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Notes for the observer

Fourier smoothing of digital photographic spectra

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Abstract. Fourier methods of smoothing one-dimensional data are discussed with particular reference to digital photographic spectra. Data smoothed using lowpass filters with different cut-off frequencies are intercompared. A method to scale densities in order to remove the dependence of grain noise on density is described. Optimal filtering technique which models signal and noise in Fourier domain is also explained.

Key words : reduction techniques—photographic spectroscopy

1. Introduction

An image recorded on a photographic emulsion is contaminated by the emulsion grain noise. This grain noise or graininess is caused by clumping or grouping of photosensitive silver halide into more or less dense clusters of grains, with areas of lower density between them. The nonuniformity of sensitivity to light over the surface of the grain also contributes to noise in the photographic data. The noise needs to be removed for the evaluation of weaker features in the data.

Noise removal can be done in the measurement domain by convolution of data with a weighting function such as a Gaussian, or in the Fourier domain by multiplying with an appropriate filter. Both the methods are equivalent, and reduce the spatial resolution. Different techniques of noise removal from photographic image are discussed here with an emphasis on spectroscopic data.

Noise in the measurement domain is assumed to be random. The convolution of the observed data $D(x)$ with a symmetric function $C(x)$ implies obtaining a weighted average of the data with the weights determined by the convolving function. The random noise gets averaged out under such a process. The smoothed data $S(x)$ obtained by such a convolution is equivalent to that obtained by multiplying the Fourier transform $\tilde{D}(\nu)$ of the data with a filter $\tilde{F}(\nu)$ in the Fourier space:

$$S(x) = D(x) * C(x) \equiv \mathrm{FT}^{-1} [\tilde{D}(\nu)\tilde{F}(\nu)].$$

The power spectrum

$$P(\nu) = \tilde{D}(\nu) \tilde{D}(\nu)^*$$

is useful in the evaluation of signal and noise in the Fourier space. The noise power decreases exponentially with frequency, whereas the signal power decreases even faster.

This fast decrease in the signal amplitude is because of the degrading effect of factors like the finite resolution of the photographic emulsion and the instrumental profile of the spectrograph. As a result of such a degrading effect, there is very little or no signal at high frequencies. At low frequencies, for a well-exposed plate, signal power dominates over the exponential noise power. Thus, in principle, in the Fourier space, a filter can be applied to average out only the noise leaving the signal relatively unaffected. Fourier filtering is advantageous over filtering in the measurement domain since it helps separation of noise and signal, and removal of noise without affecting signal. In the following, noise filtering in the Fourier domain is discussed.

The spectrogram considered as an example is that of the recurrent nova RS Ophiuchi, recorded with an image-tube spectrograph at the Cassegrain focus of the 1-m telescope at Vainu Bappu Observatory, on a IIa-D emulsion at a reciprocal dispersion of 200Å mm^{-1}. The spectrogram was digitized using the Perkin Elmer PDS 1010M microdensitometer. An aperture 10 $\mu m \times 133.3\ \mu m$ was used and the data digitized at every 5 μm at a speed of 2 mm s^{-1}. RESPECT package (Prabhu, Anupama & Giridhar 1987; Prabhu & Anupama 1990) was used for all the computations.

Figure 1 shows the noisy spectrum, not yet brought to a wavelength scale, and its power spectrum. The signal power clearly dominates the power spectrum up to the frequency 10 cycles mm^{-1}, and is noticeable upto 15 cycles mm^{-1}. The higher frequencies are dominated by the noise power. The power spectrum is folded about the Nyquist frequency of 100 cycles mm^{-1}. Thus the negative frequencies -100 to 0 cycles mm^{-1} appear as $+100$ to 200 cycles mm^{-1}.

2. Lowpass filter

The Fourier transform beyond a particular frequency ν_c contains only the noise in the data. Hence the noise can be reduced by cutting off frequencies higher than ν_c. This is achieved by multiplying the Fourier transform of the data with a filter which has a strength of 1.0 upto ν_c and a strength of 0.0 at all frequencies higher than ν_c. An inverse Fourier transform of the filter-multiplied data yields a smooth spectrum. The filter function is a square function of width $2\nu_c$. It can be denoted by

$$F(\nu) = 1 \qquad -\nu_c \leqslant \nu \leqslant \nu_c$$

$$F(\nu) = 0 \qquad \text{otherwise}$$

The Fourier transform of such a function is a sine function, which has appreciable sidelobes. The inverse transform of data after multiplication with a square filter is equivalent to convolution of the stellar spectrum with a sine function. The sidelobes of the sine function introduce oscillations or ringing in the data, which can be avoided by rounding off the edges of the filter. By rounding off, the steep fall from a strength of 1.0 to 0.0 at ν_c is avoided and the filter strength varies slowly from 1 to 0 over a few frequencies. One of the methods of rounding off is the application of a cosine-bell function such that the filter has a value 1 upto frequency ν_1 and approaches zero at frequency ν_2 as

$$1 - \cos^2\left(\pi \frac{\nu_2 - \nu}{\nu_2 - \nu_1}\right).$$

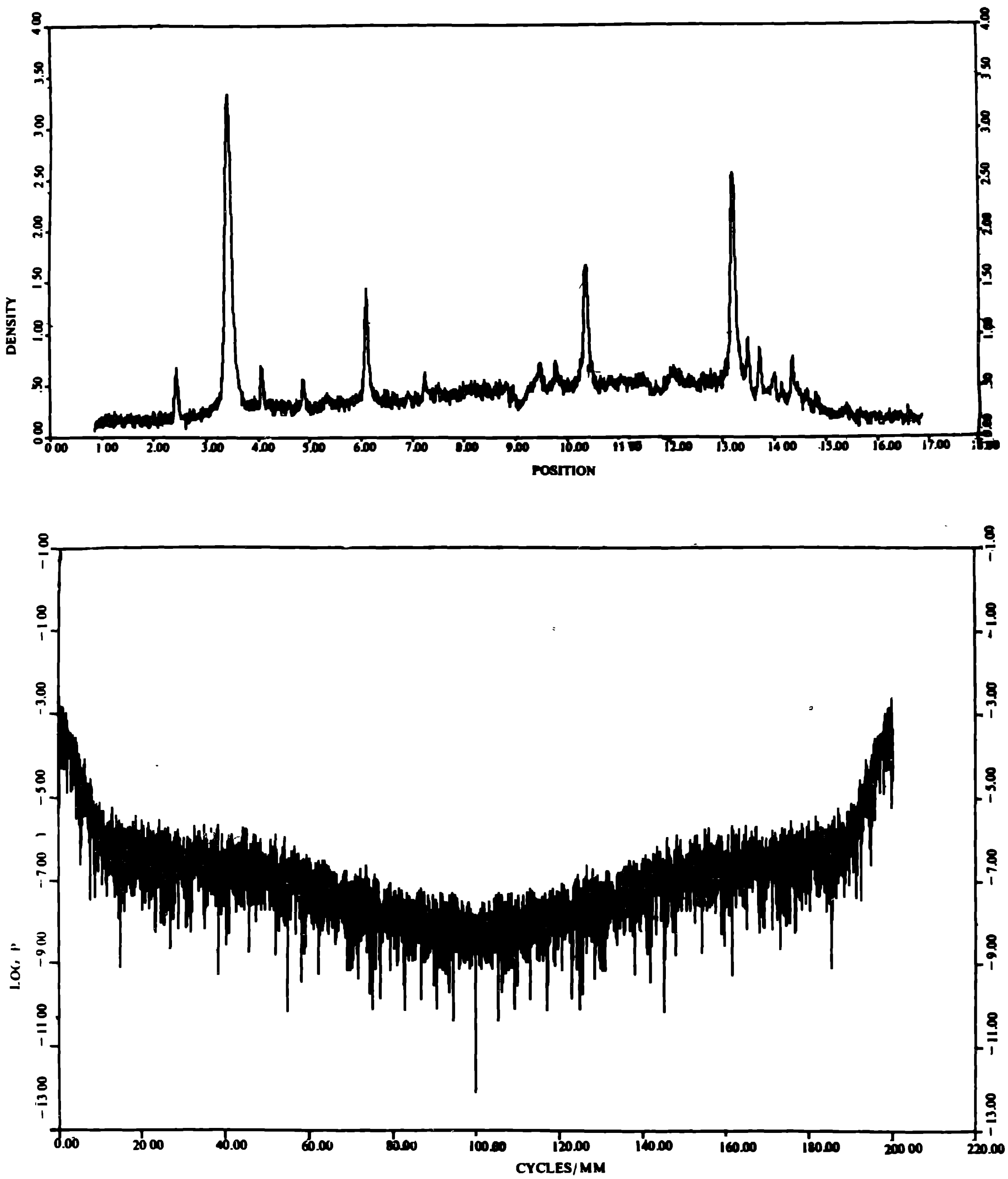

Figure 1. Image-tube spectrum of nova RS Ophiuchi recorded on a IIa-D emulsion (top); and its power spectrum (bottom). The power spectrum is noisy because of the random amplitude and phase changes of the signal with wavelength (Brault & White 1971). The power spectrum is folded about the Nyquist frequency 100 cycles mm^{-1} corresponding to the digitization step of 5 μm.

Four filters with different cutoffs (ν_c) are shown in figure 2, superposed over the power spectrum. The cosine-bell has been applied between frequencies $\nu_1 = (\nu_c - 6)$ cycles mm^{-1} and $\nu_2 = (\nu_c + 6)$ cycles mm^{-1}. Figure 3 shows a portion of the spectrum, smoothed by applying these different filters. The same portion of the original noisy spectrum is shown at the top for comparison.

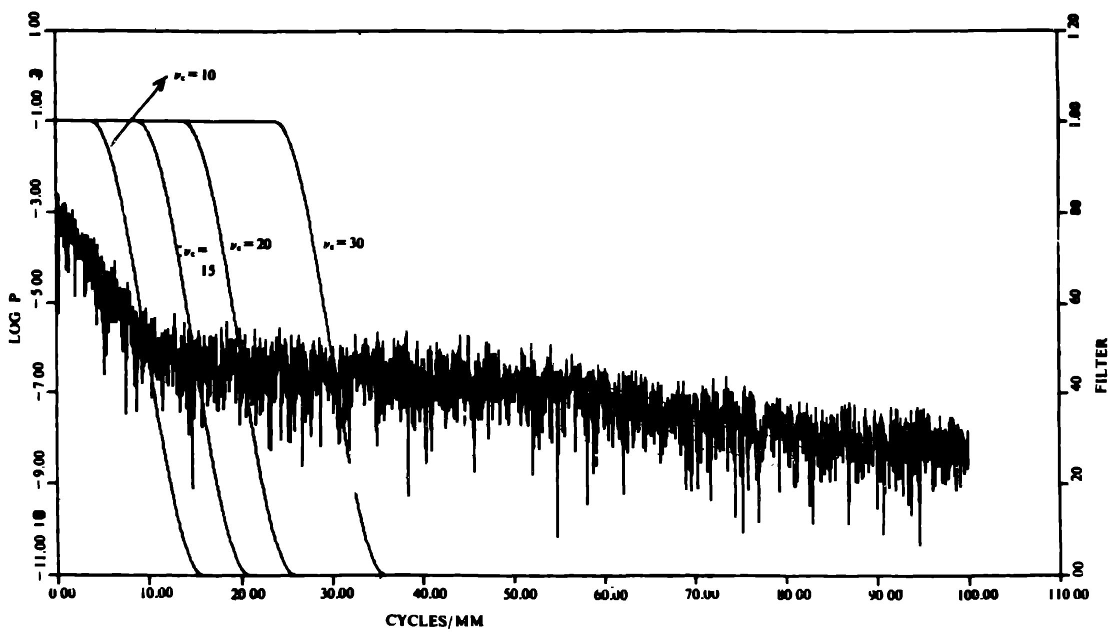

Figure 2. Power spectrum and filters with different cut-off frequencies. The edges of filters are rounded off by a cosine-bell function of width 12 cycles mm^{-1}.

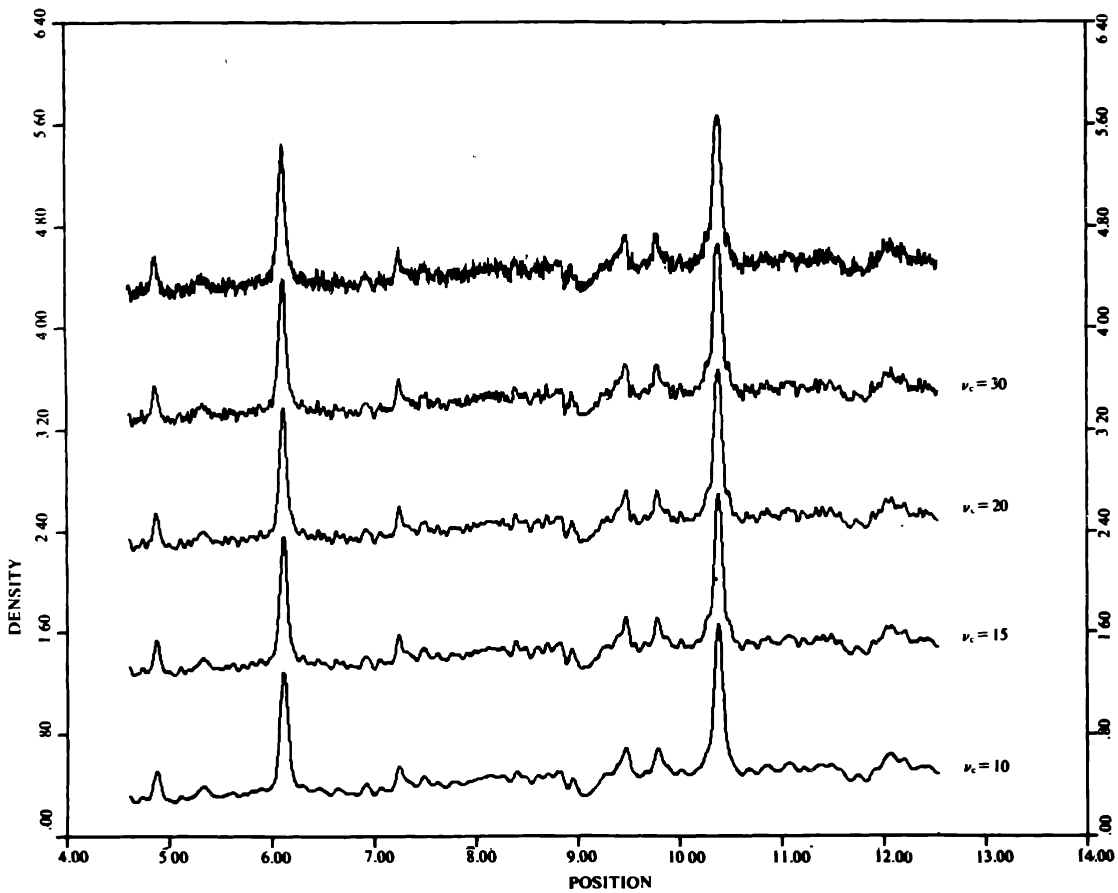

Figure 3. Spectra filtered using different lowpass filters, compared with the original at the top. Successive spectra from bottom to top are given a bias of 1 density unit.

3. Modified densities

The photographic grain noise is not constant at all density levels. The noise is greater at higher densities (as shown in figure 4). As a result, noise is not accurately removed by the application of filters. Higher accuracies in noise removal can be obtained by transforming the densities such that the noise is nearly constant at all density levels. If noise is assumed increasing linearly with density, then $\sigma_D = a + bD$ and the transformation given by Lindgren (1975)

$$D' = A \ \ln \ (a + bD) + B$$

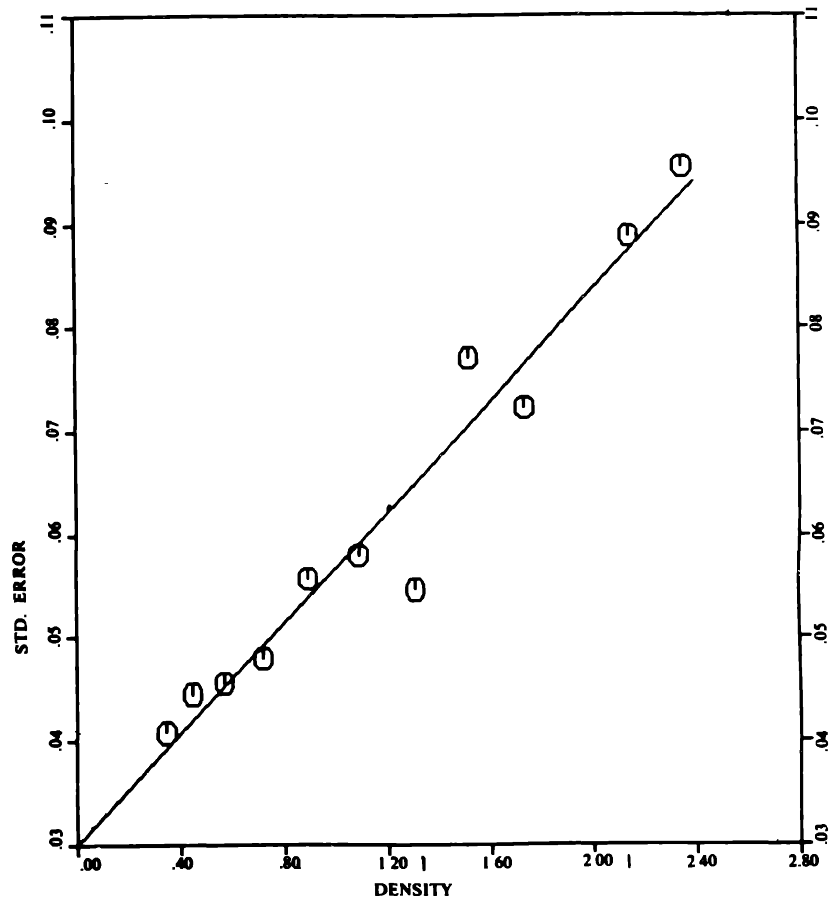

Figure 4. Standard deviation (noise) plotted as a function of density. The straight line fit is given by the equation $\sigma_D = 0.0301 + 0.0267D$. Kodak IIa-D emulsion; Sampling aperture = 1333 $(\mu m)^2$.

where A and B are arbitrary constants, will have the noise σ_D nearly independant of D. The constants A and B may be chosen so as to make the order of magnitude of D' the same as that of D. The data to be smoothed is transformed to modified desnities (MD), D', before applying filter and the smoothed data is back-transformed to ordinary densities.

The standard deviation (noise) at each density level of the calibration steps, plotted against the mean density yielded a linear fit

$$\sigma_D = 0.0301 + 0.0267D,$$

as shown in figure 4. The densities of the spectrum were then transformed using

$$D' = 2.475 \ln (0.0301 + 0.0267D) + 8.605$$

The constants A and B were obtained as

$$A = \frac{D_1 - D_2}{\ln (a + bD_1) - \ln (a + bD_2)},$$

and

$$B = D_1 - A \ln (a + bD_1),$$

where D_1 and D_2 are the maximum and minimum values in the data. Figure 5 shows the difference between spectra (i) smoothed by applying filter ($\nu_c = 15$ cycles mm^{-1}) to spectra in ordinary densities; and (ii) smoothed by applying the same filter to spectrum transformed to MD and back transformed to ordinary densities after smoothing. On comparison with the original spectrum (top), it is clearly seen that at high densities (emission lines), smoothing of ordinary densities does not remove all the noise. It is hence desirable to smooth data in MD rather than in ordinary densities.

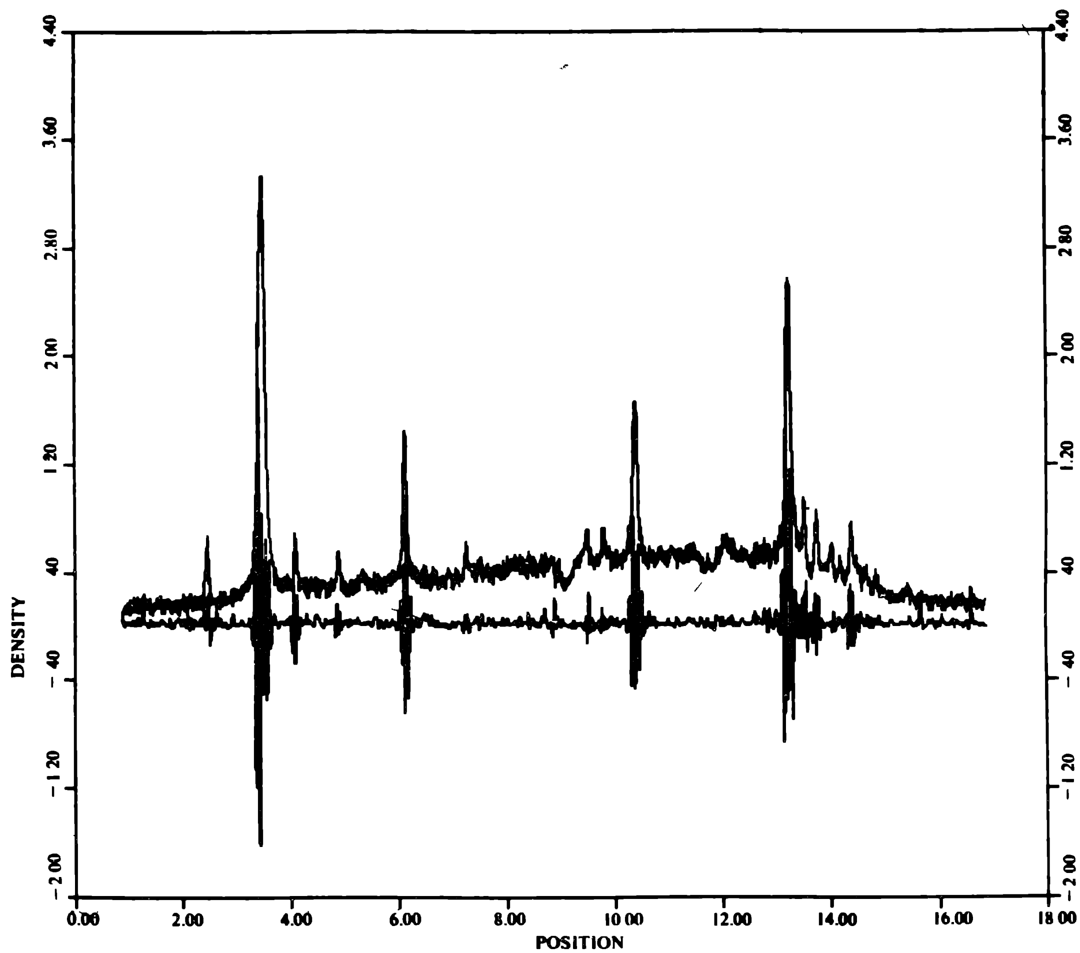

Figure 5. Difference between data smoothed in ordinary densities and data smoothed in modified densities. The original spectrum is plotted above for comparison.

4. Optimal filter

The low-pass filter, though removes the high-frequency noise, leaves the low-frequency noise intact. The use of a weighted, optimal, filter (Brault & White 1971, Lindgren 1975)

$$\Phi_\nu = \frac{P_S(\nu)}{P_S(\nu) + P_N(\nu)},$$

where $P_S(\nu)$ and $P_N(\nu)$ are the signal and noise powers respectively, removes an estimated proportion of variations caused by noise.

Low-frequency noise could arise due to largescale fluctuations in the sensitivity of the photographic emulsion, either intrinsic or introduced during the development process. In order to construct the filter $\Phi(\nu)$, models for the frequency dependence of the noise and signal powers are needed. The noise power $P_N(\nu)$ can be fitted by a straight line, *i.e.*, power decreases exponentially with frequency. The signal $P_S(\nu)$ can be modelled as the power spectrum of the mean line profile, such as Voigt function or a Gaussian, as the case may be. However, the smoothed data is not very sensitive to the actual shape of the filter as this is an optimal filter. Hence small deviations from the true filter shape results in second order errors only. It is often convenient to fit the logarithmic signal power as decreasing exponentially with frequency which fits the observed power spectrum fairly well. In the case of the nova spectrum being considered, the lines being Doppler broadened, the signal power was considered to be a Gaussian.

Figure 6 shows the noise power P_N, Gaussian approximation to the signal P_S and the constructed optimal filter Φ. Figure 7 shows a portion of the spectrum smoothed using the optimal filter. The same portion of the original spectrum is also shown for comparison.

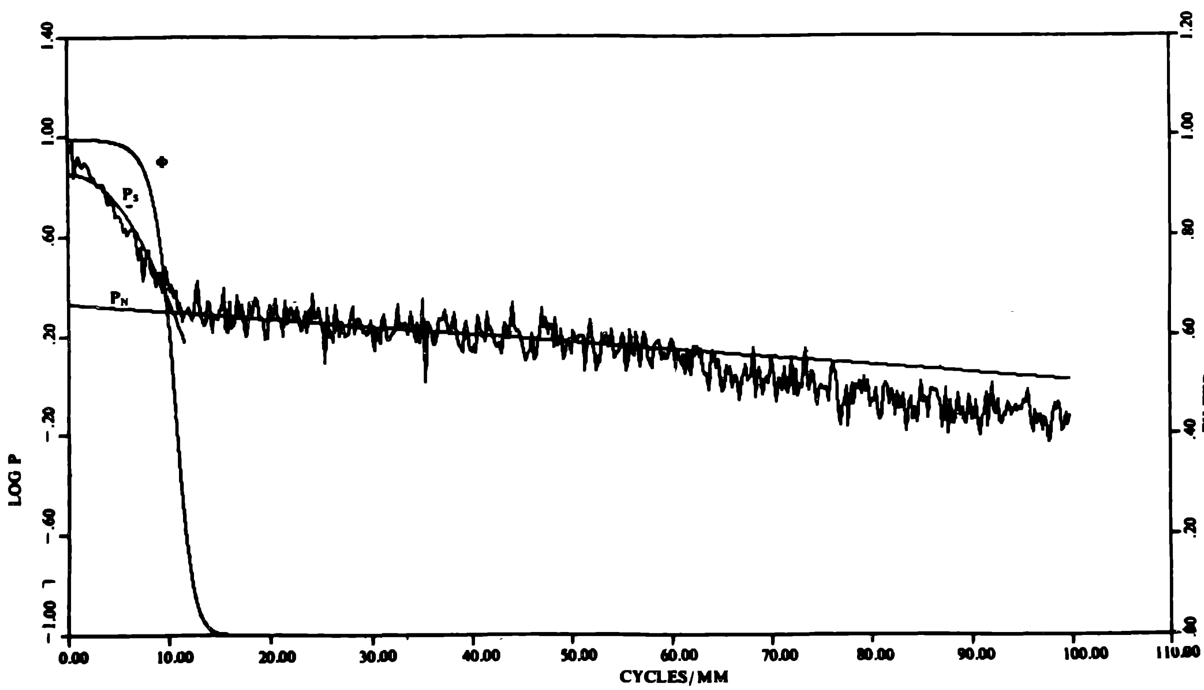

Figure 6. The optimal filter Φ, the noise line P_N and the Gaussian signal approximation P_S (parabola in log P).

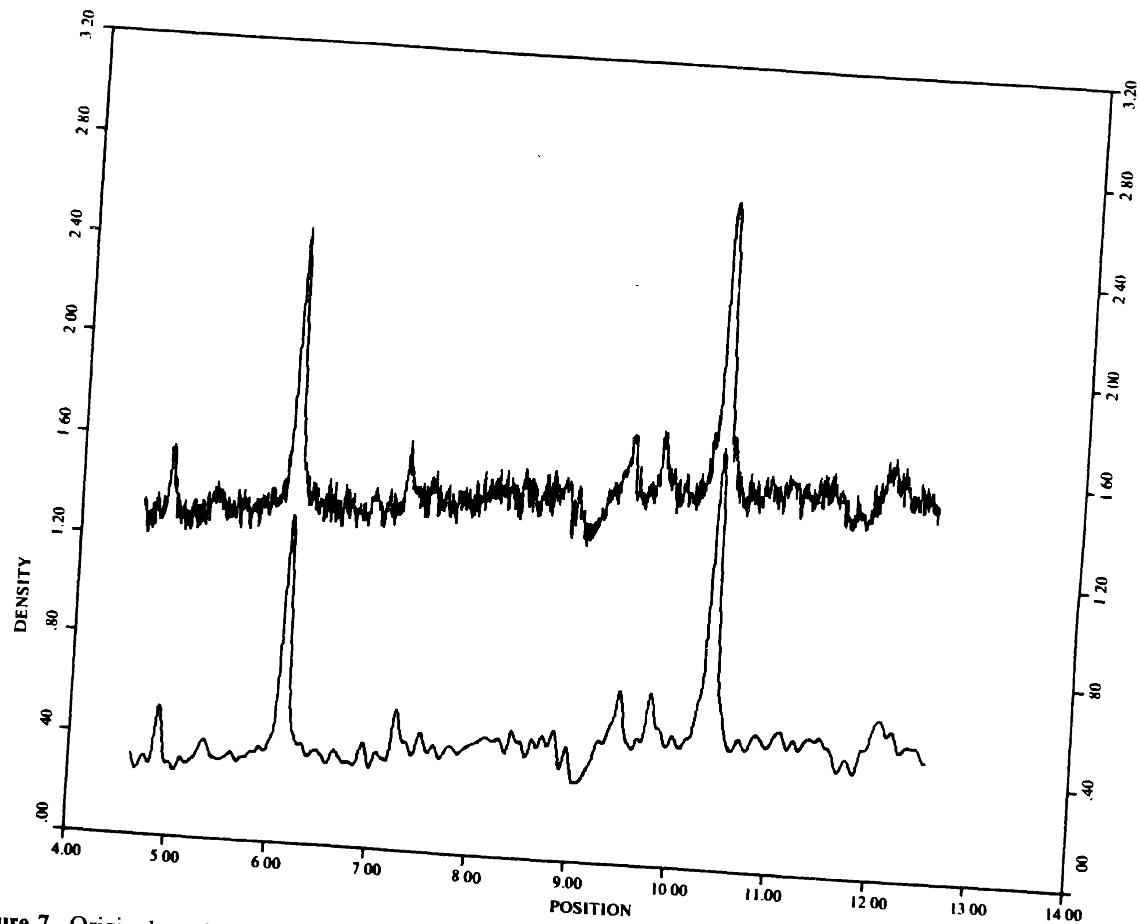

Figure 7. Original, and optimal-filtered data. A bias of 1 density unit has been given to the original spectrum on top.

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