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Binary stars as tests of gravity theory

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Abstract. The binary stars have provided testing sites for gravity theories ranging from the Newtonian law of gravitation in the last two centuries to the theory of relativity and other gravity theories today. The binary pulsar in particular holds out the promise of testing strong field effects of gravity, distinguishing between relativity and other theories. This review discusses the conceptual and observational issues involved in this process of testing.

Key words: binary stars—gravity theories—relativity

1. Introduction

The law of gravitation as originally proposed by Isaac Newton was inspired by astronomy rather than by any laboratory experiment (with due regards to the mythical story of the falling apple!). Its first testing ground was the solar-system—the Keplerian laws of planetary motion and the detailed observations of the moon. Indeed, to test the long range validity of this law there is no alternative but to have recourse to astronomical data.

In this sense the observations of binary stars provide direct proof of the validity of Newton's law at distances far greater than the solar system distances. Indeed, it would not be an exaggeration to say that binary stars have provided the longest distances out to which the law of gravitation is tested. The observations at galactic and extragalactic distances are at best tentative. For example, the observations of flat rotation curves of spiral galaxies can be interpreted either as implying dark matter or as indicators of violation of the Newtonian laws of motion and gravitation.

What about the tests at the more sophisticated level of strong gravity? Just as the motion of the perihelion of Mercury confirmed the small but necessary correction brought in by general relativity, can we look to the binaries to provide evidence of similar effects?

Until about a decade ago this looked difficult if not impossible. For strong gravity effects, one needs close, compact binaries. Further one needs accurate observations of space and time to test such effects. Also, the theoretical interpretation of any expected effects looked ambiguous. Thanks to the discovery of binary pulsars, especially the pulsar PSR 1913 + 16, these difficulties have largely gone away.

In this talk I will review the observational evidence and its theoretical interpretation in such cases. Thus I shall concern myself with strong gravity effects in binary pulsars. It is best to begin with a brief summary of observations.

2. Observational status

The presence of a pulsar in the binary provides us with an accurate clock that serves as a useful component of any experiment testing gravity theories. Table 1 summarizes the physical characteristics of four radio pulsars found in binary systems.

Table 1. Data on four binary radio pulsars*

Name	$P_{(orb)}$ (d)	e	Mass function ($M_\odot$)	Most likely companion mass ($M_\odot$)	$P_{(pulse)}$ (s)
PSR 1913 + 16	0.32	0 617	0.1322	1.40 ± 0 05	0 059
PSR 0655 + 64	1.03	0.000	0.0712	1.00 ± 0.30	0 196
PSR 0820 + 02	1232	0.012	0.00301	0 2 – 0 4	0 865
PSR 1953 + 29	~117	<0.01	0.00272	02 – 0.4	0 0061

*From van den Heuvel (1984).

Of these four cases the first one PSR 1913 + 16 has proved to be the most relevant one for testing gravity theories because of its low period, compact size, and large eccentricity. It is not surprising, therefore, that this pulsar has featured prominantly in the context of gravitational effects. The three main effects noticed are (i) the precession of the periastron of the orbit; (ii) the precession of the rotation axis of the pulsar through the spin-orbit coupling effect; and (iii) the shortening of the orbital period, possibly due to gravitational radiation (see Taylor, Fowler & McCulloch, 1979).

(i) The two masses of the system are around 1.4 $M_\odot$. Assuming that the binary orbit is as per the reduced mass of ~0.7 $M_\odot$ and using the naive extrapolation of the Schwarzschild solution for such an orbit one arrives at a precession rate for the periastron of the observed value, viz 4.23° per year. To place this value in proper perspective, the corresponding rate for planet Mercury is 43 arcsec per century, *i.e.*, about 36,000th part of the pulsar value!

(ii) The spin-orbit coupling is the force acting on a spinning massive body as it orbits around another one. This effect is due to general relativity and is yet to be measured for, say, a gyroscope orbiting the earth. The spin direction which should otherwise have remained fixed in space, precesses about a fixed axis as a result of this coupling. In the case of a pulsar the precession would result in a change in the pulse pattern, *e.g.*, a systematic change in the pulse window shape. The binary pulsar shows a precession rate of about 1° per year.

(iii) The most publicized effect seen is the steady decrease in the orbital period at the rate $\approx 3 \times 10^{-12}$ (Note that the rate of change of period is a dimensionless quantity!). The most likely cause of this effect is gravitational radiation. The two masses in a binary system form a quadrupole moment that changes with time. The classic first order theory of gravity wave emission gives a radiation rate of

$$L = \frac{G}{45\,c^5} | \dddot{I} |^2, \qquad \ldots(1)$$

where **I** is the quadrupole moment tensor and a dot denotes time derivative.

On this basis the binary loses energy and its orbit shortens in size at the rate

$$\dot{r} = \frac{-64\, G^3 m_1 m_2 (m_1 + m_2)}{5 c^5 r^3}, \qquad \ldots(2)$$

where m_1, m_2 are the binary masses and r their separation. Correspondingly the period decreases at the rate

$$\dot{T} = \frac{3}{2} \frac{\dot{r}}{r} T. \qquad \ldots(3)$$

Although the estimated rate (1) for the binary pulsar is too small to be detected by the present generation of gravity wave detectors, the consequence (3) of gravitational radiation appears to be in agreement with the observations.

Although prima facie we have evidence here for the post-Newtonian effects in gravity the theoretical end of the situation is not as clear cut as it first appeared. Indeed, there has been a considerable discussion amongst the relativists as to what are the predicted theoretical values to be compared with observations. I shall discuss these issues next.

3. The exact solution

General relativity theory has produced very few exact solutions of realistic nature. The first solution that has proved to be a classic one is the Schwarzschild solution which describes the gravitational effects of a spherical compact distribution of matter. This is essentially a one body problem. In the application of this solution to the solar system we assume the sun to be that single body and planets, the test particles. That is, the presence or motion of a typical planet does not change the underlying geometry. The fact that

$$\frac{M_\odot}{M_{\text{Planet}}} \gg 1,$$

makes this assumption a realistic one. Thus we can 'trust' the test particle orbits as representing the actual planetary orbits. Further, the 'weak field' nature of the problem facilitates an easy comparison with the Newtonian problem.

In the case of the binary pulsar the problem is a genuine two-body problem for which to date no exact solution is available in general relativity. The attempt to convert it to a one-body problem by having recourse to the 'reduced' mass and relative separation has no *locus standi* since there is no demonstration in general relativity that this can be done.

The fact that this simplification is doable in the Newtonian theory is due to the linearity of the equations of motion. General relativity on the other hand is manifestly nonlinear. The only situation in which it can have recourse to the linearized procedure is in the weak field limit where it is approximated by the Newtonian theory.

In the binary pulsar, the two masses are nearly equal and so the problem cannot be approximated by the Schwarzschild solution as in the sun-planet case. Moreover, the dimensions of the orbit are such that one cannot be sure that we can have recourse to the weak field approximation.

Thus in my opinion extreme caution has to be exercised in claiming a theoretical result that fits (or does not fit) the binary pulsar observations. In the final section of this talk I will describe the approximate procedure that is followed by the theoreticians (who themselves did not agree on a unique procedure for several years) in dealing with the strong gravity case. For a more detailed review see Damour (1987, 1989).

4. Theoretical approximations

The observed secular acceleration of the mean orbital motion of the binary pulsar has fuelled the controversy whether the formula (1) can be applied. As pointed out by Damour the only convincing way of comparing the theory with observations is to deduce from the field equations a 'complete relativistic celestial mechanical description' of a binary system of condensed bodies. Such an approach can be broken up into three steps:

(i) The derivation of equations of the binary system complete upto the level where the terms which might cause a secular acceleration arise;

(ii) The solution of the equations of motion taking into account both short term periodic effects and long term periodic and/or true secular effects;

(iii) The computation (finally) in a direct way of what is observed, namely the arrival times, on the earth, of the electromagnetic signals emitted by a spinning neutron star moving on the previously calculated orbit.

Damour has pointed out that Laplace had predicted the secular acceleration effect arising from a possible delayed (*i.e.*, noninstantaneous) propagation of gravitation. Suppose the speed of propagation of gravitational interaction is C_g. Then the effect of mass m on mass m' is to produce a small non-radial component in the latter's acceleration, given by

$$a = \frac{Gm}{R^2}\,\frac{V}{C_g}, \qquad \ldots(4)$$

where R is the relative distance between the present position of m' and the 'retarded' position of m when the gravity signal left it towards m', and V is the corresponding relative velocity of m' with respect to m. Laplace then showed that the small damping term in equation (4) will cause a secular acceleration of the motion of a planet. He even used the 'known' secular acceleration of the moon to compute C_g to find that it must exceed 7×10^6 times the speed of light!

Although the final conclusion is not correct the essence of the idea is right. Later Eddington (1924) showed that when the calculation is done correctly the Laplace effect is wiped out to first order in C_g^{-1} but survives in the second order. The full calculation of the equations of motion of two condensed objects taking into account the effect of propagation of gravity, the speed of light and higher order weak field nonlinearities has shown that the dissipative part works out as

$$\mathbf{a} = \frac{4G^2}{5c^5}\,\frac{mm'}{R^3}\left\{\mathbf{v}\left(-v^2 + \frac{2Gm'}{R} - \frac{8Gm}{R}\right) + \mathbf{N}(\mathbf{N}.\mathbf{v})\left(3v^2 - \frac{6Gm'}{R} + \frac{52}{3}\,\frac{Gm}{R}\right)\right\},$$

$$\mathbf{N} = \mathbf{R}/R. \qquad \ldots(5)$$

For detailed references to this work see Damour (1987). This is the full 'residual Laplace effect' announced by Eddington. It may also be called the gravitational radiation damping force. On the basis of this formula general relatively predicts a secular acceleration of the mean orbital motion of a binary system.

Even when the equations of motion have been written down to a sufficient degree of accuracy, one still needs to compute the directly observable quantity, the so-called pulse timing formula. This formula gives the arrival times on the earth of the pulses from the pulsar as a function

$$\tau = F(N,\ P_1,\ P_2,\ \ldots), \qquad \ldots (6)$$

of the number N of the pulses from the pulsar and certain parameters P_1, P_2, . . . of the binary system.

Several workers (Blandford & Teukolsky 1976; Epstein 1977, 1979; Haugen 1985; and Damour & Deruelle 1985, 1986) have computed τ in increasingly accurate manner. The following point emerges. The calculation involves, in principle, the mixing of strong field effects near the pulsar with the weak field ones in regions away from it and in the solar system. Of the works quoted here only the last one takes note of the strong field effects explicitly. The remarkable fact emerges that the strong field effects can be 'renormalized away' by redefining the magnitudes of the binary masses and adding a few unobservable constants. This 'effacing' of strong field effects seems to be a general feature of the theory of relativity and might not hold necessarily in other gravity theories.

I shall not go into the details of the timing formula except to state that it is calculated in a series of steps of which the first two involve calculating the pulse arrival time at the barycentre of the solar system. From these corrections are made for calculation of τ at the earth. Even the timing to barycentre contains 19 parameters; of which four come from the pulsar configuration, one from the Doppler motion of the binary system relative to the solar system, six Keplerian parameters, one 'half-post-Keplerian' parameter, five post-Keplerian parameters and two secular parameters. The half and full post-Keplerian parameters contain, respectively, corrections of the order v/c and $(v/c)^2$

5. Concluding remarks

Like the PPN (parametrized post-Newtonian) approach to testing gravity theories the above may be called the PPK approach where 'Newton' is replaced by 'Kepler'. Damour has argued that several alternative gravity theories can be cast in this form for the numerical PK parameters. In principle all the PPK parameters are measurable observationally, but one must allow for 'noise', *i.e.*, errors of observations. On the theoretical side one expects the gravity theory to lead to unique answers provided the binary system is 'clean' (*i.e.*, there are no perturbations to be added to the relativistic two-point-mass system).

Thus in practice one is not dealing with very clear cut comparisons of theoretical and observed 'points' in the parameter space but with broad strips which may or may not overlap.

It is still too early to claim a complete confirmation of general relativity or rejection of any particular alternative theory of gravity based on these calculations. Broadly one can say that general relativity is consistent with the data while more work needs to be done on other theories. In particular the important property of effacement of strong gravity effects that simplify the general relativistic prediction may not be present in other theories. Thus there is scope for further improvement of the tests both on the observational and theoretical fronts.

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Discussion

Bhat : In a two-pulsar binary system, sudden period changes, including 'glitches', become more important than in systems involving a single neutron star and will introduce significant, additional uncertainty in computing the signal 'phase' or arrival time. Is this uncertainty accounted for in present-day, theories/experiments?

Narlikar : The surface phenomenon you refer to arises from the detailed modelling of pulsars as finite objects rather than as 'point masses' assumed in the 'clean' approximation I have described. These have to be taken into account at some stage; but one begins by the simpler 'clean' theory first. That is difficult enough.

Sapre : Recent observational works seem to have found the right amount of dark matter to explain the flat rotation curves of spiral galaxies. Does this not favour the gravity theories on galactic scales?

Narlikar : The presence of dark matter is inferred by assuming that the Newtonian law of gravity holds Thus there is no independent evidence for dark matter, to my knowledge.

Kaul : From the observational point of view what is the scope of long base line laser interferometry?

Narlikar : I think some of the parameters of pulsars in binaries could be more accurately determined.