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Polarization in binaries

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Abstract. Theoretical models have been computed for estimating linear polarization from the extended dusty outer layers of the components of a close binary system whose surfaces are distorted by rotation and mutual tidal effects. We have assumed plane-parallel layers of the dusty atmospheres of the components. We have employed a wavelength dependent scattering coefficient, and Rayleigh phase function is used in solving the equation of radiative transfer. It is noticed that polarization increases with decreasing wavelength and increasing particle size. Polarization for uniform rotation is larger than that for non-uniform rotation. Polarization for single stars is always less than that for a binary component, dependent on the position of the observer.

Key words : polarization—radiative transfer—stars : binaries systems

1. Introduction

It is well known that both early and late type stars are intrinsically polarized. For a spherical star the intrinsic polarization is zero. The observation of large intrinsic polarization from a star means therefore that there is a large asymmetry in the geometry of its extended atmosphere or an internal source of polarization. To explain the observed polarization, Cassinelli (1986) has presented a variety of models for rotationally distorted and geometrically extended atmospheres. Recently, Lamers *et al.* (1978) have considered asymmetries in stellar envelopes, and winds containing blobs as in late B-supergiants such as β Ori. This leads to time variable polarization. Harrington & Collins (1968) and Collins (1970) have attempted to explain the observed polarization by considering the atmosphere in plane parallel approximation and have obtained polarization of about 2%. Peraiah (1976) has considered non-spherical systems. He has worked on early type close binary stars taking electron and molecular scattering into account. He has shown how rotation (i.e. rotation of the components about their own axes of rotation), and tidal effects due to the presence of the secondary can change the linear polarization. Buerger (1969) has computed the continuum and line radiation emitted by a rotationally and tidally distorted star which is irradiated by the light of the secondary component. The basic theory for the calculation of polarization has been developed by Brown & McLean (1977) for any axially symmetric envelope with electron scattering. McLean & Brown (1978) have presented models where the electrons are concentrated towards an equatorial zone. The models give polarization 1.0 to 1.5% for the rapidly rotating stars. Sonneborn (1982) has discussed the case of a rotating plane-parallel model of Be stars and showed that scattering is the dominant source of opacity. Shawl (1972) has, calculated polarization of late type stars by taking into account scattering from grains in a circumstellar envelope. He has found that the wavelength dependence of polarization is essentially independent of geometry (so long as there are equal number of scatterers towards and away from the observer). But the model is not sufficient for explaining high polarization.

Shawl (1974) has presented a review of the research carried out on late type stars with circumstellar envelopes. These show the presence of grains; the average size being 0.08 μm for silicate particles and slightly smaller for iron and graphite particles. Dyck & Jennings' (1971) study of some K and M giants and supergiants strongly supports the conclusion that solid particles are responsible for intrinsic polarization in these late-type stars. Dyck *et al.* (1971) has found that intrinsic polarization is always accompanied by an infrared excess, but an infrared excess is not necessarily accompanied by intrinsic polarizaiton. Shawl (1972) has found that the polarization for luminous red variables is usually increasing with decreasing wavelength, the dependence being generally given by λ^{-4}. Donn *et al.* (1966) have suggested a model for M stars where the polarization arises by Rayleigh scattering from molecular hydrogen in a highly asymmetric envelope. A maximum polarization of 5.5% is predicted by this model. Kruszewski, Gehrels & Serkowski (1968) have considered scattering from dust in asymmetric circumstellar cloud; but for polarization greater than 6%, it was necessary to stress on the star light along the observer's line of sight. Harrington (1969) has proposed a systematic pole-to-equator temperature variation of several hundred degrees. A circular polarization component has been reported by Kemp *et al.* (1972) and Gehrels (1972).

Thus to explain the observed polarizations, several authors have proposed different types of theoretical models which depend on the asymmetry of the extended envelope, on the wavelength of incident light, the size of particle, etc. None of these theoretical models however explains the polarization observations satisfactorily.

In this paper we compute theoretical models to estimate linear polarization from the extended atmospheres of the components of close binary systems whose surfaces are distorted due to rotation and tidal effects, and the outer layers contain dust grains (silicate). The grains are assumed to obey the laws of Rayleigh scattering. The models contain several parameters: (i) wavelength dependent incident light, (ii) the size of dust particle, (iii) the mass-ratio of binary components, (iv) the type of rotation, and (v) the position of observer on the equator. The aim of this paper is to see how these parameters can change the linear polarization calculated from our theoretical models of such stars.

We describe the method of calculations in section 2 and the computational procedures and results are described in sections 3 and 4 respectively.

2. Theoretical models to compute linear polarization of a close binary star

Let the close binary system have masses m_1 (primary) and m_2 (secondary); $m_1 > m_2$. The origin of the coordinate system lies at the centre of the primary; with the z-axis along the

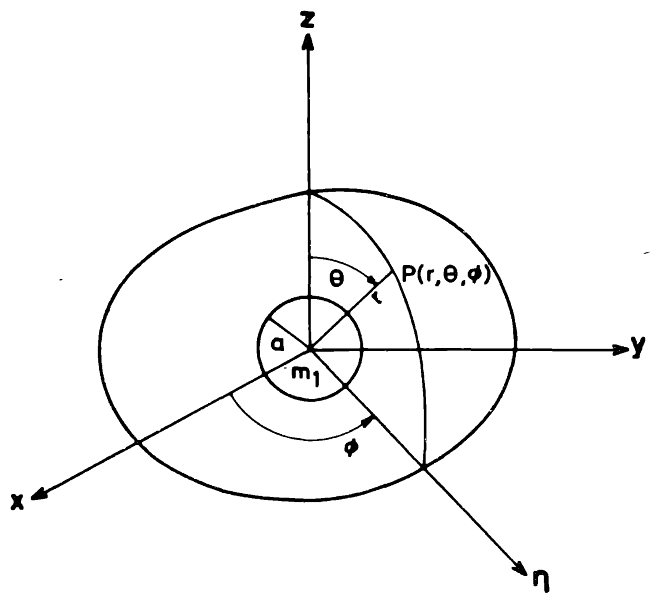

Figure 1. Distorted surface of the primary star m_1 with core radius a.

primary's axis of rotation (figure 1) and the x-axis along the line joining the centres of the binary. The equatorial planes of the components of the binary are assumed to be coincident with the orbital plane of the system. We assume that the surface of the primary is distorted (figure 1) due to rotation about its axis perpendicular to the equatorial plane and from the tidal effect due to the secondary of the system.

The equation of the surface of the distorted star is (Peraiah 1970)

$$\alpha\rho^7 \sin^6\theta + \beta\rho^5 \sin^4\theta + (\gamma \sin^2\theta + J)\rho^3 - (1 - Q)\rho + 1 = 0. \qquad \ldots(1)$$

where

$\rho = r/r_p$, r is the radiaal distance of any point, P on the equipotential surface and r_p the polar radius;

θ = colatitude of the point P;

$\alpha = (f(x-1)^2/6x^2)\,(r_p/r_e)^7$;

$\beta = (f(x-1)/2x^2)\,(r_p/r_e)^5$;

$\gamma = (f/2x^2)\,(r_p/r_e)^3$;

$J = Q(3 \sin^2\theta \cos^2\phi - 1)$;

$Q = (1/2)\ \mu_1\,(r_p/r_e)^3$;

$\mu_1 = (m_2/m_1)\,(r_e/R)^3$;

$x = \Omega_e/\Omega_p$, the ratio of the angular velocity at the equator to that at the pole;

$f = \Omega_e^2 r_e^3/Gm_1$, the ratio of the centrifugal to the gravitational at the equator.

R is the distance between the centres of the masses m_2 and m_1. r_e and r_p are the equatorial and the polar radii, and are given by

$$(r_e/r_p)^3 - \mu(r_e/r_p)^2 - (1/2)\mu_1 = 0, \qquad \ldots(2)$$

$$\mu = 1 + f(x^2 + x + 1)/6x^2 + (1/2)\mu_1(3 \cos^2\phi - 1). \qquad \ldots(3)$$

The radial distance of a point P in the units of polar distance, $\rho = r/r_p$, can be calculated from equations (1) and (2) for a given set of parameters x, f, θ, ϕ in the case of a single star, and incorporating parameters m_2/m_1 and r_e/R in the case of the components of a close binary system.

If binary stars rotate synchronously, then

$$f = 1 + (m_2/m_1)(r_e/R)^3. \quad \ldots(4)$$

The total surface gravity g

$$g = (G\, m_1/r^2)\{[1 - f_r(\theta) \sin^2\theta - 2J\rho^3]^2 + [f_r(\theta) \sin\theta \cos\theta + 6Q\rho^3 \sin\theta \cos\theta \cos^2\phi]^2 + [36Q^2\rho^6 \sin^2\rho \sin^2\phi \cos^2\phi]\}^{1/2}, \quad \ldots(5)$$

where

$$f_r(\theta) = 2\gamma\, \rho^3 + 4\beta\, \rho^5 \sin^2\theta + 6\alpha\, \rho^2 \sin^4\theta. \quad \ldots(6)$$

The surface element is

$$ds = g\, r^2 \sin\theta\, d\theta\, d\phi / g_r. \quad \ldots(7)$$

(a) *The radiative transfer*

We assume that the rotational and tidal effects are small over a sphere of radius a (*cf.* figure 1), where $a \ll r_p$ and this is always set to be the inner radius of the spherical shell (where $\tau = T$, τ is the optical depth at any point and T is the total optical depth). The specific intensities I_l and I_r at any point $\rho(r, \theta)$ on the distorted surface of the star are calculated by solving the equation of transfer in spherical symmetry corresponding to radius r (Peraiah 1976).
The equation of radiative transfer for spherical symmetry can be written as (Peraiah 1976)

$$\frac{\mu}{r^2}\frac{\partial}{\partial r}\{r^2 I(r, \mu)\} + \frac{1}{r}\frac{\partial}{\partial \mu}\{(1-\mu^2)I(r, \mu)\} + \sigma(r)I(r, \mu) = \sigma(r)\left\{[1-\omega(r)]b(r) + \frac{\omega(r)}{2}\int_{-1}^{+1} P(r, \mu, \mu')I(r, \mu')d\mu'\right\} \quad \ldots(8)$$

and

$$I(r, \mu) = \begin{bmatrix} I_l(r, \mu) \\ I_r(r, \mu) \end{bmatrix}, \quad \ldots(9)$$

where $I_l(r, \mu)$ and $I_r(r, \mu)$ refer respectively to the states of polarization in which the electric vector vibrates along and perpendicular to the principal meridian; $\omega(r)$ is the albedo for single scattering; α the extinction coefficient; $b(r)$ the source inside the medium; and $P(r, \mu, \mu')$, the Rayleigh's phase function is given by

$$P(r, \mu, \mu') = \frac{3}{4}\begin{bmatrix} 2(1-\mu^2)(1-\mu'^2) + \mu^2\mu'^2 & \mu^2 \\ \mu'^2 & 1 \end{bmatrix} = \begin{bmatrix} P_{11}(\mu, \mu') & P_{12}(\mu, \mu') \\ P_{21}(\mu, \mu') & P_{22}(\mu, \mu') \end{bmatrix}. \quad \ldots(10)$$

We shall set $\omega(r) = 1$, since we are considering pure scattering. The total optical depth T is given by (van de Hulst 1957).

$$T = L\ \alpha\ d, \qquad \ldots(11)$$

where

$$\alpha = \pi l^2\ Q_{scat}, \qquad \ldots(12)$$

$$Q_{scat} = (8/3)\ (2\pi l/\lambda)^4\ ((m^2 - 1)/(m^2 + 2))^2, \qquad \ldots(13)$$

where d is the number of dust particles per unit volume; L the total path length; l the radius of the dust particle; m the refractive index of the particle; and λ the wavelength of the incident light. The total path length L has been taken as the length between the inner radius and the outer radius of a star.

(b) *The surface integrated linear polarization*

From equation (9) we have

$$I(\mu) = \begin{bmatrix} I_l(\mu) \\ I_r(\mu) \end{bmatrix}, \qquad \ldots(14)$$

where $I_l(\mu)$ and $I_r(\mu)$ are parallel and perpendicular components to the electric vector of the emergent intensity at $P(r, \theta, \phi)$ (figure 2). We employ the roots of Gauss-Legendre quadrature points for angle quadrature for $\mu \in (0,1)$. The integrated intensities I_l and I_r can be written as

$$I_l = \sum_{j=1}^{n} a_j I_l\ (\mu_j), \quad 0 < \mu_j < 1 \qquad \ldots(15)$$

$$I_r = \sum_{j=1}^{n} a_j I_r(\mu_j), \quad 1 < j < n \qquad \ldots(16)$$

where a_j are the corresponding quadrature weights.

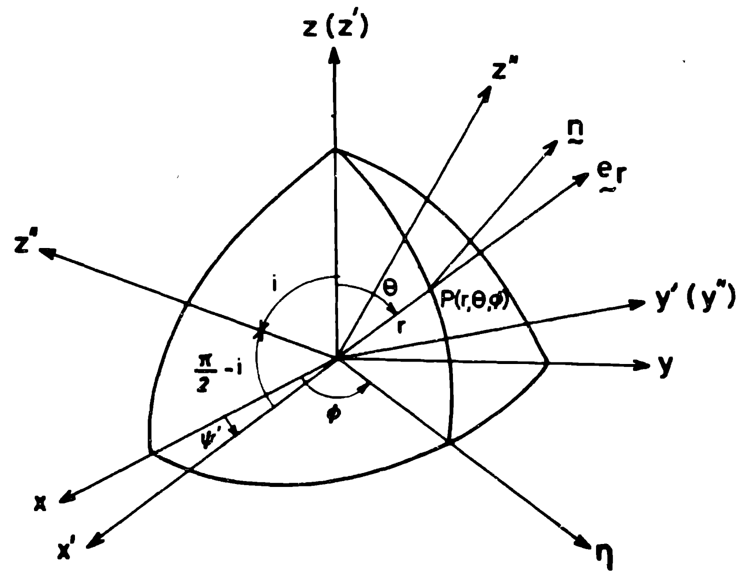

Figure 2. Geometry of rotation.

Let i, j, k be the unit vectors along x^θ, y^θ, z^θ axes respectively and e_r, e_θ, e_ϕ unit vectors along the r, θ, ϕ directions. Then

$$\mathbf{e}_r = \mathbf{i} \sin\theta \cos\phi + \mathbf{j} \sin\theta \sin\phi + \mathbf{k} \cos\theta, \qquad \ldots(17)$$

$$\mathbf{e}_\theta = \mathbf{i} \cos\theta \cos\phi + \mathbf{j} \cos\theta \sin\phi - \mathbf{k} \sin\theta, \qquad \ldots(18)$$

$$\mathbf{e}_\phi = -\mathbf{i} \sin\theta + \mathbf{j} \cos\phi. \qquad \ldots(19)$$

Following Harrington & Collins (1968), we consider two rotations (*cf.* figure 2). One is the rotation of x-y plane about the z-axis through an angle ψ', and the other rotation is of x'-z' plane about y' axis through an angle $\pi/2 - i$, where i is termed as ecliptic angle. Then

$$x'' = i \cos\psi' \sin i + j \sin\psi' \sin i + k \cos i, \qquad \ldots(20)$$

$$y'' = -i \sin\psi' + j \cos\psi' \qquad \ldots(21)$$

$$z'' = -i \cos\psi' \cos i - j \sin\psi' \cos i + k \sin i. \qquad \ldots(22)$$

If x'' is taken as the line of sight, then the observer will see the unit vector n projected on the y''-z'' plane. let α denote the angle between this projection and the z''-axis. Now

$$g\mathbf{n} \cdot \mathbf{l} = \mathbf{g} \cdot \mathbf{l} = (g_r e_r + g_\theta e_\theta + g_\phi e_\phi).\ l$$

$$\text{or } \mathbf{n} \cdot \mathbf{l} = (g_r e_r + g_\theta e_\theta + g_\phi e_\phi.l/g \qquad \ldots(23)$$

= the component of $\mathbf{n}$ along the line of sight.

Similarly, the components of $\mathbf{n}$ along the y''-axis, $n.y''$, and along the z''-axis, $n.z''$, can be written. The ratio of the y'' and z'' components is the tangent of the angle α

$$\tan\alpha = (n.y'')/(n.z''). \qquad \ldots(24)$$

Therefore, the polarization p along the line of sight is given by

$$P = \frac{\int_0^{\pi/2} \int_{\pi/2}^{\pi/2} (I_r - I_l) r^2 \cos 2\alpha \sin\theta (g\mathbf{n}.\mathbf{1})/g_r\, d\phi\, d\theta}{\int_0^{\pi/2} \int_{\pi/2}^{\pi/2} (I_r + I_l) r^2 \sin\theta\ (g\mathbf{n}.\mathbf{1})/g_r\, d\phi\, d\theta}. \qquad \ldots(25)$$

3. Computational procedure

Firstly, we have to find out r_e/r_p from equation (2) and then ρ from equation (1). To find r_e/r_p and ρ we have used Newton-Raphson method. In both cases 1.2 can be taken as the starting value. Then for each ρ, we have to solve the radiative transfer equation (18) to find I_l and I_r of equation (9). The inner polar radius of the spherical shell is taken to be 10^{12} cm. The medium above this shell is divided into 200 layers. The equation of transfer is solved by employing the discrete space theory (Peraiah 1984). From equation (25) we have calculated polarization for wavelengths 5000 Å to 10000 Å for (i) dust particles $\ell = 0.02, 0.025, 0.03\ \mu$m (*cf.* figure 3). (ii) $m_2/m_1 = 0, 0.5, 0.9$ (figure 4), (iii) ratio of the equatorial to the polar angular velocities $x = 1, 5, 10$ (*cf.* figure 5) and (iv) positions on the equator $\psi' = 0°, 35°, 70°$ (*cf.* figure 6). Other parameters for the figures have been

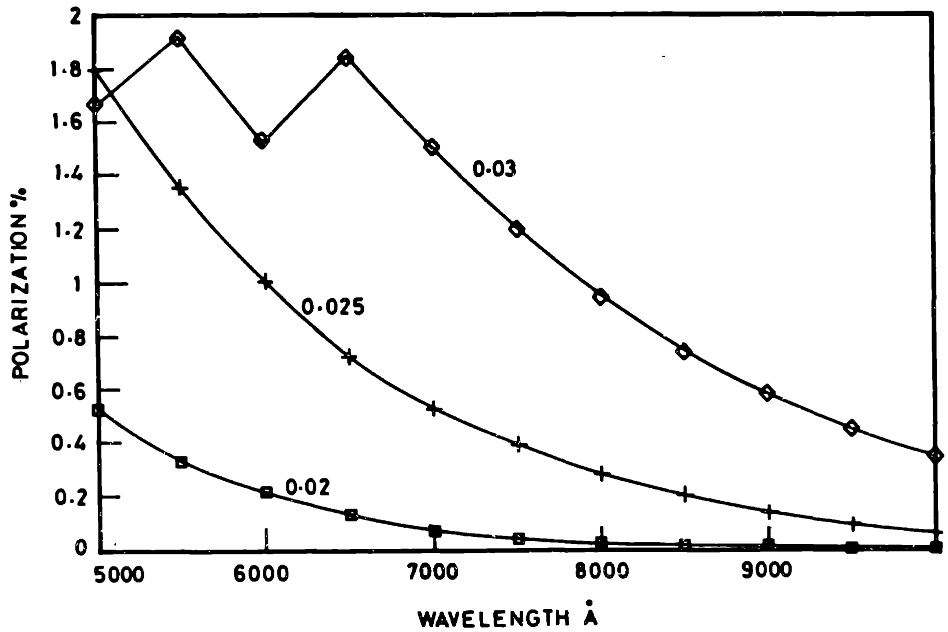

Figure 3. Polarization versus wavelength for $l' = 0.02\mu$, 0.25μ, 0.03μ (mentioned on the graph). The other parameters are $x = 1$, $m_2/m_1 = 0.5$, $r_e/R = 0.5$, $m = 1.45$, $\psi' = 0°$, $i = 90°$.

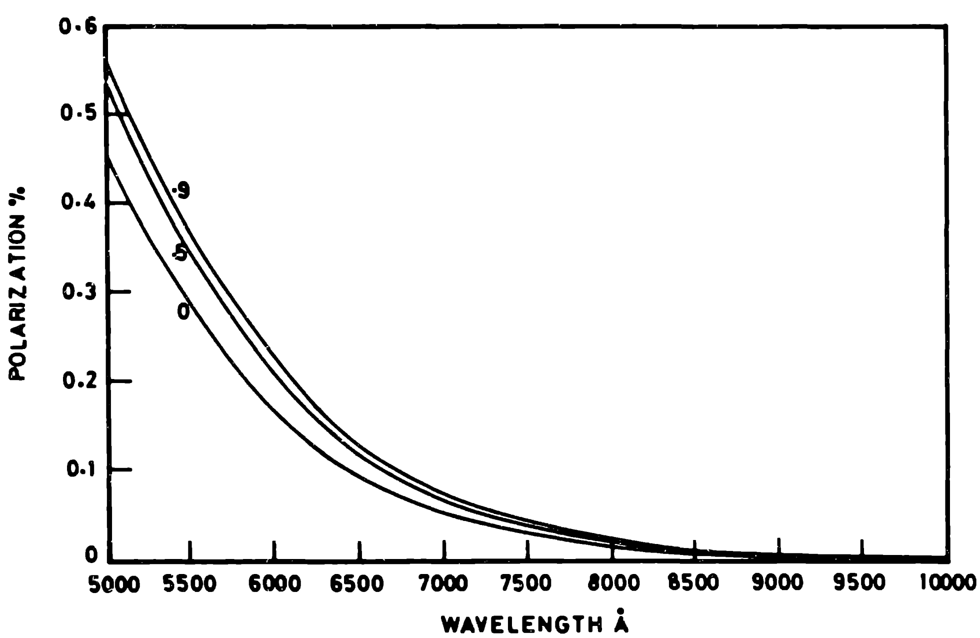

Figure 4. Polarization versus wavelength for $m_2/m_1 = 0$, 0.5, 0.9 (mentioned on the graph). The other parameters are $l = 0.02\mu$, $x = 1$, $r_e/R = 0.5$, $m = 1.45$, $\psi' = 0°$, $i = 90°$.

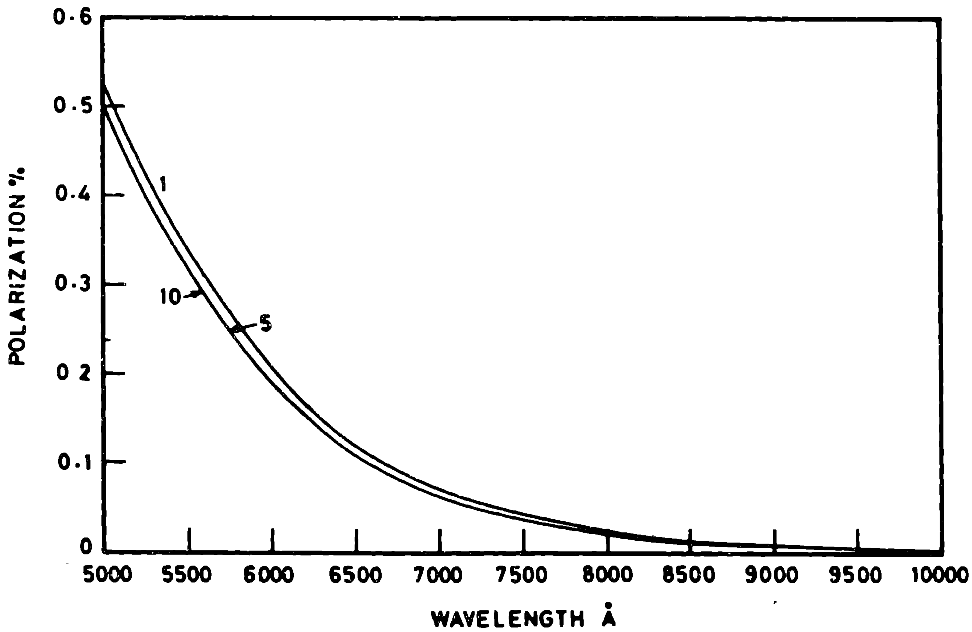

Figure 5. Polarization versus wavelength for $x = 1, 5, 10$ (mentioned on the graph). The other parameters are $l = 0.02\mu$, $m_2/m_1 = 0.5$, $r_e/R = 0.5$, $m = 1.45$, $\psi' = 0°$, $i = 90°$.

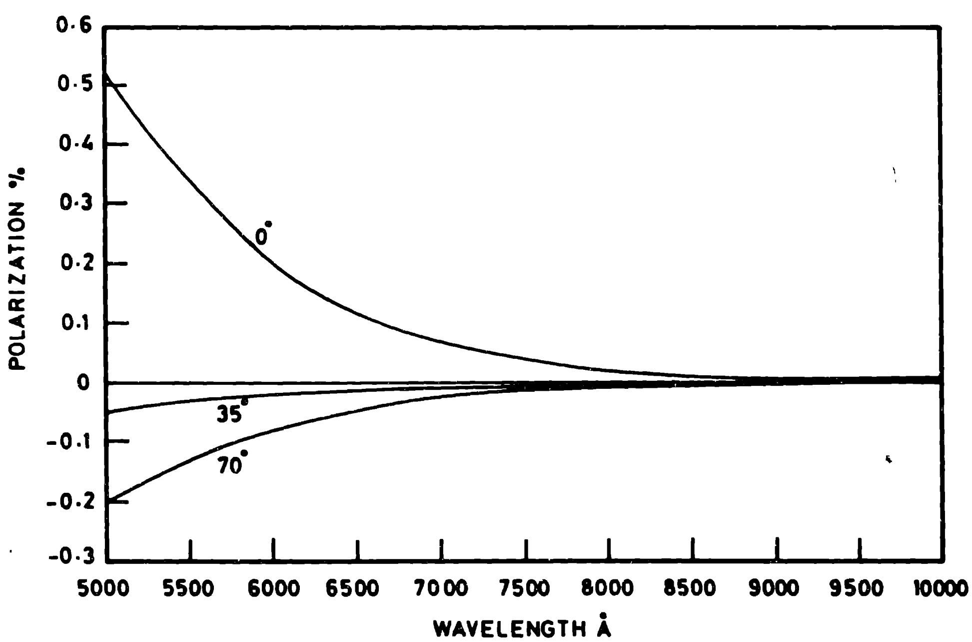

Figure 6. Polarization versus wavelength for $\psi' = 0°$, 35° and 70° (mentioned on the graph). The other parameters are $l = 0.02\mu$, $x = 1$, $m_2/m_1 = 0.5$, $r_e/R = 0.5$, $m = 1.45$, $i = 90°$.

mentioned in the figures. We have taken synchronous rotation of binaries. For simplicity, we have set $\phi^\circ = 0$.

4. Results and discussion

We have set $2\pi l/\lambda < 0.4$, since the equation (13) for Q_{scat} should be taken for $2\pi l/\lambda < 0.5$ (van de Hulst 1957). We have therefore taken small dust particles not greater than 0.3 μm for the wavelengths 5000 Å to 10000 Å. We have taken refractive index of dust particles (silicate) to be 1.45 and the number density d equal to 4 cm^{-3}. From the figures 3-6, it is clear that polarization decreases with the increase of wavelength like molecular scattering (Peraiah 1976). This is the expected variation of Rayleigh scattering. However, the dust particles with $l = 0.03$ μm show deviations from this behaviour which is not understood well. In the next work we shall use Shah's (1977) program to find Q_{scat} that works for any size of dust particles and for any combination of scattering and absorption which are reasonable for the stars of interest. With increse of particle size (figure 3), polarization has increased within the interval of wavelength. This is due to the fact that larger particles can create in the outer layers and surfaces of a star more asymmetry and distortion.

Polarization increases with mass ratio (figure 4). Polarization, when $m_2/m_1 > 0$, is greater than that when $m_2/m_1 = 0$. This is due to the fact that the primary component is becoming more asymmetric due to the presence of the secondary. This is in agreement with Peraiah's work (1976) for electron scattering. It is noticeable that uniform rotation ($x = 1$) has the maximum effect (figure 5) on polarization. Polarization for uniform rotation ($x = 1$) is greater than that for non-uniform rotation ($x > 1$). Here we are considering $x > 1$ i.e. the angular velocity at the equator is always larger than that at the pole. In our model we have $r_p < r < r_e$ which corresponds to $x > 1$.

Polarization is very much dependent on the position of the observer (figure 6). As the observer moves along the equatorial plane through $\psi' = 35°, 70°$ polarizations become negative.

We have done this work to see the effect of some parameters on the polarization by dusty outer envelope of close binary system in the line of sight. We wish to apply this model to real situations (in late type binaries) in future. For that case, we have to adjust the parameters, the size of the particle should be increased (so $2\pi l/\lambda$ may be much greater than 1) and the absorption by dust particle has to be considered. The medium should be taken as non-uniform. All these considerations are under study.

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