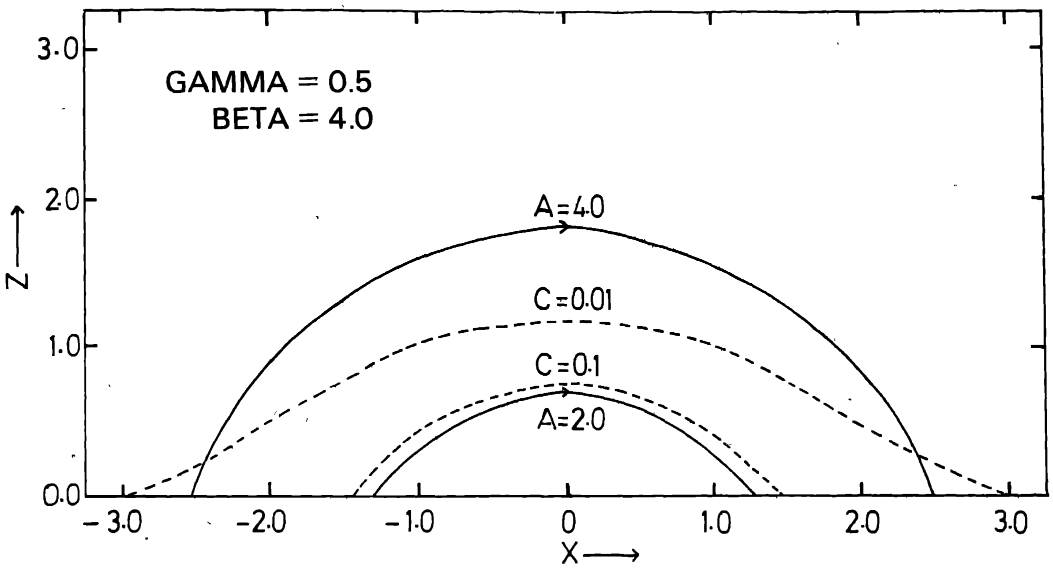

Figure 3. Figure shows the lines of force (solid lines) and pressure isobars (dotted lines) corresponding to minus (−) sign and for the values of the parameters $\gamma = 0.5$, $\beta = 4.0$ with different values of A and C. The magnetic lines of force are obviously circular arcs. The pressure isobars are distorted from the forms of the circular arcs after the inclusion of the effect of gravity.

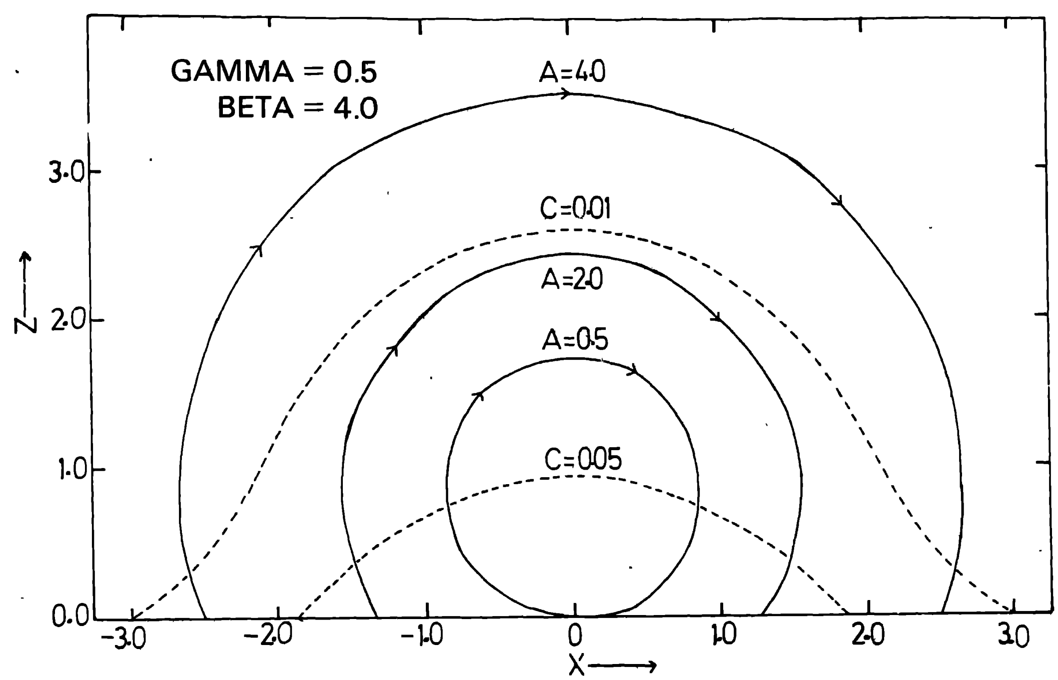

Figure 4. Figure shows the lines of force (solid lines) and pressure isobars (dotted lines) corresponding to plus (+) sign and for the values of the parameters $\gamma = 0.5$, $\beta = 4.0$ with different values of A and C. The magnetic lines of force are obviously circular arcs. The pressure isobars are distorted from the forms of circular arcs after the inclusion of the effect of gravity.

Equation (33) shows that when the effect of gravity is not accounted ($g = 0$), the pressure isobars are also circular arcs and thus, along the magnetic lines of force. After the inclusion of the effect of gravity the pressure isobars are obviously distorted.

In the present approach we found that after the inclusion of the effect of gravity the pressure isobars are distorted whereas the magnetic lines of force are unaffected. This situation is reverse to that obtained by Melville *et al.* (1983, 1984, 1986, 1987) where the magnetic lines of force were distorted and the pressure isobars remained unaffected.

In order to solve equation (33) analytically, let us consider the density ρ and the acceleration due to gravity g are constant, then the integral in equation (33) becomes $\rho g z$. Using the numerical values for constant temperature atmosphere :

Electron density

$$N_e = 10^9 \text{ cm}^{-3};\ l = 10^9 \text{ cm};\ g = 2 \times 10^4 \text{ cm/sec}^2,$$

equation (33) becomes

$$[x^2 + z^2 + 1 \pm 2z(1 - \beta\gamma^2/4)^{1/2}]^{-2} - 0.0332\ z = \text{constant} = C. \quad \text{...(34)}$$

The pressure isobars corresponding to plus (+) and minus (−) signs, for $\gamma = 0.5$ and $\beta = 4.0$, and different values of C are shown in figures 3 and 4 by dotted lines.

4. Discussion

The above details show that for the common boundary condition at the base of the corona, two magnetic configurations are possible. The total energies of these configurations defined by

$$W_\pm = \int_v \left(\frac{B_\pm^2}{2\mu} - G(z) + \frac{P_{|\pm}}{\gamma' - 1} \right) dv \quad \text{...(35)}$$

would be different. Here γ' is the adiabatic index. It can be easily shown that $W_+ \neq W_-$. Therefore, the configuration corresponding to larger energy may become unstable due to physical reasons and relax to the low energy configuration releasing an amount of energy that may be observed in the form of a solar activity. The amount of energy released would depend on the values of the parameters β and γ. When this energy is small, the solar activity may not be noticeable, but when the energy is large, the activity such as two-ribbon flares may be observable. The details for the amount of energy released would be worked out and published some where else.

5. Conclusions

We found that the magnetic lines of force are circular arcs and are unaffected after the inclusion of the effect of gravity. However, the pressure isobars are not along the magnetic lines of force. In the absence of the gravitational field the isobars are along the magnetic lines of forces. Thus the pressure isobars are distorted owing to the inclusion of the effect of gravity. But in the earlier approach discussed by some authors such as Melville *et al.* (1983, 1984, 1986, 1987) *etc.*, the pressure isobars were found to be unaffected with the consideration of the effect of gravity whereas the magnetic lines of force were found distorted.

The observations would obviously play deciding role about the relative study of the two approaches.

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