SOME ASTROPHYSICAL ASPECTS OF BLACK AND WHITE HOLES

by

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CERTIFICATE FROM THE SUPERVISOR

This is to certify that

(i) This thesis embodies the work of Shri. R.C. Kapoor . himself;

(ii) Shri.R.C. Kapoor worked under me on this thesis for not less than twentyfour months commencing from the date of his application to the Agra University for registration as a candidate for the Ph.D. Degree;

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TO MY PARENTS

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Ramesh Chander Kapoor

Dawn was spreading her saffron robe over the world, when Thunderbolt Zeus called an assembly of the gods on high Olympos. He addressed them in these words:

"Listen to me, gods and goddesses all, and
let me tell you what is in my mind . . . . "
"If I see any god going to help either Trojans
or Danaans on his own account, he shall get a
thunderstroke and go home very uncomfortable.
Or I'll catch him and throw him down into
Tartaros. A black hole that! A long way
down! A bottomless pit under the earth!.
Iron gates and brazen threshold! As far
below Hades as heaven is above the earth!
He shall discover how much stronger I am
than all the rest of you . . . ".

Homer [1]

CONTENTS

SOME ASTROPHYSICAL ASPECTS

OF

BLACK AND WHITE HOLES

SUMMARY

This thesis consists of three parts. Part I deals with certain astrophysical aspects of black holes and Part II with those of white holes. Part III contains concluding remarks. In all the problems studied, the spacetime exterior to a black hole or a white hole has been assumed to be one described by the Schwarzschild solution of the field equations of general relativity.

PART I

The spacetime surrounding an astronomical object with a sufficiently small radius is so highly curved that it folds over itself. This happens when the radius of the object is reduced to its so-called event horizon. No material particles, no photons emitted from the surface of an object of radius smaller than its event horizon can ever reach the outside world. Black hole is the name given to such an object. A brief introduction to black holes and white holes is given in Chapter 1.

In principle, black holes of any mass upward of $\sim 10^{-5}$ gm are possible in general relativity. However, a number of astrophysical considerations limit their masses within certain

mass ranges. We can have black holes formed as a consequence of gravitational collapse or of primordial origin. Thus, we can have stellar mass black holes which would form in the gravitational collapse of the cores of stars at the end of their evolution and have masses in the range ~ 1 to $\sim 100\ M_\odot$. There are reasons to believe that supermassive black holes with masses $\gtrsim 10^5\ M_\odot$ might form at the end point in the evolution of supermassive stars, galactic nuclei or highly dense star clusters. Black holes with masses as small as $\sim 10^{-5}$ gm and upwards are understood to have been produced in the earliest phases of the Universe (à la Big Bang Cosmology) by density fluctuations.

One does not know whether black holes of primordial origin exist. But with the discovery of the exotic objects, such as quasars, pulsars and X-ray sources, the hopes of discovering supermassive and stellar black holes formed as a result of gravitational collapse have increased. Stellar black holes are expected to be present in certain binary star systems, those with masses $\sim 10^{2-3}\ M_\odot$ might reside in globular clusters while the best places to find supermassive black holes ($\sim 10^{8-10} M_\odot$) are the nuclei of galaxies and probably quasars. Possibly, monster black holes ($\sim 10^{10-15} M_\odot$) might be lurking in intergalactic space [13].

But, by definition, a black hole, stellar or supermassive, is invisible. The most promising way of detection is the interaction of the black hole with mass energy in its surroundings and the subsequent release of energy which can escape to infinity and render it 'luminous'. An analytical study of radiation emitted in the forward direction by a source moving in a highly relativistic circular orbit about a black hole has been presented in Chapter 2 using geometrical optics. It is found that photons emitted in the forward direction by a charged particle moving in an orbit very close to the last unstable circular orbit at $3GM/c^2$ (the radius of the photon sphere; G = constant of gravitation, c = velocity of light and M = mass of the black hole) get highly blueshifted. This happens due to Doppler effect, as the particle velocities are close to that of light, despite an enormous gravitational redshift in the immediate neighborhood of the black hole. This analysis generalized to an ensemble of particles forming a ring around the black hole at a radius slightly in excess of $3GM/c^2$ indicates that radiation from the system has a power law spectrum of the form ν^{-1} (ν = frequency). Such a spectrum is indeed common to extragalactic radio sources and quasars.

This study was concerned with only the tangential photons emitted from a circular orbit in the forward direction. In the latter part of Chapter 2, this analysis has been generalised so as to be applicable to nontangential photons also, the source now being in an eccentric orbit, and the frequency shifts of the photons emitted at various angles are studied. For compact orbits, we predict spectral line broadening in the case of stellar mass black holes and peculiar line oscillations in the case of supermassive black holes purely on geometrical grounds.

In Chapter 3, the possibility of the gravitational recoil of supermassive black holes from galactic nuclei has been explored. The nonspherical gravitational collapse of a mass is accompanied by the emission of gravitational waves in an anisotropic fashion which carry away not only mass and angular momentum but linear momentum also. Consequently, the black hole that forms must recoil in order to conserve linear momentum, with a velocity that can possibly be as high as $\sim 10^4$ km sec^{-1}. As it advances through the galaxy, it accretes gaseous matter and stars which in due course lead to the formation of an accretion disk-stellar system about the black hole. Eventually, the black hole can become luminous enough to be observable when it emerges out of the galaxy. The phenomenon seems to be relevant to the quasar-galaxy associations observed in a number of cases by Arp. Towards the end of the Chapter, some qualitative considerations are presented about the cases of unsuccessful recoils, viz., when the velocity of recoil is $\lesssim 10^3$ km sec^{-1}, and the black hole does not have sufficient kinetic energy to emerge out of the galaxy. It should execute damped oscillatory motion about the center of the galaxy and settle finally at its center after a few oscillations. The importance of the unsuccessful recoils could be gauged in relation to the phenomenon of displaced nuclei, as observed in the case of some galaxies.

PART II

It has been suggested by many authors that white holes may serve as models for galaxies with exploding nuclei and quasars. A white hole is a time-reversed version of a black hole, i.e., material bursting out of the singularity at radius R = 0. Unlike a classical black hole that accepts everything but gives off nothing, a white hole churns out matter and radiation. Even when the radiating surface is still inside the event horizon, the photons emitted from there can leak through the horizon and reach a distant observer. Depending on the epoch of emission, the photons may even suffer a severe blueshift in their frequency because of the superseding of the enormous gravitational redshift by the Doppler blueshift. Such an object would appear as a point source according to a distant observer.

Chapter 4 deals with the analytical study of the nonradial emission of radiation from a white hole surface. It is shown that nonradial photons can leak through the event horizon of the white hole even before its surface emerges from the event horizon. The upper limit on the impact parameter is calculated under the requirement that such photons are blueshifted. The apparent angular size of the white hole determined by blueshifted photons is shown to grow so rapidly in the early stages of its expansion that it produces the appearance of superluminal expansion.

In Chapter 5, we study the motion of a particle in the background of a white hole particularly from the viewpoint of its visibility to a remote observer. We first make a study of the trajectory of the particle in peculiar motion in the frame of reference of the comoving matter. It is shown that radial as well as nonradial photons from the particle can leak through the event horizon even before the particle. As would be expected, the frequency blueshifts in this case are more severe than those of photons from the white hole surface. Consequently, matter ejection from a white hole can produce an apparent intensity enhancement.

PART III

In Chapter 6, concluding remarks are presented and a brief discussion, of the viability of the various problems studied in Chapters 2-5 in relation to quasars and related objects, is given.

PART I

CHAPTER I

AN INTRODUCTION TO BLACK HOLES AND WHITE HOLES

The present thesis deals with a few astrophysical aspects of black holes and white holes, especially in the context of quasars and extragalactic radio sources. In this chapter, we present a brief introduction to the subject of black holes and white holes, viz., their formation and some key properties relevant to the problems that we intend to study in the subsequent chapters. We shall, however, not be concerned with black holes of cosmological origin in the present work. For excellent reviews of the properties and astrophysics of black holes, one may see Misner et al. [23], DeWitt and DeWitt [70] Eardley and Press [68], Giacconi and Ruffini [87] and Davies [73]. An introduction to the role of white holes in astrophysics can be found in Narlikar [71].

1.1 BLACK HOLES

A black hole will form in the supernova explosion of a massive star in case the residual core collapses to the stage when its gravitational potential energy equals its rest mass energy, i.e., when all the matter has fallen inside the so-called Schwarzschild radius (R_s) of the mass. But not every star can produce a black hole. It has to be massive enough. The pivotal role of the mass would become clear if one traces in brief the end products of the evolution of stars of different masses.

It is now widely believed that a one solar mass star ultimately evolves to the white dwarf stage after it has consumed most of its nuclear fuel and derives its energy from the gravitational contraction. However, a star beginning with a mass exceeding the Chandrasekhar limit, M_{Ch} ($\approx 1.2\ M_{\odot}$), can not become a stable white dwarf unless it has undergone a steady mass loss through a stellar wind or produced a planetary nebula, since the balance between the forces of gravity and pressure fails to develop at the white dwarf densities ($\sim 10^{5}$ gm cm^{-3}). The collapsing core is doomed to become either a neutron star or a black hole depending on whether its mass lies in the rage $M_{OV} > M > M_{Ch}$ or $M > M_{OV}$ where M_{OV} refers to the Oppenheimer-Volkoff mass limit. An exact value of M_{OV} cannot be given; the oft quoted values lie within 1.5 - 3.2 $M_{\odot}$ and the value is sensitive to the equation of state employed for the description of the superdense matter.

Suppose now that the collapsing core has a mass in the range $M_{OV} > M > M_{Ch}$. The collapse is slow in the beginning but picks up soon and a substantial portion of the star mass implodes faster than the surrounding envelope. Actually, the stages subsequent to the onset of the collapse have a critical dependence on the generation and propagation of neutrinos and the shock waves. The core is imploding in nearly free fall to higher and higher densities. However, the electrons cannot be squeezed to high values of Fermi energy. At sufficiently large densities, they tunnel into the nuclei and interact with the protons to form neutrons in the inverse β- decay process. As a result, the pressure due to electrons, which mainly contribute to the internal pressure, drops and the core collapses further under its own weight. When a density as high as 2×10^{14} gm cm^{-3} has been reached, the neutron rich nuclei start disintegrating into free neutrons because the neutron binding energy becomes negative. The material consists mostly of neutrons with a little admixture of electrons and protons. At a little higher densities, the force of repulsion among neutrons becomes large enough (because of Pauli's exclusion principle) as to be able to balance the gravitational force of the falling layers of the core and halts the collapse at $\rho \sim 10^{15}$ gm cm^{-3} and $R \sim 10$ Km. This happens so fast that a sudden conversion of the kinetic energy of the collapse into heat produces large amounts of pressure to blow off the outer envelope, still falling in at high speeds, and accelerate the particles to very high energies (the supernova explosion). What is left is a neutron star.

The potential, GM/R, is now large enough to show its effects appreciably. One of these is the effect of pressure regeneration. In general relativity, pressure acts as a source of gravity. Thus, if the neutron star so formed or the initial core were made a bit more massive, then since pressure contributes to the effective mass of the collapsar, the latter would collapse further making the pressure still larger and so on. If the collapsar had a mass $M > M_{OV}$ or the neutron star (mass M) accretes at least a mass $\Delta M \geqslant M_{OV} - M$, the contribution of pressure to the gravity is so high that the balance between the forces of gravity and pressure fails to be established even at neutron star densities; the material is doomed to be crushed further till it all falls inside the Schwarzschild radius R_s ($= 2GM/c^2$) of the mass and go out of sight. The core has become what we call a black hole and warped the surrounding spacetime so much that it folds in over itself.

The surface $R = R_s$, defined as the event horizon, forms the boundary of all the events that are possible to be connected to the future infinity by means of photons or slower than light signals. Signals emitted from within the horizon ought to move faster than light in order to escape to infinity. Therefore the event horizon is a one way membrane, and a region of spacetime that cannot communicate with the rest of the Universe by means of photons or slower than light signals is a black hole.

What happens if the geometry of the collapsing object departs from spherical symmetry? Does a nonspherical gravitational collapse too lead to the formation of a horizon, i.e. to the black hole stage? Nonsphericity might be ushered by rotation, magnetic fields, etc. Although more likely to happen in the Universe, the nonspherical gravitational collapse of astronomical objects is a difficult problem to handle. The situation has been improved lately with some studies made by a number of workers for small departures from sphericity which reveal that collapse of a nearly spherical nonrotating mass also leads to the formation of a Schwarzschild black hole, after radiating away all the gravitational deformations in the form of gravitational waves. R.Price has shown that this happens for perturbing fields of any integral spin (S) that might be coupled to the collapsar. In fact, Price's theorem states that 'anything that can be radiated away is radiated away completely'. Capable of being radiated away are the multipoles that are not conserved; that is to say, for $\ell \geqslant S$ only, radiation is possible. The final field is then characterized by conserved quantities ($\ell < S$ multipole moments).

Highly nonspherical collapse is very poorly understood and it is not known for sure whether event horizons form in this case also. While reviewing the situation, Thorne [46] conjectures that event horizons form when and only when a mass M gets squeezed into a region whose circumference in every direction is $\lesssim 4\pi \left(\frac{GM}{c^2}\right)$. The horizon forms the boundary of all the events which can be connected to the future infinity by means of photons or slower than-light signals. Therefore, all mass energy that falls down the horizon is lost for ever from the outside. When all energy has cleared away from the exterior, the only conserved integrals left to govern the final horizon and the exterior spacetime are mass, angular momentum and possibly charge that went down the hole. The exterior field is the Kerr-Newman solution of the Einstein-Maxwell equations:

$$ds^2 = \left(\frac{\Delta - a^2\sin^2\theta}{\Sigma}\right) dt^2 + \frac{2a\sin^2\theta\,(r^2 + a^2 - \Delta)}{\Sigma}\, dt\, d\varphi$$

$$- \frac{\Sigma}{\Delta} dr^2 - \Sigma\, d\theta^2 - \frac{\left[(r^2 + a^2)^2 - \Delta a^2 \sin^2\theta\right]}{\Sigma} \sin^2\theta\, d\varphi^2$$

where (1.1)

$$\Sigma = r^2 + a^2\cos^2\theta \; ; \quad \Delta = r^2 + a^2 + e^2 - 2mr .$$

Here, m = mass, a = angular momentum per unit mass and e = charge. The units are such that G = c = 1 and all physical quantities are given dimensions of length to some power. Thus the conventional quantities M(gm), Q (esu) and J/M ($cm^2\ sec^{-1}$) are all geometrized to lengths; $m = GM/c^2$, $e = G^{1/2}\ Q/c^2$, $a = G/c^3\ (J/M)$. The above metric specializes to the Schwarzschild solution for a, e = o, to Reissner - Nordström for a = o and to Kerr for e = o. The spacetime described by eq. (1.1) appears to becomes singular at radii (the roots of the eq. $\Delta = 0$):

$$r_{\pm} = m\left[1 \pm (1 - \epsilon^2)^{1/2}\right] \qquad (1.2)$$

where

$$\epsilon = \frac{(a^2 + e^2)^{1/2}}{m}$$

The surfaces r_+ and r_- are called also the outer and inner event horizon respectively (Fig. 1/1). These are in fact coordinate singularities of the Kerr-Newman metric. Pathological violations as causality occur unless $0 \leq \epsilon \leq 1$. Objects with $\epsilon = 1$ are 'extreme' Kerr-Newman black holes with one horizon only, whereas those with $\epsilon > 1$ are naked singularities [no horizon, the physical singularity at r = 0 (and $\theta = \pi/2$ if $a \neq 0$) is visible from future timelike infinity]. For $\epsilon = 0$, $r_- = 0$ and $r_+ = 2m$ (= Rs).

Surrounding the outer horizon is an infinite redshift surface also called the stationary or ergo surface, located at

$$r_E = m + (m^2 - e^2 - a^2 \cos^2\theta)^{1/2} \tag{1.3}$$

This surface coincides with the (outer) horizon only in the nonrotating case (a = 0). Otherwise, it touches the horizon only at the points $\theta = 0, \pi$. The region bounded by ergo-surface and event horizon is called the ergosphere. It is peculiar in the sense that the 'time lines' r, θ, φ = const. are spacelike here. Here, there can be no observers at rest with respect to stationary observers at infinity; anything that stays at fixed r and θ would move in the φ-direction (dragging of inertial frames).

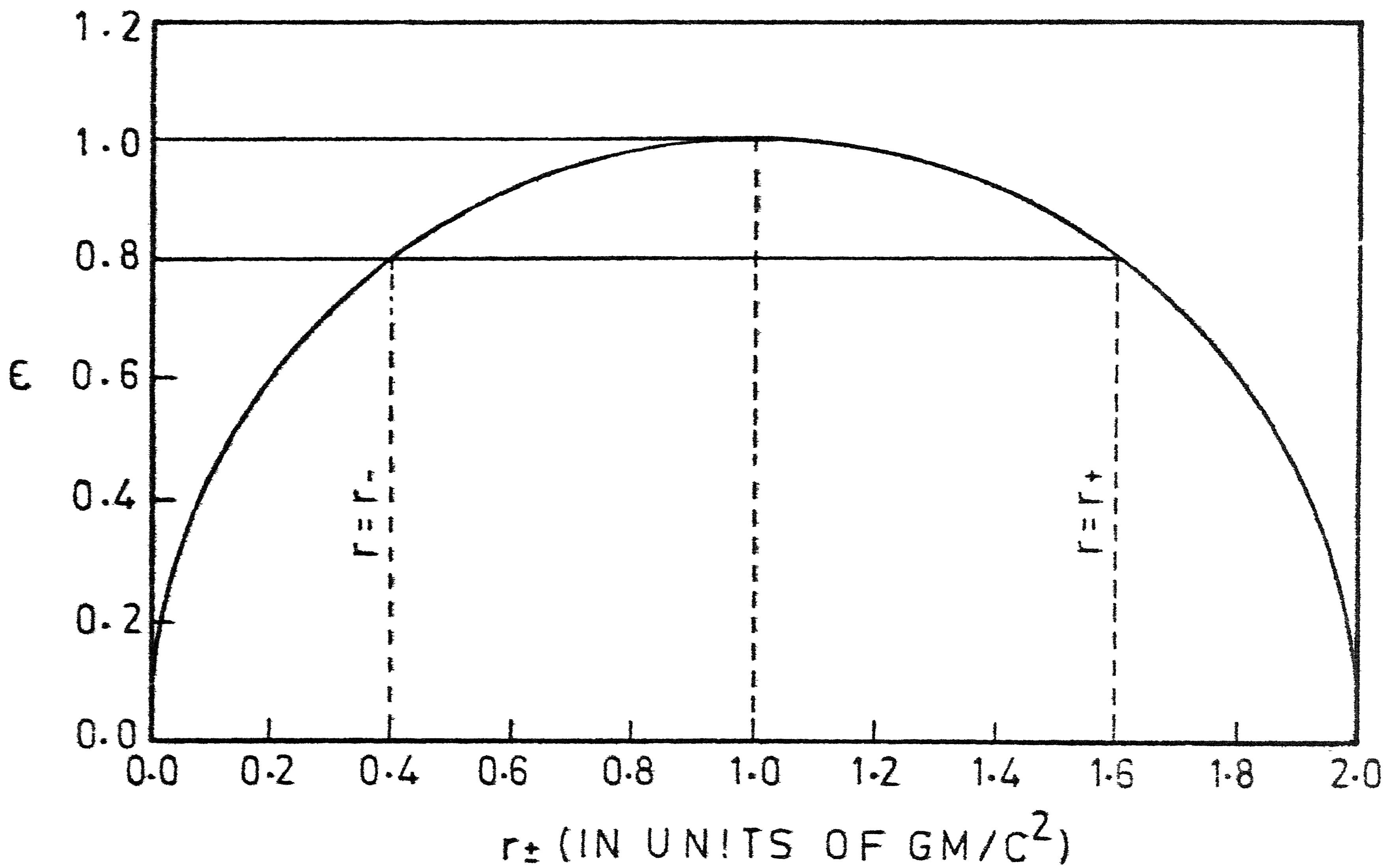


FIG. 1/1. LOCATION OF OUTER AND INNER EVENT HORIZONS, r_+, r_- IN A KERR-NEWMAN SPACE TIME FROM THE ORIGIN.

That the parameters m, a, and e alone determine uniquely the external gravitational and electromagnetic fields of a black hole, leaving no other degrees of freedom, is suggested by the various theorems of Israel, Carter and Hawking (the Israel - Carter Conjecture) [23].

So, this is a brief sketch of the possible outcomes of the ending stage of stellar evolution. In fact mass is not the only deciding parameter; rotation, steady mass loss and mixing of chemical composition of the star among its different layers also are the factors that decide whether the star must undergo supernova explosion and how much mass it would lose in the process. It is generally believed that medium mass stars ($\sim 4 - 10\ M_{\odot}$) and most heavy stars ($M \gtrsim 30\ M_{\odot}$) undergo supernova explosion to produce neutron stars and black holes, respectively, if they have not lost a large amount of mass at relatively early phases of their evolutions [72].

Black holes produced in the gravitational collapse of stars have masses in the range $\sim 1\ M_{\odot}$ to $\sim 100\ M_{\odot}$. Stars which leave cores with masses less than M_{OV} do not reach the black hole stage whereas those with masses exceeding $\sim 100\ M_{\odot}$ do not exist on account of pulsational stability. However, there are reasons to believe that stars with masses exceeding $\sim 10^{5}\ M_{\odot}$ can form in the nuclei of certain massive galaxies and dense star clusters. In the course of their evolution, such supermassive stars too would undergo gravitational collapse and become black holes.

1.2 OBSERVATIONAL PROSPECTS

One does not know whether black holes of primordial origin exist. However, with the discovery of quasars, pulsars and X-ray sources, hopes of detecting stellar and supermassive black holes have increased. Black holes of stellar mass are thought to be present in binary systems. Those which have grown to masses $\sim 10^2 - 10^3\ M_\odot$ are expected in globular clusters, whereas the best places to find supermassive black holes ($\gtrsim 10^5\ M_\odot$) are the nuclei of galaxies and probably quasars. Monster black holes ($\sim 10^{10}$ to $\sim 10^{15}\ M_\odot$) might be lurking in the intergalactic medium [13] .

But being what it is, a black hole is invisible. It is only its gravitational interaction with the surroundings and the subsequent release of energy which can escape to infinity and make it luminous. From the astrophysical point of view, the most promising source of luminosity for a black hole is accretion of gaseous matter from the surrounding (interstellar/intergalactic) space [3]. The simplest case to consider is spherically symmetric accretion of a perfectly adiabatic gas onto a Schwarzschild black hole.

At large distances from the black hole, let the gas be characterized by a temperature T_∞ or equivalently a sound speed a_∞ and density ρ_∞. At distances larger than a critical value known as the accretion radius (at which escape velocity $\sim a_\infty$)

$$r_a \sim \frac{GM}{a_\infty^2} \tag{1.6a}$$

the gas is practically at rest whereas for $r < r_a$, it is in the state of a free fall into the black hole. The rate of accretion for hydrodynamic flow can then be written as

$$\frac{dM}{dt} \approx 4\pi r_a^2 \rho_\infty a_\infty$$

$$= 10^{11}\left(\frac{M}{M_\odot}\right)^2\left(\frac{\rho_\infty}{10^{-24}\,gm\,cm^{-3}}\right)\left(\frac{T_\infty}{10^4\,K}\right)^{-3/2} gm\ sec^{-1} \tag{1.6b}$$

At $r > r_a$, $\rho \simeq \rho_\infty$ and $v \sim r^{-2}$; at $r \simeq r_a$, $v \simeq a_\infty$, whereas for $r < r_a$, $\rho \sim r^{-3/2}$ and $v \sim r^{-1/2}$. When the hole is moving through the medium with a velocity V, the accretion radius should be written as

$$r_a \sim \frac{GM}{a_\infty^2 + V^2} \tag{1.7a}$$

The accretion rate is modified likewise although the basic picture of accretion remains unchanged. When the hole velocity through the medium exceeds that of sound, a bow shock is formed, in front of the hole, at $r \sim r_a$. The energy dissipated in the shock

$$\frac{dE}{dt} \sim V^2 \frac{dM}{dt}$$

$$= 10^{24}\left(\frac{M}{M_\odot}\right)^2\left(\frac{\rho_\infty}{10^{-24}\,gm\,cm^{-3}}\right)\left(\frac{V}{10\,km\,sec^{-1}}\right)^{-1/2} ergs\ sec^{-1} \tag{1.7b}$$

is too little to be of any astronomical significance in the case of a stellar black hole.

A detailed study of the flow requires a knowledge of the effective equation of state and radiative cooling mechanisms such as free-free emission and thermal synchrotron radiation if magnetic field is present. Due to cooling, the 'effective' adiabatic index, γ, becomes smaller because compressional energy is lost to cooling. This usually makes the spherically symmetric accretion unpromising from the observational point of view.

However, if the infalling matter carries some angular momentum it would form an accretion disk around the hole. This leads to a considerable increase in the luminosity. Thus, even if the gas density is low, accretion of matter with low angular momentum can render a massive ($\gtrsim 10^5 M_\odot$) black hole extremely luminous. The main source of energy is the viscous dissipation within the disk by virtue of which the angular momentum of the gaseous matter is transported to the outer regions: the gas spirals inwards to get sucked into the black hole whereas the outward transfer of angular momentum heats up the gas. Result: the hot gas emits radiation, with a power law spectrum of the form

$$F_\nu \sim \nu^{-\alpha} \qquad (1.8)$$

where the spectral index $\alpha \approx 1$.

Most of the emission comes from the region close to the horizon. While escaping, radiation interacts with the infalling gas. When the accretion rate is large, the resulting radiation would exert appreciable pressure on the infalling material and thus the process of accretion becomes self-regulating, peaking at a critical rate

$$\frac{d\dot{M}}{dt} = \frac{4\pi G M m_p}{\sigma c} \sim 10^{17}\left(\frac{M}{M_\odot}\right) \text{gm sec}^{-1} \qquad (1.9)$$

where σ is the Thomson scattering cross section and m_p the proton mass. The corresponding luminosity is called the Eddington luminosity. A supermassive black hole, $M \sim 10^{8-9}\ M_\odot$, radiating near its Eddington limit in accord with the above equation can easily attain QSO luminosities. For stellar mass black holes, accretion rate may possibly reach critical value when it happens to be in a close binary star system.

The best place to look for a stellar black hole is thus a binary star system where it can get enough material to swallow from the companion star. Among the various binary systems suggested, the best black hole candidate is the X-ray object in Cygnus, Cyg X-1 [14] . It is a binary system consisting of a normal 09.7 Iab supergiant (HDE 226868) and a secondary a compact object emitting X rays [4]. On the basis of various arguments, a number of workers suggest a mass of 20-30 $M_\odot$ for the supergiant and $\gtrsim 5\ M_\odot$ for the secondary [14].

Rhodes and Ruffini [5] have shown that under very general constraints ($0 \leq dp / d\rho \leq 1$) on the relation between pressure p and density ρ , the maximum mass of a neutron star ($\equiv M_{OV}$) is 3.2 $M_{\odot}$. All known compact stars rotate so slowly, with periods $\gtrsim$ 33 milli sec, that the fractional enhancement of this limiting mass due to rotation works out to be

$$\frac{\delta M}{M} \lesssim 10^{-4}$$

Observations of the variations in the X ray brightness of Cyg X-1 over times $\sim$0.1 sec suggest the source size $< 3 \times 10^{4}$ km. The secondary is thus too small in size to be a normal star. However, it is too massive to be either a white dwarf (M_{Ch} = 1.2 $M_{\odot}$) or a neutron star (M_{OV} = 3.2 $M_{\odot}$). Thus, the current arguments regarding the nature of the secondary lead one to conclude that most likely it must be a black hole. There is no evidence for the presence of a third body which might destroy this conclusion. The Cyg X-1 therefore seems to be the most compelling candidate for a black hole.

In addition to Cyg X -1, there are some other X-ray sources suspected to be black holes such as, 2U 1700-37 2U0900-40, Circ X-1 and V 861 Sco [93] in our galaxy and SMC X-1 in the Small Magellanic Cloud. The first two appear similar to Cyg X-1 so far as their X-ray emission is concerned. Circ X-1 shows strong, rapid time variations on timescales down to $\lesssim$ 10 msec. The X-ray source SMC X-1 has a luminosity of $\sim 10^{39}$ erg sec^{-1} which implies that the mass of the source is about 10 $M_{\odot}$ assuming that it radiates near its Eddington luminosity.

Black holes are suspected to form in the center of globular clusters also [74, 75]. The cluster stars in the course of their movement venturing too close to the black hole may get captured and even tidally disrupted. An appreciable mass of the gas released in the process, as well as gaseous matter shed by the stars in the course of their evolution, settle down towards the center. In their fall into the black hole, their gravitational energy is released as powerful radiation. A number of globular clusters have been found to be X-ray sources [83], and, according to Bahcall and Ostriker [76], these may be breeding black holes as massive as $\sim 10^{3}$ $M_{\odot}$ which radiate X-rays by accretion. Further such a massive black hole causes the surrounding stars of the cluster to concentrate more toward the center, which should give rise to a bright central spot also. In fact, bright star like concentrations have been observed near the centers of two globular clusters that emit X-rays (NGC 6624 and NGC 7078).

Black holes of much larger masses are expected to form in the nuclei of galaxies. Lynden-Bell's [6] idea of the possible presence of supermassive black holes with accretion disks in the galactic nuclei is widely known. Attempts by a number of workers to explain Weber's observations of gravitational waves from the direction of the Galactic Center also have involved compact supermassive objects and black holes. Such models are of great interest since in them, possibly, lies the clue to an understanding of the energetics of quasars and violent activity observed in the nuclei of many galaxies. The justifications for involving supermassive black holes are: (1) small size which permits rapid brightness variations, a feature common to many quasars and related objects, and (2) the strong gravitational field of a black hole is a source of enormous energy. Some models proposed in recent years of quasars and active galactic nuclei involve supermassive black holes at the center of a system of stars in which stellar collisions and tidal breakups take place to feed the central black hole with gas and render it luminous [84]. Further, such a supermassive black hole, if present in a galactic nucleus would have observable effects on its dynamics too. Recent brightness measurements across the entire galaxy M87 (a supergiant elliptical) by Young et al [56] and spectroscopic observations all across M87 by Sargent et al [57] reveal the presence of a bright, barely resolved central luminosity spike, and a sharp increase in velocity dispersion as well as mass to light ratio toward the center. The photometric and the spectroscopic data together do not fit the standard models of King for elliptical galaxies [77] and instead can be explained by considering the presence of a central supermassive object (mass $\sim 5.10^9 M_{\odot}$, radius < 100 pc and M/L > 60, a factor of 10 times larger than that in the outer region), most likely a black hole.

1.3 WHITE HOLES

A white hole is a system in which mass energy gushes out from a highly dense state. To start with, the system is well inside its event horizon, although an observer at infinity can receive photons or slower than light signals emitted from its surface. In this manner, a white hole represents a time reversed version of a black hole.

Novikov [7] and Ne'eman [2] have suggested independently the possibility of explosion of matter from a highly dense state postulating that certain parts of the Big Bang universe did not explode at the instant $t = 0$. They remained dormant till they exploded at instants $t > 0$, hence also called delayed bangs or lagging cores.

Hjellming [8] has provided a somewhat different scenario for the white hole. It is now well known that after collapse past the event horizon, the material of a collapsing nonrotating neutral mass (case of spherically symmetric collapse) is destined to get crushed into the physical singularity at $R = 0$ of infinite density and infinite spacetime curvature. This is inevitable according to the general theory of relativity.

In the case of a nonspherical collapse, it may be that the singularity possesses nonzero but small size and all or most of the matter that falls down the event horizon may avoid being crushed to infinite density as it may get jammed to a certain maximum density. What then? It explodes into a possibly distant region of spacetime in some other topologically connected universe. The emergence 'there' of all the mass that participated in the collapse through the event horizon to produce a black hole 'here' is a great violent event and is known as a white hole.

There are examples of white holes arising in other theories also. For instance, the theory of conformal gravitation introduced by Hoyle and Narlikar [80] interprets the spacetime singularity of the Big Bang or in a black hole or a white hole as a region of a conformally transformed nonsingular manifold where the inertial masses m of material particles become zero. In this manifold a surface $m = 0$ is nonsingular but if in the usual way of general relativity one uses the conformal frame $m = \text{const}$, one arrives at a new manifold in which this surface is singular. In this sense, a closed $m = 0$ surface in a nonsingular manifold displays itself as a twice repeated combination of a black hole followed by a white hole.

The introduction of a negative energy, zero rest mass scalar field (the $\mathcal{C}$-field) à la Hoyle and Narlikar [81] also leads to the possibility of a white hole. In the standard frame work of general relativity there seems no way to reverse the gravitational collapse of a mass and avoid the singularity R = 0 once its surface has shrunk to a size smaller than its event horizon. However, in case the energy condition can be violated, the collapse may be reversed and the singularity may be avoided [82]. This, for instance, can be achieved by the introduction of the C-field. The reversed collapse of the mass (the mini bang) produces the appearance of a white hole in its expanding phase.

There have been fleated suggestions about quasars being white holes, whose high luminosity derives from the delayed explosions. According to Faulkner et al. [9] the extreme brightness of a white hole is due to the astronomically large blueshift in the frequency of photons emitted from the white hole surface in the earliest phases of its expansion. Narlikar and Apparao [10] have extended the white hole model considered by Faulker et al. and tried to establish a connection between the white holes and phenomena like transient X ray sources, gamma ray bursts, nuclei of Seyfert galaxies and cosmic rays.

There have been some arguments that white holes are too short lived a phenomenon to be astrophysically significant. According to Eardley [61], the surface of a white hole exploding in empty space while crossing the event horizon encounters a blueshift surface. The white hole therefore has a tendency to accrete ambient matter at ultra-relativistic speeds. This smothers quickly the white hole expansion; subsequently it collapses to become a black hole. However, this argument was countered by Lake and Roeder [65] who showed that in the case of white holes arising from delayed bangs, the infinite blueshift could be avoided. According to Lake [66] certain types of white holes can survive and be observable today. Such white holes (which are called grey holes), however, have to come into existence with almost zero delay. In a subsequent work, Lake and Reeder [91, 92] have also demonstrated that these white holes will be bright only for a very short duration after the beginning of the explosion. This happens as most of radiation from a white hole comes along nonradial directions and these photons remain blueshifted for a very short time after the explosion. However, these conclusions are critically dependent on the nature of the spacetime manifold near a singularity. These could be altered, for example, if the collapsing object bounces very close to a singularity [82].

CHAPTER 2

SPECTRAL SHIFT IN RADIATION FROM A SOURCE ORBITING A BLACK HOLE

2.1 INTRODUCTION

In this Chapter, we shall make an attempt to investigate certain astrophysical aspects of the frequency shift, a consequence of Doppler effect and the gravitational redshift, in the radiation from a source orbiting a black hole. We shall first confine our attention to the electromagnetic radiation emitted tangentially in the forward direction by charged particles in highly relativistic circular orbits around a black hole which might be of importance in connection with extragalactic radio sources and quasars. In the last two Sections, we generalize this analysis to the case of nontangential emission of radiation from a source orbiting a black hole, applicable also to an eccentric orbit. This gives rise to some interesting consequences when the orbits are compact enough.

2.2 THE GRAVITATIONAL SEARCHLIGHT EFFECT

Highly relativistic circular orbits about a Schwarzschild black hole lie around $r = 3GM/c^2$, the radius of the photon sphere along which a particle would be travelling with the speed of light ($v = c$). Therefore, an electric charge following an orbit slightly in excess of $3GM/c^2$ can emit synchrotron radiation. Similarly, gravitational radiation also can be emitted by a particle while following such an orbit [15, 16, 17] . However most such effects are of only theoretical significance, since they are immeasurably small.

In the following sections, we will study an effect which may have possible astrophysical applications. This is the blueshift of the electromagnetic radiation emitted in the forward direction by a source of radiation describing a highly relativistic circular orbit around a Schwarzschild black hole. The frequency blueshift is caused by the Doppler effect to such an extent that it exceeds the strong gravitational redshift suffered by the photons emitted in the forward direction by the source in the immediate vicinity of the black hole [79] . The searchlight effect exists even for a particle radiating only for an infinitesimally small duration of the coordinate time.

To begin with, we shall first study the extent of the blueshift and then its effect on the spectrum of continuum radiation emitted by a ring of sources moving round the black hole of mass M. We shall also study the geometrical properties of the null geodesics emerging from such a source.

2.3 CIRCULAR ORBITS

The spacetime exterior to a nonrotating neutral mass is given by the Schwarzschild metric

$$ds^2 = \left(1 - \frac{2m}{r}\right) dt^2 - \frac{dr^2}{\left(1 - \frac{2m}{r}\right)} - r^2(d\theta^2 + \sin^2\theta\, d\varphi^2) \tag{2.1}$$

where $m = GM/c^2$ is implied and we have set $G = c = 1$. The spacetime is singular for radii $r = 0$ and $r = R_s = 2m$. While the former is a physical singularity, the latter, also known as the Schwarzschild radius of a mass M, is a coordinate singularity which can be transformed away by using proper transformations. For an external observer, an object that has a radius

$$r = R_s \tag{2.2}$$

is a black hole, However, the following analysis is applicable to any object with its surface located at

$$2m \leq r \leq 3m \tag{2.3}$$

Let us confine our attention to orbits in the plane $\theta = \pi/2$. The general equations for geodesic motion are

$$\frac{d^2x^\mu}{ds^2} + \Gamma^\mu_{\nu\sigma} \frac{dx^\nu}{ds} \frac{dx^\sigma}{ds} = 0 ; \qquad (\mu, \nu, \sigma = 0, 1, 2, 3) \tag{2.4}$$

Note that $x^0 = t$, $x^1 = r$, $x^2 = \theta$, $x^3 = \varphi$. The azimuthal motion in the plane $\theta = \pi/2$ is

$$\frac{d^2\varphi}{ds^2} + \frac{2}{r} \frac{dr}{ds} \frac{d\varphi}{ds} = 0 \tag{2.5}$$

which solves to give a conserved quantity, the orbital angular momentum parameter, h:

$$\frac{d\varphi}{ds} = \frac{h}{r^2} \qquad (2.6)$$

Next, for the radial motion, we have

$$\frac{d^2 r}{ds^2} - \frac{m}{r(r-2m)}\left(\frac{dr}{ds}\right)^2 - (r-2m)\left[\left(\frac{d\varphi}{ds}\right)^2 - \frac{m}{r^3}\left(\frac{dt}{ds}\right)^2\right] = 0 \qquad (2.7)$$

For a circular orbit, dr = 0, so that

$$\frac{d\varphi}{ds} = \left(\frac{m}{r^3}\right)^{1/2} \frac{dt}{ds} \qquad (2.8)$$

Defining angular velocity as $\frac{d\varphi}{dt}$ = n, we have

$$n = \left(\frac{m}{r^3}\right)^{1/2} \qquad (2.9)$$

which is just the Kepler's third law of planetary motion.
From eq. (2.1), setting $\theta = \pi/2$, dr = dθ = 0, we get

$$\left(\frac{ds}{dt}\right)^2 = \left(1 - \frac{2m}{r}\right) - r^2\left(\frac{d\varphi}{dt}\right)^2$$

Hence another conserved quantity,

$$\gamma = \frac{dt}{ds} = \frac{1}{\left(1 - \frac{3m}{r}\right)^{1/2}} \qquad (2.10)$$

results, viz., the energy per unit rest mass as measured at infinity. The orbit of the charged particle can be specified by

$$r = a = 3m(1+\epsilon), \quad \theta = \pi/2, \quad \varphi_0 = n t_0 \qquad \textbf{(2.11)}$$

where (t_0, a, $\pi/2$, φ_0) are the coordinates of a particle in the orbit at the proper time parameter s. Thus when $\epsilon \rightarrow 0$,

$$\gamma \rightarrow \frac{1}{\epsilon^{1/2}} \quad , \quad n \rightarrow \frac{1}{3 . 3^{1/2} m} \qquad \textbf{(2.12)}$$

2.4 FREQUENCY SHIFT OF THE SEARCHLIGHT RADIATION

We now consider an observer O in the equatorial plane $\theta = \pi/2$ with space coordinates $r = R \gg 2m$, $\varphi = 0$. The time coordinate of O also measures its proper time in this approximation (the Schwarzschild spacetime is asymptotically flat). The typical position of the particle following the circular geodesic will be specified by the world point Q with coordinates given by eq. (2.11). In the analysis that follows, the geometrical optics approximation is used, neglecting the backscatter caused by the presence of the spacetime curvature.

Let us suppose now that a photon is emitted by Q and received by O at a time t = T. Since for a photon, $ds^2 = 0$, its motion can be specified by an affine parameter λ. The equations of motion (2.4) for a photon would then suggest

$$\frac{dt}{d\lambda} = \frac{\gamma}{1 - \frac{2m}{r}} \quad , \quad \frac{d\varphi}{d\lambda} = \frac{h}{r^2} \qquad \textbf{(2.13)}$$

The metric (2.1) for $ds^2 = 0$, $d\theta = 0$ and $\theta = \pi/2$ gives

$$0 = \left(1 - \frac{2m}{r}\right)\left(\frac{dt}{d\lambda}\right)^2 - \frac{1}{\left(1 - \frac{2m}{r}\right)}\left(\frac{dr}{d\lambda}\right)^2 - r^2\left(\frac{d\phi}{d\lambda}\right)^2$$

Therefore

$$\frac{dr}{d\lambda} = \gamma\left[1 - \left(1 - \frac{2m}{r}\right)\frac{q^2}{r^2}\right]^{1/2} \tag{2.14}$$

where

$$q = \frac{h}{\gamma} \tag{2.15}$$

is defined to be the impact parameter specifying the null geodesic. The value of the impact parameter is to be chosen by the following requirements:

(1) that the photon is emitted in the instantaneous direction of motion of the particle, and

(2) that it arrives at O.

Of course these requirements overdetermine the problem, indicating that not from all points on the circular orbit would the photon (emitted instantaneously in the forward direction) reach the observer at O. However, we will show that there exists at least one such point Q from where the photon reaches O.

The requirement of tangential emission implies that for the photon emitted from an orbit at r = a, $ds^2 = 0$, $d\theta = 0$ and dr = 0. Hence eq. (2.1) suggests

$$0 = \left[\left(1 - \frac{2m}{r}\right)\left(\frac{dt}{d\lambda}\right)^2 - r^2\left(\frac{d\varphi}{d\lambda}\right)^2\right]\Bigg|_{r=a}$$

i.e.,

$$q = \pm \frac{a}{\left(1 - \frac{2m}{a}\right)^{1/2}} \qquad \textbf{(2.16)}$$

where - sign refers to the forward emission. While escaping the neighborhood of the black hole, the emitted photon (of frequency ν_0) suffers a gravitational redshift, whereas the Doppler effect tends to shift it towards the blue. We shall show that the latter increases more rapidly than the former in the limit $\epsilon \to 0$.

The net frequency shift can be calculated either (a) from first principles, by considering two null geodesics leaving r = a at t_0 and $t_0 + \Delta t_0$ and arriving at r = R ($\gg 2m$) at T and T + ΔT, or (b) by using the following general expression due to Schroedinger [18]

$$\frac{\nu_0}{\nu} = (1+z) = \frac{\vec{u}\cdot\vec{p}\,|_{source}}{\vec{u}\cdot\vec{p}\,|_{observer}} \qquad \textbf{(2.17)}$$

Here $u^\alpha(s)$ is the source velocity, $u^\alpha(o)$ that of the observer, $p^\beta(s)$ the direction of the photon at source and $p^\beta(o)$ the same at the observer.

(a) <u>Derivation of frequency shift from first principles</u>

Consider a photon emitted from Q at $t = t_o$. The requirement that the photon arrives at O at $t = T$ indicates that

$$T - t_0 = \int_a^R f(r,q)\,dr \qquad (2.18)$$

where

$$[f(r,q)]^{-1} = \left(1 - \frac{2m}{r}\right)\left[1 - \left(1 - \frac{2m}{r}\right)\frac{q^2}{r^2}\right]^{1/2}, \qquad (2.19)$$

and

$$\phi_0 = -\int_a^R g(r,q)\,dr \qquad (2.20)$$

where

$$g(r,q) = \frac{q}{r^2}\left[1 - \left(1 - \frac{2m}{r}\right)\frac{q^2}{r^2}\right]^{-1/2} \qquad (2.21)$$

The integrals (2.18) and (2.20) are of the elliptical kind. However, it is not necessary to evaluate them explicitly for the calculation of the blue-shift. The values of these will be needed later for another purpose.

Consider now another photon emitted from the circular orbit at a world point Q' with the time coordinate

$t_o + \Delta t_o$. This photon has an impact parameter $q + \Delta q$ and arrives at O at a time $T + \Delta T$. Then we can write equations similar to (2.18) and (2.20) for this photon also. By appropriate subtraction of equations for time as well as for φ_c, we get

$$\Delta T - \Delta t_o = \Delta T - \gamma \Delta s = \Delta q \int \frac{\partial f}{\partial q} dr \tag{2.22}$$

and

$$-\Delta \varphi_o = -\eta \Delta t_o = -\gamma \eta \Delta s = \Delta q \int \frac{\partial g}{\partial q} dr \tag{2.23}$$

Δs being the proper time of the source corresponding to the time interval Δt_o. Eliminating Δq, we get

$$\frac{\Delta T}{\Delta s} = \gamma \left[1 + \eta H(q) \right] \tag{2.24}$$

where

$$H(q) = \frac{\int \frac{\partial f}{\partial q} dr}{\int \frac{\partial g}{\partial q} dr} \tag{2.25}$$

Since

$$\frac{\partial f}{\partial q} = \frac{q/r^2}{\left[1-\left(1-\frac{2m}{r}\right)\frac{q^2}{r^2}\right]^{3/2}}, \quad \frac{\partial g}{\partial q} = \frac{1/r^2}{\left[1-\left(1-\frac{2m}{r}\right)\frac{q^2}{r^2}\right]^{3/2}} \tag{2.26}$$

eq. (2.25) reduces to

$$H(q) = q = \pm \frac{a}{\left(1 - \frac{2m}{a}\right)^{1/2}} \tag{2.27}$$

The frequency blueshift can be written as

$$1 + b = (1+z)^{-1} = \frac{\nu}{\nu_0} = \frac{\Delta s}{\Delta T} \tag{2.28}$$

which in view of eqs. (2.24) and (2.27) reads as

$$1 + b = \frac{(1 - 3\epsilon/2)^{1/2}\left[2(1-\epsilon)\right]^{1/2}}{\left[2(1-\epsilon)\right]^{1/2} - \epsilon^{1/2}} \tag{2.29}$$

where we have written

$$\epsilon = \frac{2m}{a} \tag{2.30}$$

(b) <u>Derivation using eq. (2.17):</u>

Writing the Schwarzschild line element in the general form

$$ds^2 = g_{\alpha\beta}\, dx^\alpha dx^\beta$$

$$g_{00} = \left(1 - \frac{2m}{r}\right);\ g_{11} = -\left(1 - \frac{2m}{r}\right)^{-1};\ g_{22} = -r^2;\ g_{33} = -r^2\sin^2\theta \tag{2.31}$$

the various quantities in eq. (2.17) can be expressed as

$$u^\alpha(s) = \frac{dx^\alpha}{ds} = \left(\frac{dt}{ds}, 0, 0, \frac{d\varphi}{ds}\right) = (\gamma, 0, 0, \gamma\dot{\varphi})\ ; \tag{2.32}$$

$$p_\alpha = g_{\alpha\beta} p^\beta = \left[\left(1 - \frac{2m}{r}\right) \frac{dt}{d\lambda} \, , \, - \frac{1}{\left(1 - \frac{2m}{r}\right)} \frac{dr}{d\lambda} \, , \, 0 \, , \, - r^2 \frac{d\chi}{d\lambda} \right], \quad (2.33)$$

and

$$u^\alpha(0) = (1, 0, 0, 0) \quad (2.34)$$

Now

$$\vec{u} \cdot \vec{p} \,\big|_s = u^\alpha p_\alpha \big|_s = \gamma^2 (1 + nq)$$

and

$$\vec{u} \cdot \vec{p} \,\big|_0 = u^\alpha p_\alpha \big|_0 = \gamma$$

Hence

$$1 + b = \left[\gamma (1 + nq) \right]^{-1}, \quad (2.24)'$$

and therefore eq. (2.29) follows.

In case the observer is at rest at r = R ($\neq \infty$),

we have

$$u^\alpha(0) = \left(\frac{dt}{ds} \Big|_R \, , \, 0 \, , \, 0 \, , \, 0 \right)$$

where

$$\frac{dt}{ds} \Big|_R = \left(1 - \frac{2m}{R}\right)^{-1/2} \quad (2.35)$$

from eq. (2.1). The frequency blueshift in such a case is

$$1 + b = \frac{1}{\gamma(1+nq)\left(1 - \frac{2m}{R}\right)^{1/2}} \qquad (2.36)$$

It is easy to see that eq. (2.29) in the limit $\xi_1 \to 2/3$, i.e. $\epsilon \to 0$, suggests very large blueshifts; using the approximation we get

$$b \simeq \frac{2}{3}\, \epsilon^{-1/2} \qquad (2.37)$$

2.5 PROPAGATION OF PHOTONS

How does the propagation of the photons depend on ϵ ? In order to see this, it is convenient to reduce eq. (2.20) to the following form:

$$\varphi_0 = \int_0^1 \frac{dv}{\left[(1-\xi_1) - (1-\xi_1 v)\, v^2\right]^{1/2}} \qquad (2.38)$$

where v = a/r and ξ = 2m/a. We have set R = ∞ as the lower limit to the integral for the sake of convenience since there is only a slight change in φ_0 when R >>2m is replaced by R = ∞. From eq. (2.38), it is apparent that $\varphi_0 = \pi/2$ for $\xi = 0$; this is the simplest case of a photon in flat Minkowski space. As ξ increases from 0, φ_0 increases and in fact as $\xi_1 \to 2/3$, the photon makes several rounds of the black hole before arriving at the distant observer.

How does γ_0 behave as $\epsilon \to 0$ ($\xi \to 2/3$) ? We notice that at $\xi = 2/3$, the integrand in eq. (2.38) has a factor $(1-v)^{-1}$ and the integral diverges logarithmically. Therefore γ_0 is expected to diverge as $\xi \to 2/3$, largely because of the contributions to the integral at $v \gtrsim 1$. For convenience, let us write

$$\xi = \frac{2}{3} - \Delta \ , \quad \Delta \approx \frac{2}{3}\epsilon \quad \text{as} \quad \epsilon \to 0 \tag{2.39}$$

Eq. (2.38) then assumes the form

$$\gamma_0 \simeq \left(\frac{2}{3} - \Delta\right)^{-1/2} \int_0^1 \frac{dv}{\left[(1-v)(v-v_1)(v_2-v)\right]^{1/2}} \ , \tag{2.40}$$

where

$$v_1 + v_2 = -v_1 v_2 = \frac{(1-\xi)}{\xi}$$

Hence

$$v_1 \simeq (1 + 3\Delta) \ , \quad v_2 \simeq -\frac{1}{2}\left(1 + \frac{3\Delta}{2}\right) \tag{2.41}$$

neglecting powers of Δ higher than unity as $\Delta << 1$. We now use the transformation

$$v = v_1 \sin^2\theta + v_2 \cos^2\theta \tag{2.42}$$

so that eq. (2.40) now has the form

$$\gamma_c \cong 2\int \frac{d\theta}{\left[1 - \left(\frac{v_1 - v_2}{1 - v_2}\right)\sin^2\theta\right]^{1/2}} \tag{2.43}$$

This is brought into the standard elliptic integral form by the transformation

$$\sin\psi = \left(\frac{v_1 - v_2}{1 - v_2}\right)^{1/2}\sin\theta \tag{2.44}$$

Hence

$$\gamma_c \simeq 2\int_{\psi_1}^{\pi/2} \frac{d\psi}{\left[1 - (1 - 2\Delta)\sin^2\psi\right]^{1/2}}$$

$$\cong 2\left[\int_0^{\pi/2} \frac{d\psi}{\left[1 - (1 - 2\Delta)\sin^2\psi\right]^{1/2}} - \int_0^{\psi_1} \frac{d\psi}{\left[1 - (1 - 2\Delta)\sin^2\psi\right]^{1/2}}\right]$$

$$\simeq 2\,K(\sqrt{1 - 2\Delta}) - 2\,\ln(\sec\psi_1 + \tan\psi_1)\,, \tag{2.45}$$

where

$$\psi_1 = \sin^{-1}\left(\frac{v_2}{v_2 - 1}\right)^{1/2}$$

But as $\Delta \to 0$, $\sin\psi_1 \to 1/3^{\frac{1}{2}}$ and therefore

$$\sec\psi_1 + \operatorname{Tan}\psi_1 \simeq \frac{1 + \sqrt{3}}{\sqrt{2}}$$

Also

$$K(\sqrt{1 - 2\Delta}) \simeq \ln\frac{4}{\sqrt{2\Delta}} \tag{2.46}$$

Hence

$$\varphi_0 \simeq \ln \frac{8(2-\sqrt{3})}{\Delta} \simeq \ln \frac{12(2-\sqrt{3})}{\epsilon} \qquad (2.47)$$

which demonstrates the logarithmic divergence of φ_0 with ϵ .

Eq. (2.38) has been numerically integrated. In Table I, we give the values of φ_0 for various values of ξ_r close to 2/3. We see that, as indicated by eq. (2.38), the photon makes several rounds of the black hole before arriving at the distant observer. The values in Table I correspond to $\epsilon = 10^{-n}$ $(2 \leq n \leq 9)$. For such cases the change in the values of φ_0 introduced by decreasing Δ by a factor 10 is given by

$$\delta \varphi_0 \simeq \ln 10 \simeq 2.30 \qquad (2.48)$$

This is borne out by the numerical integration.

Let us now consider what happens at a finite but large value of R. For $r \to R$, eq. (2.21) is approximated by

$$\frac{d\varphi}{dr} \simeq \frac{q}{r^2} . \qquad (2.49)$$

With O as the origin, we choose rectangular Cartesian coordinates (x, y) in the plane $\theta = \pi/2$. This is possible, since far away from the black hole the spacetime is nearly flat. At O,

$$dy \simeq R\, d\varphi \quad , \quad dx \simeq dr$$

so that

$$\frac{dy}{dx} \simeq \frac{q}{R} \qquad (2.50)$$

TABLE I

THE DIVERGENCE OF γ_c WITH ε

n	$\varepsilon = \frac{2}{3}(1-10^{-n})$	γ_c radians	Number of revolutions around the black hole
2	0.66	5.776	0.92
3	0.666	8.076	1.28
4	0.6666	10.378	1.65
5	0.66666	12.681	2.02
6	0.666666	14.934	2.38
7	0.6666666	17.230	2.74
8	0.66666666	19.542	3.11
9	0.666666666	21.867	3.48

In other words the angle made by the propagation vector of a photon reaching O with the line of sight through the center of the black hole is

$$\alpha = \left| \frac{q}{R} \right| \simeq \frac{3\sqrt{3}\, m}{R} \qquad (2.51)$$

This determines the angular radius of the orbit as seen by the observer at O.

The integral for $(T-t_o)$ in the limit of $R \to \infty$ reads as

$$T - t_o \quad \frac{2m(1-q_1)^{1/2}}{q_1} \int_0^1 \frac{dv}{(1 \;\; q_1 v)\, v^2 \left[(1-q_1) - (1-q_1 v)\, v^2\right]^{1/2}} \qquad (2.52)$$

However, for any finite but fixed R, we can study its dependence on ϵ as $\epsilon \to 0$. Here again the integral diverges logarithmically. Reducing eq. (2.52) to a form similar to eq. (2.40), a simple calculation shows that if ϵ and $\epsilon + d\epsilon$ are two neighboring values very small compared to unity, the difference between the corresponding values of $T-t_e$ is given in terms of the differential of φ_o as follows

$$\delta (T - t_o) \simeq 3\sqrt{3} \left(\frac{GM}{c^3}\right) \delta \varphi_o \qquad (2.53)$$

which indicates a slow dependence of T on ϵ .

Table II gives representative numerical results on $(T-t_0)$ - integral for R = 100 a and small values of ϵ. This choice of the values of R is somewhat arbitrary; the only criterion being that R should be large enough so that the spacetime at O can be considered almost Minkowskian. From Table II one can see that as ϵ becomes smaller and smaller, the time taken by the photon to reach the observer at r = R diverges logarithmically.

Owing to this slow dependence of T on ϵ, even values of ϵ as small as 10^{-10} will not significantly alter the time scale for the light ray to emerge from the vicinity of the object. We emphasize this point here, because in the following Section, we shall be generalizing our results to an ensemble of particles going in a ring at radii very close to a = 3m, the unstable circular orbit.

2.6 THE EMISSION SPECTRUM

We now imagine the central object to be surrounded by a ring of particles moving in circular orbits with different $\epsilon \ll 1$. The volume of the ring will be determined by the overall range of ϵ. The volume element between ϵ and $\epsilon + d\epsilon$ is

$$dV = 2\pi r \left(1 - \frac{2m}{a}\right)^{-1/2} H\, dr \qquad (2.34)$$

where H is the thickness of the ring (= r $d\theta$). Since

TABLE II

THE DIVERGENCE OF $(T - t_o)$ WITH ϵ

n	ϵ	$(T-t_o)/(GM/c^3)$
2	0.66	33.03
3	0.666	34.49
4	0.6666	35.69
5	0.66666	36.87

$$\left(1 - \frac{2m}{a}\right)^{-1/2} \longrightarrow \sqrt{3}\,(1-\epsilon)$$

and

$$dr \longrightarrow 3m\,d\epsilon$$

as $\epsilon \to 0$, we have

$$dV \simeq 18\sqrt{3}\,\pi m^2 H\,d\epsilon \qquad (2.55)$$

Now suppose the ring emits N photons of frequency ν_0 per unit volume, per second in the rest frame. The number of photons emitted in time ds is (N.ds.dV). At a large distance r = R ($\gg$ 2m), these will be distributed over a sphere of surface area 4π R^2. Although most of them will be emitted in the equatorial plane, the average over the whole sphere gives a flux (per unit area per unit time) as

$$\frac{N\,ds\,dV}{4\pi R^2\,dT} = \frac{N\,\nu\,dV}{4\pi R^2\,\nu_0} \qquad (2.56)$$

where

$$\frac{\nu}{\nu_0} = \frac{\Delta s}{\Delta T} \qquad (2.57)$$

The average flux of energy is therefore

$$\frac{N\,h\,\nu_0\,dV\,(1+b)^2}{4\pi\,R^2} = \frac{E(\nu_0)\,dV\,\nu^2}{4\pi\,R^2\,\nu_0^{\,2}} \qquad (2.58)$$

where E(ν_0) is the volume emissivity at the emission frequency ν_0 . Now we use eqs. (2.55) and (2.37) to get

$$
\begin{aligned}
dJ &= \frac{\sqrt{3}}{2}\,\pi R_s^2\, H\, \frac{d\epsilon}{d\nu}\, d\nu \\
&= 4\sqrt{3}\,\pi R_s^2\, H\, \nu_0\, \frac{d\nu}{\nu^3}
\end{aligned}
\qquad (2.59)
$$

so that eqs. (2.58) and (2.37) can be combined to give the flux S(ν) dν in the frequency range ν and $\nu + d\nu$ as follows

$$
S(\nu) = \frac{\sqrt{3}\cdot R_s^2\, H\, E(\nu_0)}{\nu} = \frac{A(\nu_0)}{\nu} \qquad (2.60)
$$

Thus the ring of particles at r $\simeq$ 3m emits radiation with a power law spectrum of the form

$$
S(\nu) \sim \frac{1}{\nu}, \qquad \nu \gg \nu_0 \qquad (2.61)
$$

purely on geometric grounds. The ν^{-1} dependence of the spectrum is common to many extragalactic sources of radio emission. Chitre and Narlikar [20] have discussed the astrophysical consequences of such a model with special reference to extragalactic radio sources and quasars.

2.7 NULL GEODESICS: THE CASE OF NONTAGENTIAL EMISSION

The analysis of the null geodesics presented in the following sections is a byproduct of the one developed in the foregoing sections, applicable to photons emitted nontangentially as well. Also, we generalize our results to the case of the source following an eccentric orbit. The source of photons could be an ordinary star orbiting a supermassive black hole, a hot spot or a flare produced in an accretion disk around one (stellar/supermassive) due to generation and growth of instabilities, or a stellar outburst that may take place in the 'vicinity' of the black hole.

It is possible to have a (solar type) star in a high energy orbit around a black hole of mass M without tidal breakup if its density exceeds a critical value

$$\rho_c \sim 3 \times 10^{14} \left(\frac{M_\odot}{M} \right)^2 \ gm\ cm^{-3} \qquad (2.62)$$

According to Sunyaev [21], hot spots can form in the accretion disks around black holes which can last for many revolutions. These can form near the event horizon too and may be quite luminous in a certain wavelength range.

It is reasonable to expect eccentric orbits in some of the cases mentioned above. The emission of the gravitational waves however can circularize the orbit in a period shorter than 10^{10} yrs depending on the masses involved and their separation. Assuming the validity of a Landau-Lifshits type formula [85], the time of circularization of an eccentric orbit can be given as

$$\tau = \frac{15}{304}\,\frac{a^4 F(e_1,e_2)}{m_1 m_2 (m_1+m_2)} \; ; \quad F(e_1,e_2) = \int_{e_1}^{e_2} \frac{(1-e^2)^{5/2}\,de}{e\left(1+\frac{121}{304}e^2\right)}$$

Here m_1 and m_2 are in geometrical units, e_1 and e_2 are initial and final eccentricities respectively, a the semi-major axis of the orbit and $F(e_1, e_2) \sim 1$ for $e_1 \simeq 1$ and $e_2 \simeq 0$. Hence for instance, for a system consisting of a $M_1 = 10^9 M_\odot$ black hole and a $M_2 = 10\ M_\odot$ star, a distance about $10GM_1/c^2$ apart, $\tau \sim 10^8$ yrs if $e_1 \simeq 1$ and $e_2 \simeq 0$. In fact when the orbit is compact enough, a circular orbit also can shrink because of the emission of gravitational waves, according to relations

$$a(t) \simeq a_0\left(1-\frac{t}{T_s}\right)^{1/4} \; ; \quad T_s = \frac{5 a_0^4}{256\, m_1^2 m_2} \,,$$

where T_s is the spiral-in time. Choosing the same parameters as in the example above, $T_s \sim 10^7$ yrs. It is obvious that for smaller ratios of m_1/m_2, τ and T_s decrease appreciably. Liebes [86], Campbell and Matzner [19] and Cunningham and Bardeen [24] have worked out in detail the optical appearance of a source in the background of a gravitating mass, and in orbit round a Schwarzschild or a Kerr black hole. It what follows, we shall be mainly concerned with the frequency shifts.

Now we proceed to write the equations of motion of the source and the photons it emits. The formulation is exact as long as the geometrical optics and the test particle approximation (source mass $\ll$ black hole mass) can be considered valid. Let us suppose that the eccentric orbit is specified by an eccentricity e and a semi-latus rectum a. Let us take $\frac{d\theta}{ds} = 0$ and assume that the observer stays far away from the hole at r = R ($\gg$ 2m) in the plane of the orbit ($\theta = \pi/2$). According to McVittie [22], the metric (2.1) and the eqs. of geodesic motion (2.4) solve together to give

$$\frac{dr}{ds} = \pm\left[\gamma^2 - \left(1-\frac{2m}{r}\right)\left(1+\frac{h^2}{r^2}\right)\right]^{1/2} \tag{2.63}$$

$$\frac{d\theta}{ds} = 0 \tag{2.64}$$

$$\frac{d\varphi}{ds} = \frac{h}{r^2} = \frac{(ma)^{1/2}}{r^2}\left[1-\frac{(3+e^2)m}{a}\right]^{-1/2} \tag{2.65}$$

$$\frac{dt}{ds} = \frac{\gamma}{1-\frac{2m}{r}} = \frac{\left[1-\frac{4m}{a}+4(1-e^2)\frac{m^2}{a^2}\right]^{1/2}}{\left(1-\frac{2m}{r}\right)\left[1-\frac{(3+e^2)m}{a}\right]^{1/2}} \tag{2.66}$$

Here γ is the energy per unit rest mass of the source and h its orbital angular momentum per unit rest mass as measured at infinity. For a circular orbit, e=0, r=a and dr/ds = 0 also. The angular velocity of the source about the hole is defined as before as n = $d\varphi$ /dt.

For the photons, the various components of the 4- momentum are given as

$$\frac{dr}{d\lambda} = \gamma \left[1 - \left(1 - \frac{2m}{r}\right) \frac{h^2/\gamma^2}{r^2} \right]^{1/2} \qquad (2.67)$$

$$\frac{d\theta}{d\lambda} = 0 \qquad (2.68)$$

$$\frac{d\phi}{d\lambda} = \frac{h}{r^2} \qquad (2.69)$$

$$\frac{dt}{d\lambda} = \frac{\gamma}{1 - \frac{2m}{r}} \qquad (2.70)$$

The source emits photons in all directions but only those emitted in the plane of the orbit will be able to reach the distant observer. In view of eqs. (2.63-2.70), we write

$$u^{\alpha}(s) = \left[\frac{\gamma}{1 - \frac{2m}{r}} , \frac{dr}{ds} , 0 , \frac{d\phi}{ds} \right] \qquad (2.71)$$

$$u^{\alpha}(0) = (1, 0, 0, 0) \qquad (2.72)$$

and

$$p_{\beta} = \left[\gamma , -\frac{1}{1 - \frac{2m}{r}} \frac{dr}{d\lambda} , 0 , -r^2 \frac{d\phi}{d\lambda} \right] \qquad (2.73)$$

Hence, if r_o refers to the instantaneous location of the source with respect to the origin of the system of coordinates, then from eq. (2.17) we can write the frequency shift as

$$1+z = \frac{\gamma}{1-\frac{2m}{r}}\left[1+\frac{1}{\gamma^2}\frac{dr}{ds}\frac{dr}{d\lambda}+\gamma q\right]\Bigg|_{r=r_0} \qquad (2.74)$$

where q is the impact parameter of the photon $= h/\gamma'$.
In order to evaluate q, let us consider the two physical components v_r and v_φ of the photon velocity as according to an observer at arest in the Schwarzschild field with respect to his proper reference frame:

$$v_r = \left(\frac{-g_{11}}{g_{00}}\right)^{1/2}\frac{dr/d\lambda}{dt/d\lambda} = \frac{1}{\gamma}\frac{dr}{d\lambda} \qquad (2.75)$$

$$v_\varphi = \left(\frac{-g_{33}}{g_{00}}\right)^{1/2}\frac{d\varphi/d\lambda}{dt/d\lambda} = \frac{\left(1-\frac{2m}{r}\right)^{1/2}}{r}\,q \qquad (2.76)$$

such that

$$v_r^2 + v_\varphi^2 = 1 \qquad (2.77)$$

Therefore it is reasonable to write

$$v_r = \cos\delta \quad , \quad v_\varphi = \sin\delta \qquad (2.78)$$

where δ is the angle at which the photon is emitted with respect to the radius vector of the source through the origin of the system of coordinates; δ increases in the direction opposite to that of motion of the source. Thus, for a radially ingoing photon $\delta = \pi$ whereas for one going outwards $\delta = 0$. In the case of gravitational searchlight (tangential emission), $\delta = \frac{3\pi}{2}$ so that $v_r = 0$ and $v_\phi = -1$. However, in the case of an eccentric orbit, δ is not necessarily $\frac{\pi}{2}, \frac{3\pi}{2}$ for tangential emission. In view of eqs. (2.67-2.70), the line element (2.1) for $ds^2 = 0$ gives

$$q = q_0 \left[1 - \frac{1}{\gamma^2} \left(\frac{dr}{d\phi} \right)^2 \right]^{1/2} = q_0 \sin\delta \qquad (2.79)$$

where

$$q_0 = \frac{r_0}{\left(1 - \frac{2m}{r_0}\right)^{1/2}} \qquad (2.80)$$

characterizes a photon emitted at $\delta = \pi/2$, in the direction opposite to that of motion of the source. The frequency shift is therefore

$$1 + z = \frac{\gamma}{1 - \frac{2m}{r}} \left[1 \pm \frac{\cos\delta}{\gamma} \frac{dr}{ds} + mq \right] \Bigg|_{r = r_0} \qquad (2.81)$$

$$r_0 = a(1 + e\cos\phi)^{-1}$$

Here, + sign refers to $dr/ds < 0$ and vice versa. For $\delta = \frac{3\pi}{2}$ and $e = 0$, this equation reduces to (2.29) for the gravitational searchlight. For large circular orbits, eq. (2.81) approximates to

$$z \simeq \left(\frac{m}{a} \right)^{1/2} \sin\delta \qquad (2.82)$$

It can be noted that Doppler blueshift overcomes the gravitational redshift for forward emission ($\pi < \delta < 2\pi$) from a large circular orbit.

2.8 PROPAGATION AND FREQUENCY SHIFTS OF PHOTONS EMITTED AT DIFFERENT ANGLES:

Photons emitted by the source at different angles not only suffer different frequency shifts but also different amount of gravitational bending. A photon emanating from the source at an instantaneous position $r_o > 3m$ will be captured by the black hole unless the impact parameter satisfies the condition

$$q > q_{lim} = 3\sqrt{3}\, m \tag{2.83}$$

(see, for instance, ref. 23) A photon emitted at an impact parameter slightly in excess of q_{lim} would make numerous rounds of the central black hole before arriving at a distant detector. Consequently, such a photon takes a longer time to arrive there compared to the one emitted with a q greater than q_{lim}. The limiting angle corresponding to $q = q_{lim}$ is therefore

$$\delta_o = \sin^{-1} \frac{3\sqrt{3}\, m}{q_o} \tag{2.84}$$

Photons emitted at an angle $\delta < \pi - \delta_0$ and $\delta > \pi + \delta_0$ escape whereas those emitted at $\pi + \delta_0 \geqslant \delta \geqslant \pi - \delta_0$ are captured. In an eccentric orbit, photons emitted at a certain λ may get captured when r_o is such that q falls below its limiting values and may escape at another instant when r_o becomes such that q exceeds q_{lim}. Table III gives the values of the limiting angle δ_0. corresponding to different values of eccentricity and semilatus rectum, when the source is at the apastron and periastron of its orbit.

An evaluation of the integrals

$$T - t_0 = \int_{r_0}^{\infty} \frac{dt/d\lambda}{dr/d\lambda}\, dr \tag{2.85}$$

and

$$\gamma_0 = -\int_{r_0}^{\infty} \frac{d\gamma/d\lambda}{dr/d\lambda}\, dr \tag{2.86}$$

can tell how long it takes for a photon emitted at a certain angle δ ($< \pi - \delta_0$ or $> \pi + \delta_0$) to reach a distant observer and how much bending it suffers. If we denote

$$\epsilon = \frac{2m}{r_0} \quad , \quad v = \frac{r_0}{r}$$

the integrals read as

$$T - t_0 = \frac{2m(1-\epsilon)^{1/2}}{\epsilon} \int_0^1 \frac{dv}{(1-\epsilon v)\, v^2 \left[(1-\epsilon) - (1-\epsilon v)\, v^2 \sin^2\delta \right]^{1/2}} \tag{2.87}$$

TABLE III

LIMITING ANGLE VARIATION WITH SEMILATUS RECTUM AND ECCENTRICITY

a	e = 0	e = 0.5		e = 1
	δ_0 (degrees)	δ_c max (degrees)	δ_c min (degrees)	δ_c max (degrees)
100 m	2.949	4.403	1.481	5.844
40 m	7.274	10.801	3.677	14.269
10 m	27.695	40.701	14.269	53.609
6 m	45.000	66.716	23.284	90.000
4 m	66.716	r_c min < 3m	34.229	--
3 m	90.000	"	45.000	--

and

$$\varphi_c = [illegible] \int_0^1 \frac{du}{\left[[illegible] - [illegible] \right]^{1/2}} \qquad (2.88)$$

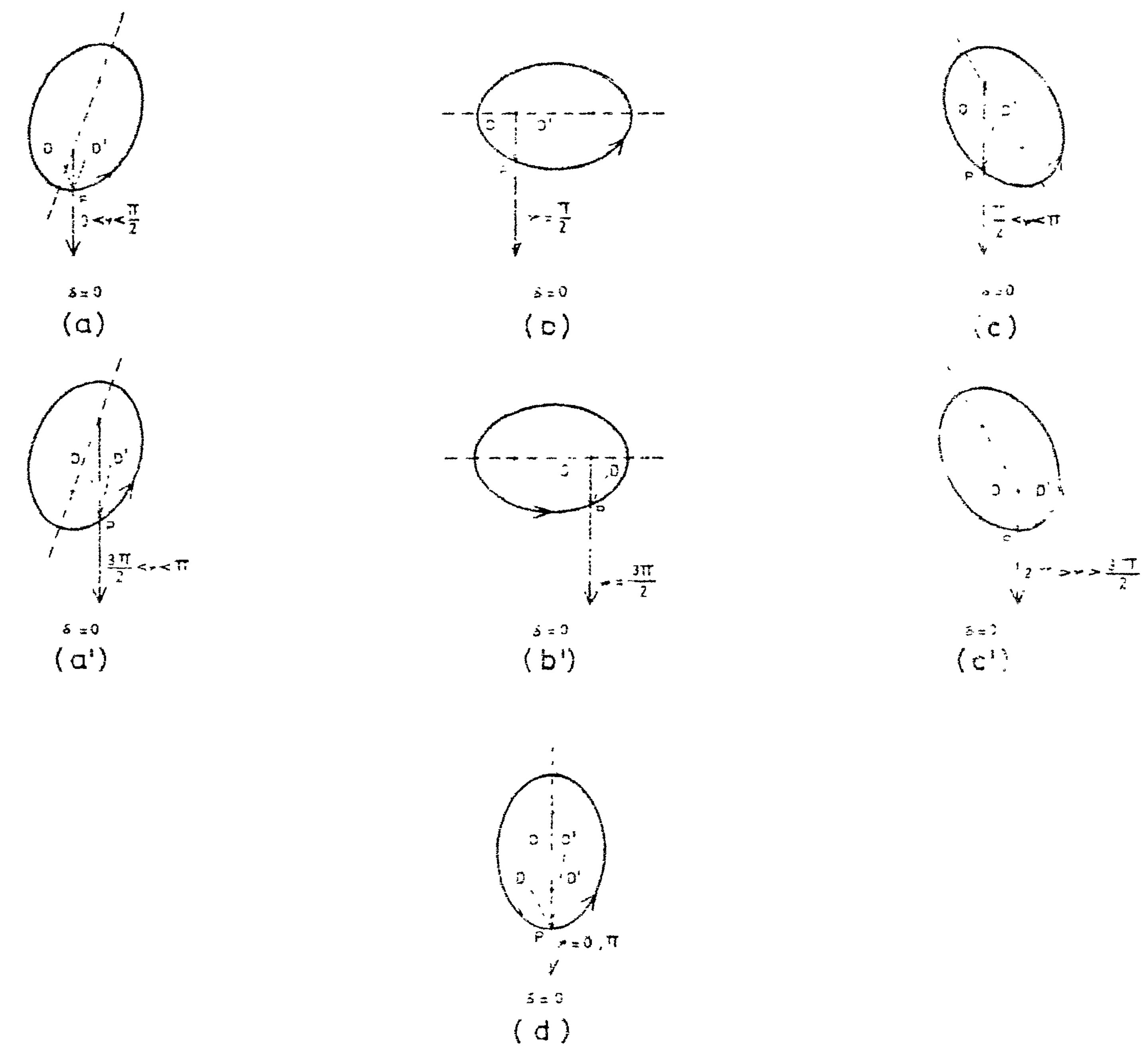
The elliptic integrals in these equations can be transformed into the standard Legendre form by employing suitable transformations. Although, φ_c is asymmetrical with respect to $\pm\delta$, the gravitational bending for $\pm\delta$ photons remains numerically the same. As $r_0 \rightarrow 3m$, eq. (2.88) predicts $\varphi_c \rightarrow \infty$ for $\delta = \pm\pi/2$ photons. The numerical evaluation of eqs. (2.87-2.88) however is of no interest to us here.

Let us focus our attention on the emission of photons by the source at different values of δ. To start with, we consider the simplest case, one of perfect alignment. The center of the black hole, the source and the observer fall on the same line (line of sight). Fig. 2/1 depicts various possible orientations of the orbit with respect to the line of sight. Photons emitted by the source within a cone D'PD are captured by the black hole. The apex angle of the cone, δ_0, varies with r_0 in accord with eq. (2.84). Photons emitted at $\delta = 0$ are received by the remote observer after every complete revolution, with a frequency shift amounting to

$$1 + z(\delta = 0) = \frac{\gamma}{1 - \frac{2m}{r_0}} \left[1 \pm \frac{1}{\gamma} \left\{ \gamma^2 - \left(1 - \frac{2m}{r_0}\right)\left(1 + \frac{h^2}{r_0^2}\right) \right\}^{1/2} \right] \qquad (2.89)$$

FIGURE 2/1

Shows various possible orientations of the orbit with respect to the line of sight for the cases of perfect alignment of the black hole, the source and observer.



$\delta = 0$
(a)
$\delta = 0$
(b)
$\delta = 0$
(c)
$\delta = 0$
(a')
$\delta = 0$
(b')
(c')
$\gamma = 0, \pi$
$\delta = 0$
(d)

FIG. 2/1.

In this equation, - sign refers to $dr/ds > o$ and vice versa, and

$$\gamma^2 \geq \left(1 - \frac{2m}{r_0}\right)\left(1 + \frac{L^2}{r_0^2}\right) \qquad (2.90)$$

corresponds to the timelike trajectory of the source. In this equation, the sign of equality refers to the circular case, or, the source at its periastron or apastron, so that

$$1 + z(\delta = 0) = \frac{1}{\left(1 - \frac{3m}{a}\right)^{1/2}} = \text{constant} > 1 \qquad (2.91)$$

Obviously, these photons are always redshifted and are received whenever the source comes in between the black hole and the observer such that $\frac{dr}{ds} = 0$. However, in general $1 + z\ (\delta = 0)$ may be $>$ or < 1, i.e., the $\delta = 0$ photons may be red- or blueshifted, according as $dr/ds < 0$ or > 0.

In the case of an accentric orbit we have to take into account also the fact that the periastron advances significantly when the orbit is compact enough. The pariastron advance would introduce a secular change in the frequency shift of a photon emitted at a certain δ, $=0$ in the case under consideration. Suppose to start with, the major axis of the orbit coincides with the line of sight and that the source is at its apastron and in between the black hole and the observer. Let T_p be the time taken by the periastron to shift by 2π. Generally, $T_p \gg$ orbital period. Then, in an interval $T_p/2$, the black hole, the source and the observer are once again exactly aligned, the source this time being at its periastron and in between the black hole and the observer. Since $dr/ds = 0$ at $r_{o\,max}$ and $r_{o\,min}$, the respective frequency shifts would be

$$1+z_{min}(\delta=0) = \frac{\gamma}{1-\frac{2m}{r_{o\,max}}} \quad ; \quad 1+z_{max}(\delta=0) = \frac{\gamma}{1-\frac{2m}{r_{o\,min}}} \tag{2.92}$$

the magnitude of the difference between z_{min} and z_{max} amounting to

$$|\Delta z_1| = \frac{4me\gamma}{a\left[1-\frac{4m}{a}+\frac{4(1-e^2)}{a^2}m^2\right]} \tag{2.93}$$

For instance, for $e = 0.5$ and $a = 10m$, $z_{min} = 0.08$, $z_{max} = 0.39$. Hence, a 6000 A line will be shifted by amounts 480 A and 2340 A respectively towards the red. Δz_1 is larger if e is larger. For a large value of a and/or e, $\gamma \rightarrow 1$ and

$$|\Delta z_1| \simeq \frac{4me}{a}\left(1+\frac{4m}{a}\right) \tag{2.94}$$

How much does the frequency shift of a photon emitted at an angle $\delta = 0$ change, after the periastron has advanced by an amount $\Delta\varphi$, in one complete revolution? In order to find this out, let us refer to Fig. 2/2. There are in fact two cases to be considered: (a) when the source is at its periastron to start with ($\varphi = 0$), the semimajor axis coinciding with the line of sight; after once going round the black hole, the source intersects the line of sight again, its radius vector now making an angle $\varphi = -\Delta\varphi$ with the semimajor axis, (b) when the respective angles are $\varphi = \pi$ (source at its apastron) and $\varphi = \pi - \Delta\varphi$. We notice that the change in the frequency shift in case (a) is not the same as that in the case (b). These are given respectively as

$$1+Z_{max}(\varphi=0) = \frac{\delta}{1-\frac{2m}{r_{o\,min}}}\,, \quad 1+Z(\varphi=-\Delta\varphi) = \frac{\delta}{1-\frac{2m}{r_o}}\left[1+\frac{1}{\delta}\frac{dr}{ds}\right]\Bigg|_{r=r_o} \tag{2.95}$$

for case (a), and

$$1+Z_{min}(\varphi=\pi) = \frac{\delta}{1-\frac{2m}{r_{o\,max}}}\,, \quad 1+Z(\varphi=\pi-\Delta\varphi) = \frac{\delta}{1-\frac{2m}{r_o}}\left[1+\frac{1}{\delta}\frac{dr}{ds}\right]\Bigg|_{r=r_o} \tag{2.96}$$

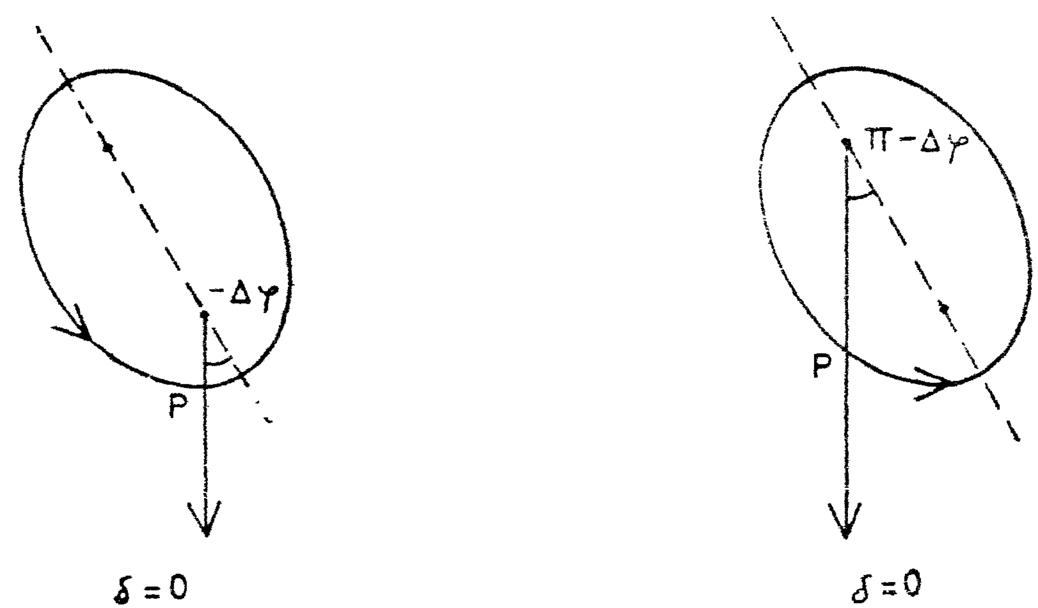
for case (b). To the first order in $\Delta\varphi$, we therefore have

$$|\Delta Z_{2}|_{\pm} = \frac{e\,m\,\Delta\varphi}{\left[ma-2m^2(1\pm e)\right]^{1/2}\left[1-\frac{(3+e^2)m}{a}\right]^{1/2}} \tag{2.97}$$

In eq. (2.97), + sign refers to case (a), whereas − sign

FIGURE 2/2

Shows the cases of perfect alignment after one revolution when the periastron advances by a small angle.



case(a) case(b)

FIG.2/2.

to case (b). Furthermore, $|\Delta z_2|_+ > |\Delta z_2|_-$. For example, for e = 0.5 and a = 10 m, $|\Delta z_2|_+ \simeq 0.023\Delta\rho$ and $|\Delta z_2|_- \simeq 0.020\,\Delta\rho$. In the derivation of the foregoing equations, we have neglected any change in 'e' and/or 'a' due to gravitational radiation.

The photons emitted at various angles, as well as those emitted close to the limiting angle can give rise to the formation of luminous concentric rings about the black hole also. This happens whenever the black hole, the source and the observer are exactly aligned; the observer can receive the photons in any plane which includes the line of sight. One therefore sees a number of concentric rings about the central black hole. When the source is in between the black hole and the observer, it appears as a red or a blue point (vide eq. 2.89 and the following discussion) at the center of the ring system. The innermost ring corresponds to photons which were emitted at δ close to $\pi \pm \delta_c$. They suffer the largest amount of bending and also take the longest time to reach the distant observer. Following the reasoning of the Section 2.4, it is then easy to show that the angular radius of the innermost ring is

$$\phi \simeq \frac{q}{R} = \frac{3\sqrt{3}\,m}{R} \qquad (2.98)$$

where R is the location of the observer ($\gg$ 2m). Here, we would like to stress that the ring system so formed is not as simple as that formed in the case where the source is stationary, not gravitationally bound to the black hole. In the latter case the rings have a uniform thickness, intensity and color. In the present case, the additive and subtractive contributions ofthe Doppler effect and the photon emission off the plane of the orbit render the appearance of the ring system complex. The ring system consists of redshifted as well as blueshifted photons. For the innermost ring, the difference in the frequency shifts amounts to

$$|\Delta z_0| = \frac{2m\tilde{\epsilon}\,(3r_0 - m)}{\left(1 - \frac{2m}{r_0}\right)} \qquad (2.99)$$

which varies with r_o. When the orbit is a compact one, the frequency shift difference is large and consequently the intensity of the rings would be mainly contributed by the blueshifted photons. As the source moves off the line of sight, the rings start degenerating into a number of distorted ghost images of the source appearing around the black hole, though not symmetrically placed and nonuniform in color. When the source goes behind the black hole, the rings appear again, the central red/blue point now missing.

For a compact orbit, the sensitive dependence of the frequency shift and the gravitational bending on give rise to an interesting effect, viz., spectral line broadening in the case of a stellar mass black hole, and, peculiar oscillations of a number of lines across the spectrum in the case of a supermassive black hole. To illustrate this, let us suppose that the source moves in a circular orbit, emitting monochromatic radiation at frequency ν_0 . Now, wherever the source be in its orbit, owing to a large gravitational bending, there would always be some photons emitted at various angles in the forward as well as backward direction reaching the observer at a given instant. As the source moves in its orbit, photons, emitted at progressively changing values of δ are received at $r = R$. The net result is the oscillations of a number of spectral lines about ν_0 in a peculiar manner. In the case of a stellar mass black hole, the orbital period of a source in a compact orbit is $\ll 1$ sec. Therefore, the source goes round the black hole so quickly that the spectral region between $z(\pi/2)$ and $z(-\pi/2)$ is apparently occupied, giving the impression of a broadened spectral line. The wings of the spectral line correspond to emissions at $\delta = \pm \pi/2$, separated by an amount

$$|\Delta z_4| = 2\left[\frac{m a}{(a-3m)\,(a-2m)}\right]^{1/2} \tag{2.100}$$

According to the Liouville theorem, the observed (I_ν) and the emitted (I_{ν_0}) monochromatic intensities are related through

$$[illegible] \qquad (2.101)$$

so that we can express the ratio of the intensities near the wings as

$$[illegible] = \left(\frac{1 + \nu q_0}{1 - \nu q_0} \right)^3 \qquad (2.102)$$

For instance, when $a = 20\ m$, $I_{blue}/I_{red} \simeq 4$ whereas for $a = 10\ m$, $I_{blue}/I_{red} \simeq 9$. The ratio thus gets steeper as a decreases. Therefore, for a compact orbit, the line profile would be highly asymmetrical. In the case of the source orbiting a supermassive black hole, the situation is rather complicated; the orbital period is comparatively much larger and one expects peculiar oscillations of a number of spectral line across the spectrum, the bluer lines being comparatively the brighter.

CHAPTER 3

GRAVITATIONAL RECOIL OF MASSIVE BLACK HOLES FROM GALACTIC NUCLEI

3.1 INTRODUCTION

Since 1966, Arp [11,25, 26, 27, 37 and refs therein] has been presenting evidence of the quasar - galaxy association in a number of cases. In many cases the members of the pair are joined by luminous intergalactic matter. The nucleus of the galaxy appears eruptive, suggesting ejection of material with considerable speeds in roughly opposite directions in the form of a coherent massive object and blobs of gas. The most puzzling feature of such associations is that the redshift of the ejected object which also happens to be a radio source exceeds that of the peculiar galaxy. Particularly, whenever the radio source is a quasar, it shows up with a much larger redshift and is dynamically or evolutionarily younger. A few BL Lacertae type objects too have been seen associated with certain galaxies [55, 88].

The absence of independant confirmation of Arp's observations and the anomalous redshift of the quasars in such associations have been the main points against the ejection hypothesis proposed by Arp. Statistical arguments have occasionally been put forth, for as well as against the hypothesis [see, e.g., ref. No.12] . However, the statistical analyses suffer from the scanty data and can not at present settle the controversy.

Arp's recent work [27, 28] on the isophotal tracings of some of such galaxies reveals disturbances in the inner isophotes extended fairly close in the direction of the quasar. Had the associations been chance juxtapositions, no disturbance in the galaxy and luminous bridges connecting the galaxy and the quasar would have been observed. We are, therefore, tempted to consider ejection to be real.

Now the question of the cause of ejection and the anomalous redshift remains unsettled. Only a few mechanisms are known which could eject different kinds of objects. Shklovsky [40] has tried to interpret Arp's observations of associations by postulating the ejection of a massive magnetoid (a large rotating magnetic plasma body) from the nuclear region of a galaxy as a consequence of anisotropic emission of energetic particles. Harrison [45] has also proposed a similar hypothesis. Saslaw [43] has conjectured the possibility of a gravitational sling-shot mechanism operating in the center of a galaxy which ejects a pair of compact massive objects in one direction and a third one in the opposite direction.

In this Chapter, we propose yet another mechanism of ejection, viz., the gravitational recoil of a supermassive black hole from the center of a galaxy. This can occur because of the anisotropic emission of gravitational waves in the non-spherical gravitational collapse of a supermassive body in the nucleus of a certain galaxy. The ejected black hole captures gaseous matter and stars on its way out and can become luminous enough to be observable at the time of its emergence out of the galaxy [35, 44]. It is then natural to expect some observational prospects and the only question is whether we can identify such a phenomenon with an observed quasar-galaxy association?

In order to be able to eject a massive black hole, with mass $M \sim 10^{8-9} M_\odot$ considered here, the galaxy must be comparatively a massive one. The likely seats of this kind of activity may be the nuclei of spiral, elliptical and Seyfert galaxies. According to Wolfe and Burbidge [42], the observational evidences in the case of elliptical galaxies, such as masses, mass to light ratios (~ 70) larger than a normal galaxy, stellar composition and the occurrence of violent explosions, suggest that in these galaxies a large amount of mass should be present in the form other than stars and perhaps in the form of central black holes with masses $\lesssim 10^{10}\ M_\odot$. The central black hole is crucial to an explanation of the violent explosions in their nuclei and high mass to light ratios.

How can a supermassive black hole form in the nucleus of a galaxy? For instance, Lynden-Bell and Wood [36] and Spitzer [41] have proposed that a galaxy may develop a dense nuclear region as a consequence of a thermal runaway or a lack of energy equipartition between light and heavy mass stars. When a stage of so high a central density is reached that a photon coming from the center of the system is redshifted by an amount $\gtrsim 0.51 - 0.73$, [31, 32, 34], the supermassive body may collapse to the black hole stage because it becomes unstable against radial pulsations. In general, the collapse would be nonspherical. When a nonspherical gravitational collapse takes place, there is an emission of gravitational waves in an anisotropic manner. These carry away not only energy and angular momentum, but linear mementum also. Consequently, the black hole (or an object with its surface very close to its event horizon) must recoil in order to conserve linear mementum, in the frame of reference in which the collapsing mass was originally at rest. The linear momentum is radiated in lowest order as a quadrupole - octupole co-operative effect. The stellar case has been worked out by Bekenstein [29] who has shown that the recoil phenomenon may impart speeds as high as $\gtrsim 1000$ Km sec^{-1} to the black hole independent of its mass and consequently lead to consequences like break-up of a binary star system upon collapse of one of its components, escape of black holes from globular clusters and the Galaxy, etc. In what follows, we outline the basic idea applicable to the nonspherical gravitational collapse of a supermassive body in the nucleus of a certain galaxy and the resulting recoil of supermassive black hole from the center of the galaxy. The order of magnitude estimates show that the recoil effect has something interesting to offer.

3.2 THE RECOIL OF THE SUPERMASSIVE BLACK HOLE

The collapse can be nonspherical owing to rotation, magnetic field and the asymmetrical nature of the nuclear regions of the galaxy, etc. Even if the collapsing body is nonrotating, highly nonspherical collapse is expected since asymmetries tend to grow as $(R)^{-1}$ in a dynamical collapse which may begin at $\lesssim 10^3\ R_s$ [68]. The collapsing body emits gravitational radiation in an anisotropic manner during the collapse at a rate [Ref. 23, pp. 980]:

$$\frac{dE}{dt} \sim \frac{1}{G}\left(c\,\frac{R_s}{R}\right)^5 \quad \mathrm{erg\ sec^{-1}} \qquad (3.1)$$

where $R_s = 2GM/c^2$ is the Schwarzschild radius of the collapsing mass M (in gms), R is its instantaneous radius, and G and c have their usual meaning. According to Arp (private communication), the velocities of emergence of the ejected quasars are estimated to be 10^2 km $\mathrm{sec}^{-1} \lesssim v \lesssim 10^4$ km sec^{-1}. Thus, large velocities of ejection are suggested. In the gravitational recoil, large velocities can be achieved only when the collapse is considerably nonspherical and proceeds upto the black hole stage, as most of the gravitational radiation is emitted only during collapse between radii $R = 3\,R_s$ and $R = R_s$. Highly nonspherical gravitational collapse is very poorly understood and it is not known for sure whether event horizons would form in such cases also. Thorne [46] has conjectured that horizons (probably) form when and only when a mass M gets confined into a region whose proper circumference in every direction is $\lesssim 4\pi\left(\frac{GM}{c^2}\right)$. When all the 'dust' and gravitational waves have cleared away from the exterior, the only conserved integrals left to govern the final horizon and the exterior spacetime are mass, charge and angular momentum that went down the black hole. The exterior spacetime is described by the Kerr-Newman solution (eq.1.1) of the field equations.

If a considerably nonspherical gravitational collapse produces a black hole, the energy emitted in the form of gravitational waves over a dynamical time scale can be as high as $E = \alpha Mc^2$ with $\alpha = 0.01 - 0.9$ [23]. The equivalent linear momentum carried by the waves in this process is αMc. Because of the anisotropic emission, the black hole recoils in a direction of minimum flux in order to conserve linear momentum. Its net momentum is βMc and a recoil speed $v = \beta c$. An exact range of β can be adjudged only by rigorously working out the details of the nonspherical collapse. It depends, among other things, on the geometry of the collapsing body. For the purpose of illustration, the range of the recoil velocity is chosen to be $0 < \beta c \lesssim 10^4$ km sec^{-1}, and $M \sim 10^9 M_\odot$.

The maximum frequency of the emitted gravitational waves is

$$\nu_{max} \sim \frac{1}{\tau} \sim 10^{-5} \; Hz \,, \tag{3.2}$$

with a bandwidth $\Delta\nu$ of the same order. Since the gravitational energy $E(\sim GM^2/R)$ of the collapsing body changes as $\frac{dE}{dt} \sim -\frac{E}{R}\frac{dR}{dt}$, in accordance with eq. (3.1), the characteristic dimension of the collapsing mass evolves in the following approximate manner:

$$R(t) \sim R_0 \left[1 - \frac{t}{T(R_0)}\right]^{1/4}, \quad T(R_0) = \frac{2R_0^4}{cR_s^3} \tag{3.3}$$

where $T(R_0)$ is the collapse time, R_0 the initial dimension and $T(R_0) \sim \tau$ coresponds to $R_0 \sim R_s$. Since asymmetry of the collapsing mass enhances with the collapse, the stars in the nuclear region of the galaxy may acquire some angular momentum also in the process.

The flux F_ν of gravitational radiation received at the earth in the case of ejection of the black hole along the direction away from the observer is maximum and in the caseof ejection towards the observer at the earth it would be minimum. Therefore,

$$F \lesssim \frac{\alpha M c^2}{4\pi d^2 (1+z')^2} \simeq 10^9 \alpha \ \mathrm{erg\ cm^{-2}\ sec^{-1}\ Hz^{-1}} \qquad (3.4)$$

for d ~ 100 Mpc. In eq.(3.4), z' is the mean frequency shift of the pulse of gravitational waves which consists of the cosmological redshift (z_c), the Doppler shift (z_d) and the gravitational redshift (z_g); $d(1 + z_c)$ is defined as the luminosity distance to the ejecting galaxy. The order of equality sign in eq. (3.4) refers to the case of ejection of the black hole away from the observer. However, in this case of ejection, the 'object' even after its emergence from the galaxy is eclipsed; but the occurrence of such an event would make the nuclear region of the galaxy appear eruptive partly due to the ejection of matter towards the observer and partly due to the fact that passage of strong gravitational waves disturbs the intervening matter. For arbitrary directions of ejection, one might hope to see the lumps of gaseous matter and the 'object' in roughly opposite directions, the latter joined to the ejecting galaxy by a luminous bridge of stars and gaseous matter. The bridge can form as a consequence of tidal interaction between the 'object' and the galaxy. Rees [69] has suggested that strong gravitational waves emitted during the collapse might lead to the directional ejection of matter (immediately outside the collapse) at speeds comparable to that of light [68] .

However, the motion of the ejecta ('object' and the gaseous lumps) in either direction is damped, due to regular and irregular forces they are subject to, while within the galaxy. Therefore, depending on factors like size and confinement, the structure of the gaseous lumps is likely to change due to tidal effects within the galaxy and internal motions in the lumps. This might as well make a binary association (object-galaxy) more probable than a multiple one.

3.3 FLIGHT OF THE HOLE THROUGH THE GALAXY

Let us assume that the collapse takes place in an elliptical galaxy. The density distribution of stars in the elliptical galaxies is considered almost isothermal and, herein, we assume a simple exponential density law, without taking into account the high energy cut off and angular momentum which make the density fall more steeply in the outer regions:

$$n(r) = n_0 \exp\left[-\frac{U(r)}{kT}\right] , \qquad (3.5)$$

where n_0 is the central star density, U(r) the potential of a star at a distance r from the center, T the temperature of the system and k the Boltzmann constant. While an assumption of spherical symmetry in the stellar distribution is self-contradictory, as it rules out the allowance for nonsphericity in the galactic nucleus responsible for the gravitational recoil effect, it has been adopted for the sake of simplicity to make order-of-magnitude estimates. From the theory of isothermal gas spheres [47]

$$\psi = \frac{\mu\varphi}{kT} = \frac{1}{6}\xi^2 - \frac{1}{120}\xi^4 + \cdots \quad ; \quad \xi = \frac{r}{a} \tag{3.6}$$

corresponds to the smooth density distribution of stars with an average mass μ, where a is the scale height defined by

$$a = \left(\frac{kT}{4\pi G \mu n_0}\right)^{1/2} = \left(\frac{\langle V^2 \rangle}{4\pi G \mu n_0}\right)^{1/2} \tag{3.7}$$

and $\langle V^2 \rangle$ is the mean square velocity of the stars. The recoiling black hole captures gaseous material and stars from the surroundings while on its way out. The cross section for capture varies as β^{-2} and hence, different structures can develop depending on the values of β. However, the outgoing black hole is subject to deceleration due to (1) attraction by the galactic mass $\mathcal{M}(\xi)$ within radius ξ and (2) the cumulative effect of stellar encounters. These decelerations are given by

$$\left.\frac{d\beta(\xi)}{dt}\right|_1 = -\frac{G\,\mathcal{M}(\xi)}{a^2\xi^2 c} \tag{3.8}$$

and

$$\left.\frac{d\beta(\xi)}{dt}\right|_2 = -\frac{2\pi G^2 m(\xi)\, M}{\beta^2(\xi)\, c^3}\ \ln(1 + x_1^2) \qquad (3.9)$$

respectively, where

$$m(\xi) = 4\pi \mu m_0 a^3 c^2 \frac{d\psi}{d\xi} \ ; \qquad x_1 = \frac{\beta^2(\xi)\, c^2 p_1}{GM} \qquad (3.10)$$

and p_1 is the maximum impact parameter [38] and is $\sim R_g$ (the radius of the galaxy). As will be seen later in this Chapter, the contribution of the stellar encounters to the damping becomes appreciable only when the black hole velocity becomes small. Therefore, for the time being, we shall take

$$\left.\frac{d\beta}{dt}\right|_1 \gg \left.\frac{d\beta}{dt}\right|_2$$

Equation (3.8) can now be written as

$$\beta(\xi)\,\frac{d\beta(\xi)}{d\xi} = -\frac{\langle v^2 \rangle}{c^2}\,\frac{d\psi(\xi)}{d\xi} \qquad (3.11)$$

which integrates to

$$\beta(\xi) = \beta\left[1 - k^2\psi\right]^{1/2}, \quad k^2 = \frac{2\langle v^2 \rangle}{\beta^2 c^2} \qquad (3.12)$$

Here β is the initial speed and K measures the strength of the recoil. Different configurations and structures in the black hole - galaxy system can be expected depending on whether K is large or small with respect to unity. The former case, viz., K $\gtrsim$ 1 can be called the failed recoils which conform to Wolfe and Burbidge's [42] model of a massive black hole in the center of the galaxy. In this case, $\beta(\xi)$ would fall short of the escape velocity, which, in general, is given by

$$\beta_{esc}(\xi) = \left[\frac{2\langle v^2\rangle}{c^2}\,\xi\,\frac{d\psi}{d\xi}\right]^{1/2}, \qquad (3.13)$$

before the black hole emerges out of the galaxy. It would eventually settle down in the center of the galaxy after executing damped oscillatory motion.

The solution ofthe isothermal equation given by Chandrasekhar and Wares [30] has been used to determine variation of $\beta(\xi)$ with ξ. A comparison with $\beta_{esc}(\xi)$ suggests that only those black holes which have recoil velocities $\gtrsim$ 2000 km sec^{-1} can manage to come out of the galaxy. It is convenient to define a function

$$F(\xi) = \frac{\beta_{esc}(\xi)}{\beta(\xi)} = k\left[\frac{\xi\,\frac{d\psi}{d\xi}}{1-k^2\psi}\right]^{1/2} \qquad (3.14)$$

Fig. 3/1 shows F(ξ) vs. ξ, K being the parameter. For the black hole to be able to emerge out of the galaxy, F (ξ) must be less than unity over the range of ξ to be

FIGURE 3/1

Shows the run of $F(\xi)$ with ξ , K being the parameter. For a successful recoil, $F(\xi)$ should be less than unity over $0 < \xi \lesssim R_g/a$.

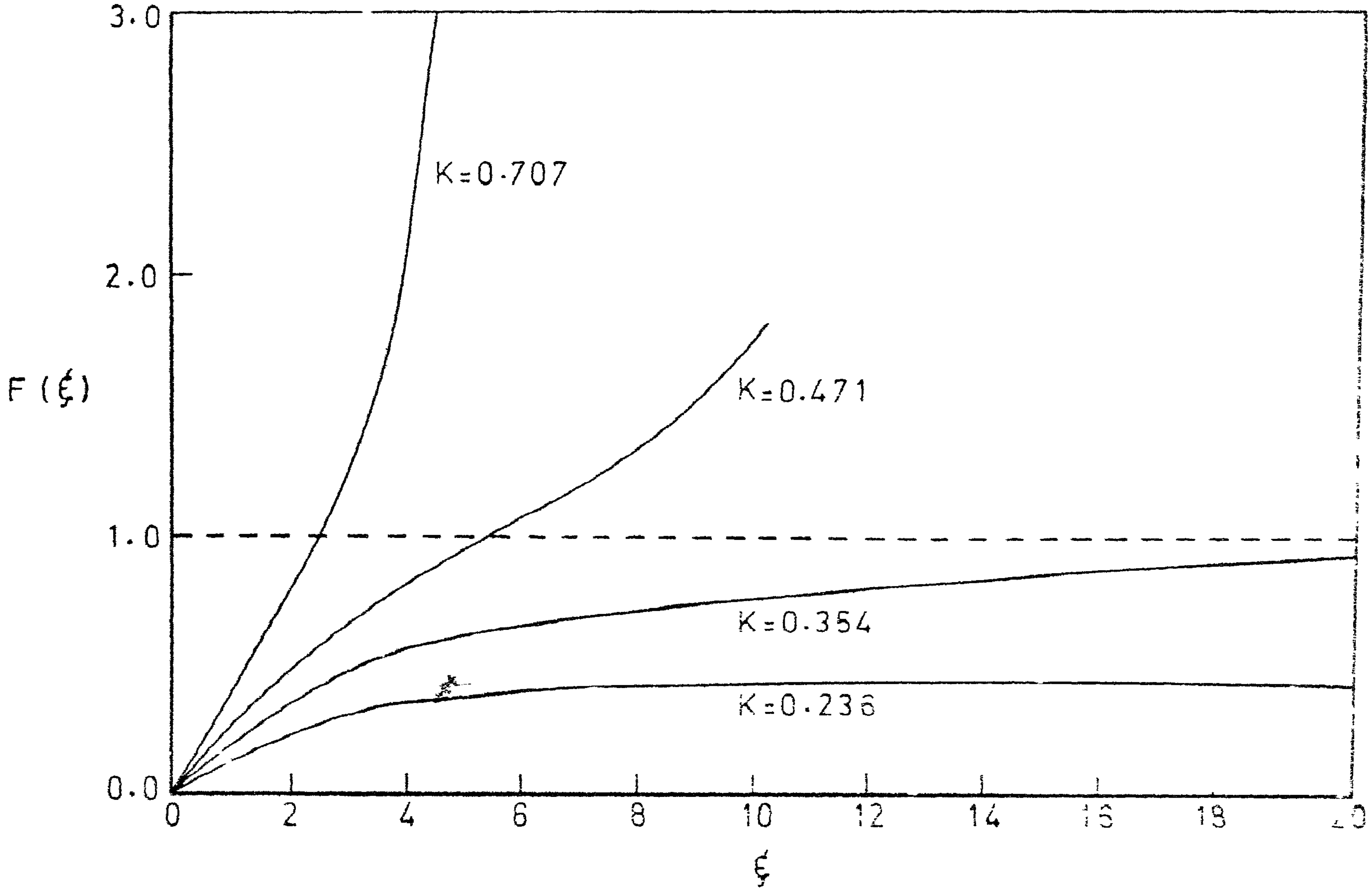


FIG. 3/1.

considered, i.e., $0 \leq \xi \leq R_g/a$. The black hole can reach infinity, if and only if $\beta_{em} \geq \beta_{esc}(R_g/a)$, or

$$\beta \geq \beta_{esc}(\xi)\left[1+\frac{2\langle v^2\rangle}{\beta^2_{esc}\left(\frac{R_g}{a}\right)c^2}\,\psi\left(\frac{R_g}{a}\right)\right]^{1/2} \qquad (3.15)$$

For example, if $\psi(R_g/a) \simeq 5$, $\beta c \gtrsim 2500$ km sec^{-1}. However, for $\xi \gg 1$, $\beta_{esc} = \left(\frac{2\langle v^2\rangle}{c^2}\right)^{1/2}$ i.e., the escape velocity as predicted by eq. (3.13) for $\xi \gg 1$ is independent of ξ and turns out to be an overestimate compared to observational values. Therefore, a black hole with a still lesser velocity of recoil would be able to come out of the galaxy and escape to infinity.

The black hole captures stars of the galaxy at rate

$$\frac{dN(\xi)}{dt} = n(\xi)\,\beta(\xi)\,c\,\sigma(\xi)\;; \quad \sigma = \pi\,\ell_{\pm}^{2} \qquad (3.16)a$$

where $\ell_{\pm}$ is the impact parameter [33]

$$\ell_{\pm} = \frac{4GM}{\beta(\xi)c^2}\,f_{\pm}(\epsilon)\;, \quad f_{\pm}(\epsilon) = \left[1+(1\pm\epsilon)^{1/2}\right] \qquad (3.16)b$$

Above, $\epsilon = a_1/m$, a_1 = the specific angular momentum of a Kerr black hole in geometrical units, $m = GM/c^2$ and ϵ has a range $0 \leq \epsilon \leq 1$. The + sign in eq. (3.16)b refers to the capture of stars by the black hole in the counterrotating manner while - sign to the captures in the corotating manner. Thus, if the black hole were extreme Kerr ($\epsilon \simeq 1$) to start with, $(\sigma_+/\sigma_-) \simeq 6$, i.e., about 14% stars are captured in the corotating manner. The total number of stars captured over the distance range $0 \lesssim \xi \lesssim R_g/a$ is

$$N \lesssim \pi \left[\frac{2GM}{c^2} f_+(\epsilon)\right]^2 \frac{n_0 a}{\beta^2} \int_0^{R_g/a} \frac{e^{-\psi} d\xi}{\left[1 - k^2 \psi\right]} \qquad (3.17)\text{a}$$

Most of the stars are captured by the black hole in its flight between $0 < \xi \lesssim 1$ so that eq. (3.17)a reduces to

$$N \lesssim \frac{\pi R_s^2 n_0 a}{\beta^2} \qquad (3.17)\text{b}$$

Using $\beta \sim 10^{-2}$, $n_0 \sim 10^6$ stars pc^{-3}, $a \sim 500$ pc and $M \sim 10^9 M_\odot$, one has $N \lesssim 10^5$. As we shall see later, these stars ultimately lead to a system of an accretion disk and stars about the black hole, referred to as object henceforth, with dimensions

$$\ell_+ \sim \frac{2GM}{\beta_{em} c^2} f_+(\epsilon) \qquad (3.18)$$

where β_{em} is the dimensionless velocity of the object at the time of its emergence out of the galaxy:

$$\beta_{em} = \beta\left[1 - k^2 \psi\left(\frac{R_g}{a}\right)\right]^{1/2}; \; \beta c \gtrsim 2000 \, km \, sec^{-1} \qquad (3.19)$$

The time of flight of the object through the galaxy is of the order

$$t_f - \frac{\Delta \beta}{\langle d\beta/dt \rangle} \qquad (3.20)$$

where $\Delta\beta$ is the total change in the velocity of the object, i.e., $\beta - \beta_{em}$, and

$$\langle \frac{d\beta}{dt} \rangle = \frac{a}{R_g} \int_0^{R_g/a} \frac{d\beta}{dt}\, da \qquad (3.21)$$

In view of eq. (3.11), we get

$$\langle \frac{d\beta}{dt} \rangle = \frac{\langle V^2 \rangle}{R_g c} \psi\left(\frac{R_g}{a}\right) \qquad (3.22)$$

The kinetic energy lost by the object amounts to

$$\Delta E = \frac{1}{2} M c^2 (\beta^2 - \beta_{em}^2) = M\langle V^2 \rangle \psi\left(\frac{R_g}{a}\right) \qquad (3.23)$$

For $\langle V^2 \rangle^{1/2} \sim 500$ km sec^{-1}, $M \sim 10^9 M_\odot$ this works out to $\Delta E \sim 10^{58}$ ergs which is comparable to the internal energy of the galaxy. This energy is used up into enhancing the rms speed of the stars in the galaxy; consequently the galaxy may expand and evaporation of stars can take place. The time of dissipation of energy is obviously the flight time which is $\sim 10^7$ yrs. This is much smaller than the relaxation time of the galaxy. Therefore, any disturbances in the galaxy caused by the formation and recoil of the black hole through the galaxy would not decay soon. It would, therefore, be interesting to make a detailed study of the changes in the structure of the galaxy.

During its flight through the galaxy, the recoiling black hole can acquire some orbital angular momentum due to the asymmetrical nature of the structure of the galaxy. The object therefore would get deflected too from its original direction of ejection, depending on the asymmetry and β . This problem has been considered recently by Saslaw [43] .

If ejection of a massive object is the answer to the observed quasar - galaxy association, then as suggested by the present model, it would be worthwhile to look for disturbances in the galactic structure in roughly the direction of ejection, because the observed speeds of emergence of the quasars imply flight times ($\sim 10^7$ yrs) far smaller than the relaxation time of galaxies.

As a consequence of tidal interaction, formation of luminous bridge in the wake of the outgoing object is quite likely in this model. The recent work by Arp and coworkers [28] on the isophotal tracings of a few galaxies near quasars does reveal disturbances in the inner isophotes extended fairly close in the direction of the quasar which can be interpreted as due to an event that occurred in the galaxy $\sim 10^7$ yrs previously.

3.4 A QUALITATIVE MODEL OF THE EJECTED OBJECT:

The 'luminosity' of the black hole, while within the galaxy is contributed by the captured stars as well as by the glowing shock front that forms due to the accretion of the interstellar matter during the flight outwards. Outside the galaxy the contribution to luminosity due to the front becomes negligible compared to that made by the gaseous matter from the system of stars. It is the supersonic motion of the object through the interstellar medium that leads to the formation of the shock front during the flight through the galaxy where a sizeable fraction of the kinetic energy released in the shock is radiated away. Novikov and Thorne [3] have shown that the shock front, located at a distance $\beta^{-2}R_s$ from the center of the black hole, glows with a luminosity

$$L \approx 10^{24} \left(\frac{M}{M_\odot}\right)^2 \left(\frac{\rho}{10^{-24}\,gm\,cm^{-3}}\right) \left(\frac{\beta c}{10\,km\,sec^{-1}}\right)^{-1/2} erg\ sec^{-1} \quad (3.24)$$

Thus, if $\beta c \sim 10^3$ km sec^{-1}, $\rho \sim 10^{-26}$ gm cm^{-3} in the case of ellipticals considered here, $L \approx 10^{40}$ erg sec^{-1}.

Now we shall present a brief sketch of the activity in the stellar system around the black hole. We have already estimated that a total number of $\gtrsim 10^5$ stars are captured by the black hole. The average stellar density

$$\eta = \frac{3N}{4\pi \ell_+^3} \sim \frac{3\beta^3 N}{4\pi R_s^3} \qquad (3.25)$$

in a volume of radius l_+ works out to be $\gtrsim 10^{10}$ stars pc^{-3}. In a cluster with such a high stellar density, collisions, coalescences and tidal interactions among the stars themselves as well as those between the stars and the black hole become important soon after the recoil has taken place. This happens as most of the stars are captured during the early stage of the flight of the black hole.

The velocity dispersion in the system of stars around the black hole is

$$\langle v^2 \rangle \sim \frac{2GM}{\ell_+} \approx \beta c^2 \qquad (3.26)$$

Let each star be characterized by a mean radius R ($\sim R_o$) so that the collision cross section σ_{coll} is $\pi (2R)^2$. Hence, the mean free path in the system is

$$\lambda \sim \frac{1}{\eta \pi (2R)^2} = \frac{\ell_+^3}{3N R^2} \qquad (3.27)$$

A star in the system of mean stellar density η makes collisions with other stars at a rate ($\eta\, v\, \sigma_{\text{coll}}$) per unit time. Therefore the total collision rate for the entire system would be

$$N \frac{dN}{dt} = \frac{3N^2 R^2 c\, \beta^{7/2}}{2 R_s^3} \simeq 10^{-1}\ \mathrm{yr}^{-1} \tag{3.28}$$

for N $\simeq 10^5$ and $\beta \sim 10^{-2}$. The collision time-orbital time ratio is

$$\frac{\tau_c}{T_0} \sim \frac{\lambda / v}{\ell_+ / v} \sim \frac{1}{3N}\left(\frac{R_s}{\beta R}\right)^2 \sim 10^5 \tag{3.29}$$

Most of the collisions taking place in the stellar system are very violent. For velocities of collision $\lesssim 10^3$ km sec^{-1}, the collisions lead to coalescence, whereas for velocities $\gtrsim 10^3$ km sec^{-1}, the collisions are disruptive. Since $\langle v^2 \rangle \sim \beta c^2$, collisions would be disruptive for a recoil velocity exceeding $\simeq 0.003$ c. If, in each collision, an average mass $\langle \mu_c \rangle$ is released, then the entire system liberates mass at a rate

$$\frac{dM}{dt} = \langle \mu_c \rangle N \frac{dN}{dt} = \frac{3N^2 R^2 c\, \beta^{7/2}}{2 R_s^3} \langle \mu_c \rangle \tag{3.30}$$

It is known from the work of Spitzer and Saslaw [48] and Sanders [49] that about 5% of the star mass is liberated in the event of a coalescence. Stars in the system also shed some gaseous material in the course of their evolution. In fact, some of the stars may gather enough material to evolve within a time t_f and undergo supernova explosions, liberating large amount of mass and leaving behind collapsed stars (unless they also become runaway objects due to gravitational recoil). Thus, coalescences and disruptions together with some contribution from stellar evolution and interstellar/intergalastic media should be able to yield about 10^{-2} M_o yr^{-1}, if we take $\langle\mu_c\rangle \sim 10^{-1}$ $M_\odot$. Eq. (3.30) then attributes a luminosity of $\gtrsim 10^{41-42}$ ergs sec^{-1} to the object, produced by the conversion of mass into energy in the process of accretion onto the black hole.

It should be borne in mind that the stellar encounters and collisions in the system make massive stars tend to fall towards the black hole. In case the system of stars rotates about the black hole, the effect of stellar encounters within the system is to make it ellipsoidal in shape. As aresult, massive stars tend to concentrate not only towards the black hole but towards the equatorial plane of the sytem as well. This is further assisted by the gas released in the violent physical processes going on in the system. The gaseous material goes into orbits round the black hole to form ultimately an accretion disk and tends to damp out the motion of the stars in the system. The supersonic motion of the stars, in high energy orbits about the black hole, through the gas itself has interesting consequences: shocks formed in the high energy collisions of stars when most of the stellar matter interacts supersonically, convert much of the kinetic energy of the stellar motion into thermal energy to be radiated away. The relaxation time of the stellar system

$$T_R \simeq \frac{2}{\sqrt{3}} \frac{\langle v^2 \rangle^{3/2}}{3\pi G^2 \mu^2 \Lambda \eta} \tag{3.31}$$

where $\Lambda = \ln (N/2) \sim 10$ is the gravitational Coulomb logarithm, is of order $\sim 10^{19}$ sec. This is the order of time over which the stellar orbits in the system change through two-body collisions [50]. Eq. (3.30) suggests that the black hole would eat up the entire system in a period of about 10^{7-8} yrs $\ll T_R$.

A number of workers have recently focussed attention on the astrophysics of a stellar system around a black hole [84]. These studies have been mainly motivated by the discovery of X-ray sources in some globular clusters, Hills' model [51] for QSOs and the possibility of the presence of supermassive black holes ($\sim 10^{8-9} M_\odot$) at the centers of certain galaxies. In these models the main source of luminosity is the gas released in stellar collisions and tidal disruption of stars by the central (stationary) black hole. The tidal radius within which a star of mass μ and radius R would get disrupted by a black hole is

$$R_t = R\left(\frac{M}{\mu}\right)^{1/3} \tag{3.32}$$

For $M > 3 \times 10^8\ M_o$, $R_t < R_s$. In the case of black holes of mass $< 3 \times 10^8\ M_o$, the tidal disruption of stars caused by the black hole is important; the black hole consumes the debris through a 'loss cone' [52]. The loss cone is conical in velocity space in which stars are tidally disrupted within one orbital period when they venture into one tidal radius of the black hole after being scattered into highly eccentric orbits. The loss cone consumption according to Young et al [53] and Shields and Wheeler [54] is so strong for black hole masses $> 10^7\ M_o$ that QSO luminosities can not be attained. The same difficulty is encountered in Hills model [51] for quasistellar objects. However, further work is needed to understand the fate of such models (see also Ozernoi and Reinhardt [89]).

In the black hole mass range of our interest, viz. $\sim 10^{8-9} M_o$, the stellar consumption through the 'loss cone' does not take place. The stars would get tidally deformed as a result of their interaction with the black hole, or, be consumed in toto. However, the tidal interactions among the stars themselves are not to be neglected. In the present context, therefore, the sources of gaseous material are the collisions of the stars, mass loss in the course of stellar evolution, the tidal interactions between the stars themselves and those between stars and the black hole and the interstellar and intergalactic media. Thus a possible model is one of a supermassive black hole at the center of an accretion disk with a number of stars, including perhaps collapsed stars too, moving around in low energy orbits. The gas in the accretion disk is expected to carry a magnetic field which would get sheared and tangled because of the complicated motions in the gas. Synchrotron radiation and thermal bremsstrahlung can, therefore, contribute to the luminosity of the object when it emerges out of the galaxy. The net radiation is expected to have a power law spectrum, resembling that of quasars.

Thus, it seems tempting to attach an importance to the phenomenon of gravitational recoil which is natural to happen, especially in view of the observations of association of quasars and BL lacertae type objects with galaxies. This mechanism has the advantage over those proposed by earlier by others [39, 40] that the black hole, given an appropriate recoil velocity, always moves out of the galaxy and can collect vast amount matter while in the latter cases, the object ($\gtrsim 10^5\ M_\odot$) can collapse during its flight through the galaxy and one is never sure whether the resulting black hole will undergo gravitational recoil in a direction other than that of its motion. It may fail to leave the galaxy or be too faint to be observable at the time of its emergence.

The observed anomaly in the redshifts of the quasar-galaxy associations remains unexplained in all the models of ejection of massive objects. A possibility that the present model can suggest is regarding the total redshift of the object as partly gravitational (z_g) partly peculiar Doppler (z_d) and partly cosmological (z_c) common to the quasar and the galaxy as follows

$$1+z = (1+z_c)(1+z_d)(1+z_g) \qquad (3.33)$$

Several attempts [6, 42] have been made to interpret the quasar-like activity of the nuclei of certain galaxies in terms of gigantic black holes swallowing gaseous matter from their surroundings. There is now compelling evidence for the galaxy M87 breeding a supermassive black hole at its center [56, 57]. The present model would suggest that in these systems one is actually looking at the objects which failed to recoil gravitationally. One wonders whether or not in the case of field quasars on the one hand, and in the case of quasars seen in association with galaxies and active galactic nuclei on the other, different seats of activity are at work. We already have seen that quasars ejected from the galaxies have a life time of at least $\sim 10^{7-8}$ yrs. This is larger than that of the field quasars by an order of magnitude or two. This implies that an ejected quasar, which need not be superluminous, emits $\sim 10^{55-56}$ ergs over its life time. On the other hand, a field quasar emits about $\sim 10^{60-61}$ ergs and is supposed to live for $\sim 10^{6}$ yrs. Thus, two classes of quasars can be suggested; the class A consists of field quasars whose seat of activity is not yet known and the class B consists of the ones found associated with or in the nuclei of certain galaxies. These are comparatively nearby, subluminous ($\sim 10^{41-42}$ erg sec^{-1}), live longer than the field quasars and probably involve super massive black holes.

3.5 FAILED RECOILS

When the velocity of recoil is not large enough, say $\lesssim 10^{3}$ km sec^{-1}, the black hole would not have sufficient kinetic energy to make the recoil successful. After the recoil, the black hole stops at a certain distance $\xi = \xi_1$ ($0 < \xi_1 \leq R_g/a$) and retraces its path; it would execute damped oscillatory motion about the center of the galaxy.

As will be demonstrated, for smaller velocities of recoil, the contribution due to the cumulative effect of stellar encounters to the damping of motion of the black hole also needs to be taken into account. Because of a very large mass M of the black hole compared to that of an individual star in the galaxy, the cumulative effect of the stellar encounters is important only in the direction of its movement. Hence, from eqs. (3.8) and (3.9), we derive the equation of motion of the black hole as

$$\frac{d^2\xi}{dt^2} + \frac{\langle V^2 \rangle}{a^2}\frac{d\psi}{d\xi} + \frac{\langle V^2 \rangle}{2a^5}\,\frac{e^{-\psi} G M \ln(1+x_1^2)}{(d\xi/dt)^2} = 0 \qquad (3.34)$$

valid for $0 \lesssim \xi \lesssim R_g/a$. It can be seen from the above equation that the third term does not diverge as $\frac{d\xi}{dt} \to 0$, since then $\ln(1 + x_1^2) \to x_1^2$ and so it goes as $\sim (d\xi/dt)^2$. In what follows, we study the motion of the black hole restricted to the radii in the range $0 < |\mathbf{r}| \lesssim R_g$.

It is convenient to study such motion by rendering first eq. (3.34) into a dimensionless form. Let

$$\frac{d\xi}{dt} = p\,, \qquad p^2 = u\alpha \qquad (3.35)$$

where

$$\alpha = \frac{2\langle V^2 \rangle}{a^2}$$

we write eq. (3.34) as

$$\frac{du}{d\xi} + \frac{d\psi}{d\xi} + \frac{GM}{4\langle V^2 \rangle a}\cdot\frac{e^{-\psi}\ln(1+x_1^2)}{u} = 0 \qquad (3.36)$$

Behavior at small ξ

For $\xi << 1$, $e^{-\psi} \rightarrow e^{-\xi^2/6}$. When the black hole has just recoiled, $e^{-\xi^2/6} \gtrsim 1$ and $\ln x_1 \simeq$ const., to a fairly good approximation. Hence, eq. (3.36) can be simplified to the form

$$\frac{du}{d\xi} + \frac{\xi}{3} + \frac{2\gamma_1}{u} = 0 \tag{3.37}$$

where

$$\gamma_1 = \frac{GM\langle V^2\rangle \ln x_1}{3a^5\alpha^2} \sim 10^{-1} \tag{3.38}$$

The solution of eq. (3.37) can be expressed as power series

$$u = u_0 - \frac{2\gamma_1\xi}{u_0} - \frac{1}{2!}\left(\frac{1}{3} + \frac{4\gamma_1^2}{u_0^3}\right)\xi^2 - \cdots \tag{3.39}$$

where u_0 corresponds to the initial kinetic energy (in dimensionless form) of the black hole. Hence to a first approximation, the velocity of the black hole, for $\xi << 1$, evolves according to

$$\frac{d\xi}{dt} \simeq \frac{\beta c}{a}\left[1 - \frac{2\gamma_1\alpha^2 a^4}{\beta^4 c^4}\xi\right]^{1/2} \tag{3.40}$$

The black hole would retrace its path at, say $\xi = \xi_1$ ($<< 1$), where

$$\xi_1 \approx \frac{3a\beta^4c^4}{GM\langle v^2\rangle \ln x_1} \tag{3.41}$$

For instance, if $\beta \simeq 300$ km sec^{-1}, $\xi_1 \approx 0.1$, i.e., about 50 pc (for a 500 pc). When the recoil velocity of the black hole is so small that $\ln(1 + x_1^2) \to x_1^2$ (which of course implies that $\beta \lesssim 30$ km sec^{-1}), eq. (3.36) reduces to

$$\frac{du}{d\xi} + \frac{\xi}{3} + 2\gamma u = 0 \tag{3.42}$$

where

$$\gamma = \frac{\langle v^2\rangle R_g^2}{GMa} \sim 10^4$$

This is a nonhomogeneous linear equation with a series solution of the form

$$u = u_0 - 2\gamma u_0\xi + \frac{1}{2!}\left(4\gamma^2 u_0 - \frac{1}{3}\right)\xi^2 - \cdots \tag{3.43}$$

Thus, to a first approximation,

$$\frac{d\xi}{dt} \approx \frac{\beta c}{a}\left(1 - 2\gamma\xi\right)^{1/2} \tag{3.44}$$

In this case the black hole retraces its path at

$$\xi_1 \approx \frac{1}{2\gamma} \sim 10^{-4} \tag{3.45}$$

Behavior at large ξ

A black hole with a large recoil velocity, $\sim 10^3$ km sec^{-1}, would be able to reach large ξ. At $\xi \gg 1$, $e^{-\xi} \rightarrow 2/\xi^2$, and eq. (3.36) assumes the form

$$\frac{du}{d\xi} + f_1(\xi) + \frac{f_2(\xi)}{u} = 0 \qquad (3.46)$$

$$f_1(\xi) = \frac{2}{\xi}, \quad f_2(\xi) = \frac{6\gamma_1}{\xi^2}$$

This equation can be reduced to Abel's differential equation of the first kind. Its analytical solution is quite involved [see, for example, ref. 58]. However, it is easy to see from a comparison of the second and third terms,

$$\frac{f_2(\xi)}{f_1(\xi)\,u} = \frac{1}{\xi}\left[\frac{500\ \mathrm{km\ sec^{-1}}}{a\,d\xi/dt}\right]^2, \qquad (3.47)$$

that, at large ξ, the effect of stellar encounters becomes comparatively pronounced as the velocity of the outgoing black hole falls below ~ 500 km sec^{-1}.

When the velocity falls below ~ 30 km sec^{-1}, eq. (3.36) can be expressed as

$$\frac{du}{d\xi} + \frac{2}{\xi} + \frac{2\gamma u}{\xi^2} = 0 \qquad (3.48)$$

Its solution can be written as

$$u_1 = c_1 \exp\left(\frac{2\gamma}{\xi}\right) \tag{3.49}$$

where

$$c_1 = 2\left[E_1(\xi_0) - E_1(\xi)\right]$$

and $E_1(x)$ is an exponential integral

$$E_1(x) = \int_x^{\infty} \frac{e^{-x}}{x}\, dx$$

$E_1(x)$ tends to 0 as x tends to infinity, and to infinity as $x \rightarrow 0$. Thus, approximately

$$\left.\frac{d\xi}{dt}\right|_{\xi > \xi_0\ (\gg 1)} \simeq \frac{2\langle v^2\rangle^{1/2} e^{\gamma/\xi}}{a}\left[E_1(\xi_0) - E_1(\xi)\right]^{1/2} \tag{3.50}$$

where ξ_0 is an arbitrary point.

Eq. (3.36) has been integrated numerically and the results are shown in Fig. 3/2 for different velocities of recoil. For comparison, we have also plotted the solution of eq. (3.8) (broken lines in Fig. 3/2), written in the dimensionless form

$$u = u_0 - \psi(\xi) \tag{3.51}$$

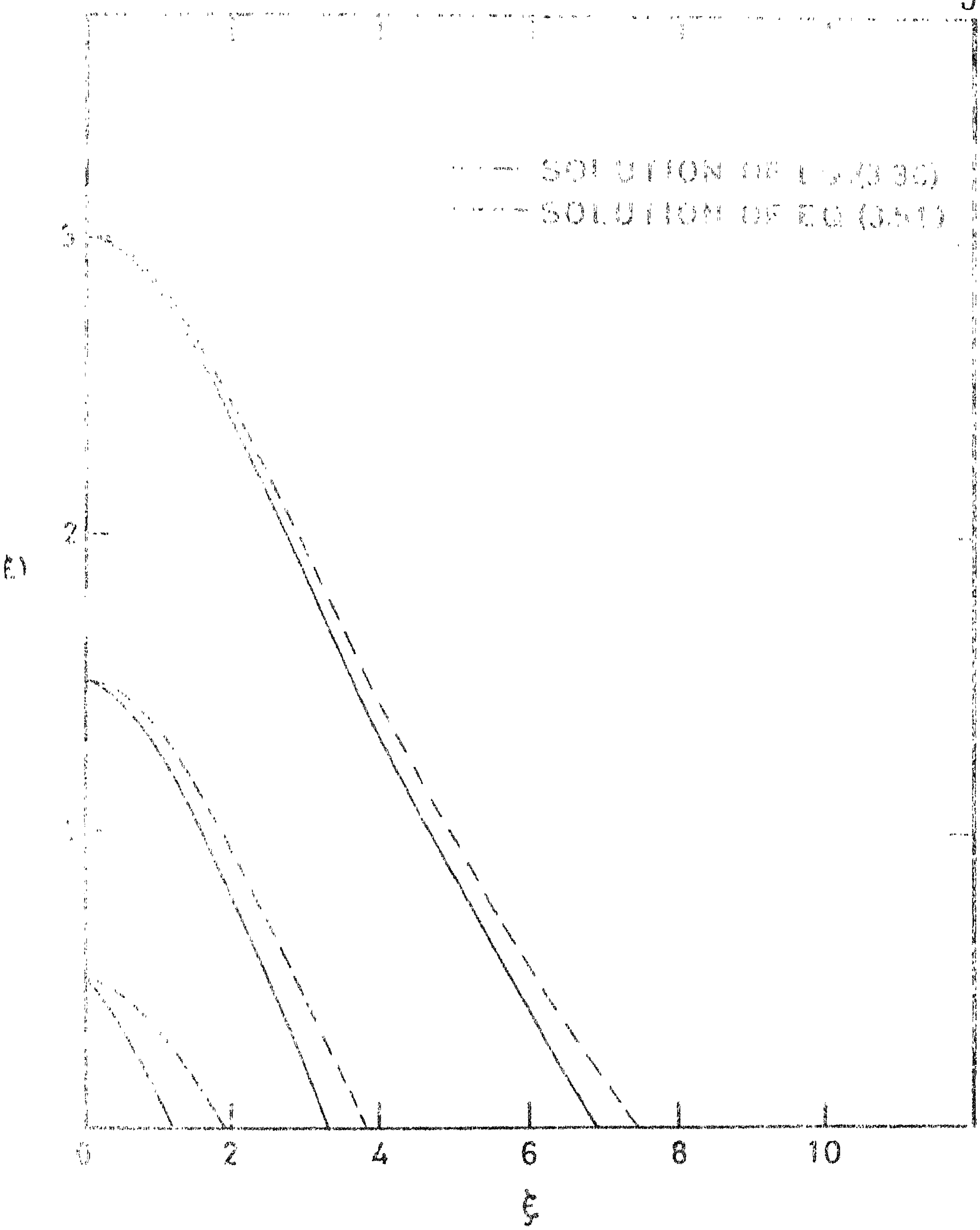


FIG. 3/2

From Fig. 3/2, it is apparent that the cumulative effect of stellar encounters on the motion of the black hole in the galaxy becomes pronounced when the hole moves at a low velocity, and more so in the (denser) nuclear regions.

It is interesting to note that the oscillatory motion of the black hole, according to eq. (3.36), is free of damping as the numerical integration reveals the curves in Fig. 3/2 to be symmetrical about the u (ξ) - axis. The damping is brought about by the tidal interaction between the black hole and the galaxy. As a consequence of the damping, the black hole would eventually settle down in the region of the lowest gravitational potential, i.e., the center of the galaxy. The oscillatory motion, particularly when the velocity of recoil is such that the black hole can reach large values of ξ , would do quite a damage to the galaxy. The kinetic energy lost by the black hole goes in accelerating the stars of the galaxy, rendering the velocity distribution anisotropic. The structure of the galaxy would be changed considerably, especially for a very massive black hole, with perturbations roughly along the direction of motion. Also, the internal energy, U, of the galaxy can change to such an extent that ultimately $\frac{\Delta U}{U} \gtrsim 1$. It would therefore be interesting to study the problem of structural changes in the galaxy as a consequence of tidal interaction with the black hole, in great detail.

In the rest of the Section, we shall make some general observations of the effect of the failed gravitational recoils on the galaxy. Let us tract the course of the black hole from the instant it starts its journey from the center of the galaxy. On its way out it captures a number of stars, given by

$$N \gtrsim \frac{4\pi R_s^2 n_0 a c^2}{\langle V^2 \rangle} \int_0^{\xi_{11}} \frac{e^{-y}\, d\xi}{u(\xi)} \qquad (3.52)$$

where the capture cross section is $\sigma \sim \pi l_+^2$ and $l_+ \sim R_s/\beta(\xi)$. At $\xi = \xi_{11}$, the black hole retraces its path. For instance, when $\beta c \simeq$ 1200 km sec^{-1}, $\xi_{11} \simeq 7.0$ (Fig.3/2), and the black hole captures abowt $\approx 10^7$ stars in one oscillation. But most of these are captured in the region $0 < |\xi| \lesssim 1$. Thus, after making a few oscillations about the center of the galaxy, the black hole would capture quite a sizable population of stars. However, this situation is complicated by two factors: (1) much activity goes on in the system of stars around the black hole and, (2) the black hole loses some of the captured material on its way while falling towards the center of the galaxy and passing through it. Because of a larger number of stars but a somewhat smaller β for the black hole, there is no drastic change in the prediction of eq. (3.30) about the accretion rate. The processes that go on in the stellar system are essentially the same as those described in the previous Section. The only difference is that, although at each passage through the center of the galaxy, the black hole loses material, it also captures fresh material, processes it and radiates by virtue of accretion. The time taken to reach $\xi_1 \sim \frac{R_g}{a}$ is $\sim 2.10^7$yr. Thus if the black hole makes $\sim$ five oscillations about the center of the galaxy, for a recoil velocity $\sim 10^3$km sec^{-1}, the total time taken would be reughly $\sim 10^8$ yrs for it to finally settle down at the center of the galaxy. This is also roughly the time to consume about $\sim 10^{6-7}$ stars.

The system of stars about the black hole is one in which contractive and expansive effects are operative. The former arise because of the fact that while falling towards the center of the galaxy the dimension of the system gets smaller (the capture cross section β^{-2}). The stars initially in the low energy orbits around the black hole evaporate away and the rest of the system gets more tighly bound to the black hole. This in fact implies that in every round of its oscillatory motion, the black hole tends to bring distant stars nearer towards the center of the galaxy. The expansive effects in the system arise from the tidal interaction between the black hole and galaxy. However, one does not really know what happens to the "gas plus star system" after passage of the object through the center. In the course of its passage through the nucleus of a dwarf spherical galaxy, a globular cluster (typical size $\sim$ 10pc, mass $\sim 10^5$ $M_\odot$) is known to disrupt totally [59]. In the present context, the star sys tem is tightly bound to the black hole, and it may as well be that the disruption is only partial.

The black hole while settling down in the center of the galaxy modifies the star distribution in its surrounding. This fact can be demonstrated by the following consideration and applies to the case when the collapse of the supermassive body is taking place or is more or less spherical, resulting in so slow a recoil that the black hole is unable to move sufficiently away from the center of the galaxy. When collapse has progressed sufficiently, both the effects of the gravitational field of the stars in the galaxy and that of the supermassive collapsar have to be taken into consideration. Then the form of $U(r)$ to be used in eq. (3.5) is

$$U(r) = U_1(r) + U_2(r) \tag{3.53}$$

where $U_1/kT = \psi(\xi)$ and U_2 (r) can be found out by considering the potential of a star at a distance r $= x + R(t)$ from the center of the collapsar with a radius R(t) at a time t afterthe onset of the collapse. Thus, we write

$$U_2(r) = -\frac{GM}{x+R(t)} \tag{3.54}$$

when $R(t) << r$, $x \simeq r$ and

$$U_2(x) \simeq -\frac{GM}{r}\left(1-\frac{R(t)}{r}\right) \tag{3.55}$$

We arrive at the stellar density distribution of the form

$$n(\xi, t) = n_0 \exp\left[-\psi(\xi) + \frac{a'}{a\xi}\right] \tag{3.56}$$

with

$$a' = \frac{GM}{\langle V^2 \rangle}\left[1 - \frac{R_0[1 - t/T(R_0)]^{1/4}}{a\xi}\right]$$

For the case of a failed recoil, or, when a black hole has almost settled down into the center of the galaxy after executing oscillation motion, the stellar distribution can be described by

$$n(\xi) \simeq n_0 \exp\left[-\psi(\xi) + \frac{R_a}{a\xi}\right] \tag{3.57}$$

Within a radius $R_a \sim \frac{GM}{\langle V^2 \rangle}$, stars remain bound to the black hole whereas for $r >> R_a$, the distribution is almost isothermal.

It is interesting to note that there are reports of the observation of asymmetrical location of nuclei in some galaxies as well as of split nuclei [60]. The stellar absorption lines in the spectra of galaxies are broadened because of the random velocities of the stars. The presence of a massive black hole tends to increase the rms velocities of the stars in the neighborhood. Therefore, larger velocity dispersions near a displaced or a split nucleus and disturbances in the isophotes of the galaxy would be worth looking for.

PART II

CHAPTER 4

WHITE HOLES: THE ANGULAR APPEARANCE

4.1 INTRODUCTION

The concept of white holes has been with us for the last 14-15 years [2, 7]. Originally proposed to explain the energetics of the quasars, white holes have been invoked in one context or the other to explain certain high energy phenomena in astrophysics. For instance, Narlikar and Apparao [10] have pointed out the viability of the white holes in connection with a number of phenomena such as transient X-ray sources, X-ray background radiation, γ-ray bursts, active galactic nuclei, high energy cosmic rays and radio sources. Their conclusions were based on the extension of the model of a white hole considered first by Faulkner et al [9]. According to them photons emitted in the early stages of the expansion of the white hole not only get out of its event horizon, but do so with large blueshifts. The blue-shifted radiation can manifest itself as a γ-ray burst or as a transient x-ray emission.

In the present Chapter, we intend to study the non-radial emission of photons from a white hole surface. The main purpose is to investigate how the angular appearance of the white hole changes with its expansion, especially in the early stages when its surface is still inside the event horizon. We will demonstrate the importance of the role played by the impact parameter of the photons in determining the angular size of the white hole [90]. For this purpose we adopt the model of canonical white hole studied by Narlikar and Apparao [10].

4.2 MODEL OF THE WHITE HOLE:

The canonical white hole that we shall be concerned with here is a spherical object of uniform density $\rho(t)$ and zero pressure in the frame of reference comoving with the outward moving particles. The interior of the white hole is assumed to be Friedmannian except for small density fluctuations superposed thereupon. The object emerges from a singular state and subsequently obeys Einstein's field equations.

A dust model ensures geodesic motion of all the particles, which can be inferred from the Friedmann metric that describes the white hole interior in terms of the constant coordinates (r, θ, φ) of a comoving particle:

$$ds^2 = c^2 dt^2 - S^2(t)\left[\frac{dr^2}{1-\alpha r^2} + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)\right] \quad (4.1)$$

$$r \leq r_b$$

Here, t is the proper time of a comoving observer and $S(t)$ the scale factor. At the boundary, $r = r_b$, a dust model ensures zero pressure so that the metric matches perfectly with an exterior Schwarzschild metric

$$ds^2 = c^2 dt^2\left(1 - \frac{2GM}{Rc^2}\right) - \frac{dR^2}{\left(1 - \frac{2GM}{Rc^2}\right)} - R^2\left(d\theta^2 + \sin^2\theta\, d\phi^2\right) \tag{4.2}$$

In cosmology, r=0 refers to any assigned particle, but in the present case, r is defined such that the scale factor S=1 when the expansion of the white hole is complete. Then, r=o refers to the center of the object. The scale factor $S(t)$ varies with time according to

$$\frac{dS}{dt} = \pm\left(\frac{8\pi G\rho}{3}S^2 - \alpha c^2\right)^{1/2} \tag{4.3}$$

where + sign refers to explosion, and ρ , the matter density, is time dependent, evolving according to

$$\rho = \frac{\rho_o}{S^3} \tag{4.4}$$

ρ_o being the density corresponding to the epoch when expansion is complete (S=1). The parameter α , a constant, is given by

$$\alpha = \frac{8\pi G\rho_o}{3c^2} \tag{4.5}$$

Eq. (4.3) integrates to give

$$t = \frac{1}{c\alpha^{1/2}}\left[\sin^{-1} S^{1/2} - S^{1/2}(1-S)^{1/2}\right] \tag{4.6}$$

The proper time for explosion from S=0 to S=1 is

$$t_0 = \int_0^1 \frac{dS}{\dot S} = \frac{\pi}{2c\alpha^{1/2}} \tag{4.7}$$

The white hole boundary crosses the event horizon at an epoch:

$$S(t_s) = \alpha r_b^2 = \sin^2 \Sigma \quad \text{(say)} \tag{4.8}$$

The matching of the two metrics (4.1) and (4.2) across the boundary of the white hole requires that on the surface r = r_b,

$$R = r_b S(t) \;; \quad \frac{dT}{dt} = \frac{(1-\alpha r_b^2)^{1/2}}{\left(1 - \frac{2GM}{R_b c^2}\right)} \tag{4.9}$$

and that the mass of the white hole is given by

$$M = \frac{\alpha r_b^3 c^2}{2G} = \frac{t_0 \sin^3 \Sigma \, c^3}{\pi G} \tag{4.10}$$

In the present work, we are concerned with extragalactic white holes in the supermass range. Table IV gives values of t_0 and ρ_0 assuming various values for Σ and M. The time of expansion as measured by a distant observer is generally of the order of t_e [10] .

Here we wish to emphasise that the canonical white hole described above is an idealisation in many respects. The

TABLE IV

EXPANSION TIME AND FINAL DENSITY VALUES FOR DIFFERENT WHITE HOLE MASSES

Σ	$M = 10^8 M_\odot$		$M = 10^9 M_\odot$		$M = 10^{10} M_\odot$	
	t_o sec	ρ_o gm cm^{-3}	t_o sec	ρ_o gm cm^{-3}	t_o sec	ρ_o gm cm^{-3}
$\frac{\pi}{1000}$	$5.0x10^{10}$	$1.8x10^{-15}$	$5.0x10^{11}$	$1.8x10^{-17}$	$5.0x10^{12}$	$1.8x10^{-19}$
$\frac{\pi}{300}$	$9.1x10^{8}$	$2.4x10^{-12}$	$9.1x10^{9}$	$2.4x10^{-14}$	$9.1x10^{10}$	$2.4x10-^{16}$
$\frac{\pi}{100}$	$5.0x10^{7}$	$1.8x10^{-9}$	$5.0x10^{8}$	$1.8x10^{-11}$	$5.0x10^{9}$	$1.8x10^{-13}$
$\frac{\pi}{30}$	$1.4x10^{6}$	$2.4x10^{-6}$	$1.4x10^{7}$	$2.4x10^{-8}$	$1.4x10^{8}$	$2.4x10^{-10}$
$\frac{\pi}{10}$	$5.3x10^{4}$	$1.6x10^{-3}$	$5.3x10^{5}$	$1.6x10^{-5}$	$5.3x10^{6}$	$1.6x10^{-7}$

real explosions are not spherically symmetric. Anisotropics could be present in the form of shear and rotation. The initial state may be highly dense rather than singular as presumed here, and explosion may not be in empty spacetime. As exploding objects of the type discussed in [10] are usually found in a medium of tenuous density, the model described above may be regarded as a crude approximation to reality at least in the not very anisotropic explosions.

4.3 NONRADIAL NULL GEODESICS

In this section, we shall write down the equations of motion of nonradial photons emitted from the white hole surface $r = r_b$. Without loss of generality, we can set $\theta = \pi/2$ for the plane of the trajectory. The dynamics of test particles in the Schwarzschild spacetime yields the following first integrals in terms of an affine parameter λ,

$$\frac{d\phi}{d\lambda} = \frac{h}{R^2} \tag{4.11}$$

and

$$\frac{dT}{d\lambda} = \frac{h}{qc\left(1 - \frac{2GM}{Rc^2}\right)} \tag{4.12}$$

where h and q are constants; h is the orbital angular momentum parameter and q the impact parameter of the nonradial photon. The metric (4.2), in view of eqs. (4.11 - 4.12) suggests

$$\frac{dR}{dT} = \left(1 - \frac{2GM}{Rc^2}\right) c\, F(R,q) \qquad (4.13)$$

and

$$\frac{d\varphi}{dR} = \frac{q}{R^2 F(R,q)} \qquad (4.14)$$

where

$$F(R,q) = \left[1 - \left(1 - \frac{2GM}{Rc^2}\right)\frac{q^2}{R^2}\right]^{1/2} \qquad (4.15)$$

Suppose that initially $\varphi = \varphi_1$, $R = R_1 = r_b S(t_1)$, $T = T(t_1) = T_1$ where t_1 is the proper time of the surface dust particle at the time of emission. Finally the photon arrives at a remote receiver with $R = R_2 >> 2GM/c^2$, $\theta = \pi/2$, $\varphi = 0$, $T = T_2$ (say). Then from eqs. (4.13) and (4.14) we get

$$T_2 - T_1 = \frac{1}{c}\int_{R_1}^{R_2} \left[F(R,q)\left(1 - \frac{2GM}{Rc^2}\right)\right]^{-1} dR \qquad (4.16)$$

and

$$\varphi_1 = q\int_{R_1}^{R_2} \frac{dR}{R^2 F(R,q)} \qquad (4.17)$$

In Fig. (4/1) we show the schematic track of a photon as it goes round the white hole before finally coming out at $R = R_2$. At this point the photon makes an angle ϵ with the line of sight through the center of the white hole (the radial direction $\varphi = 0$). This angle could be determined in a manner similar to that adopted in Chapter 2 for the case

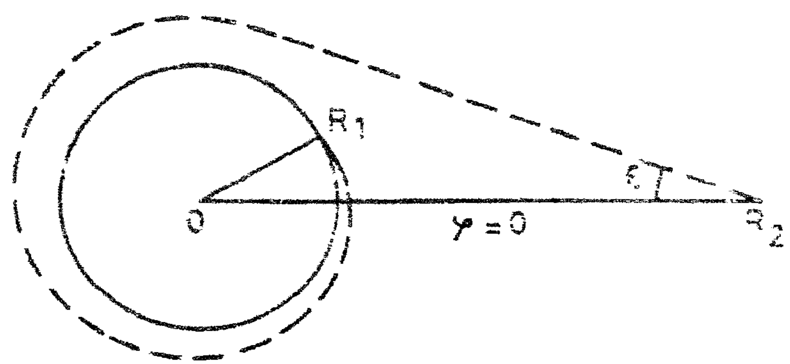


Fig. 4/1. The track of a non-radial photon from the surface of a white hole to a remote observer.

of the gravitational search light by setting up a cartesian co-ordinate system at R_2. The result is

$$\epsilon \simeq \frac{q}{R_2} \tag{4.18}$$

Once again the impact parameter determines the angle along which the photon reaches $R = R_2$. It should be noted that in the derivation of the last equation, we have assumed $\epsilon << 1$, so that $q << R_2$. The equation does not hold for the case when the observer stays very near the white hole. Thus for instance in the case of an observer at the earth, looking at a galactic or an extragalactic white hole, equation (4.18) is a good approximation.

4.4. THE SPECTRAL SHIFT:

We now proceed to calculate the frequency shift of a nonradially emitted photon described in the previous Section. This can either be done (1) from first principles, viz. by considering the null geodesics emanating from $r = r_b$ at $t = t_1$ and $t = t_1 + \Delta t_1$ and arriving at the remote observer at $T = T$ and $T = T + \Delta T$ respectively, or (2) by using Schroedinger's general formula:

$$1+z = \frac{\nu_0}{\nu} = \frac{\vec{U}.\vec{p}\,|_{source}}{\vec{U}.\vec{p}\,|_{observer}} \tag{4.19}$$

Here ν_0 refers to the frequency of photon at the time of its emission, ν that at reception and

$$u^\alpha|_{obs} = (1,0,0,0) \; ; \quad u^\alpha|_{source} = \left(\frac{dT}{ds}, \frac{dR}{ds}, 0, 0\right)$$

$$p^\alpha = \left(\frac{dT}{d\lambda}, \frac{dR}{d\lambda}, 0, \frac{d\varphi}{d\lambda}\right) \qquad \textbf{(4.20)}$$

λ **being an affine parameter, and** p^α **refers to the** 4**- momentum of the photon.**

In view of eqs. (4.9) we have

$$\frac{dR}{ds} = \frac{r_b \dot{S}(t_1)}{c}, \quad \frac{dT}{ds} = \frac{\partial T}{\partial t}\frac{dt}{ds} = \frac{(1-\alpha r_b^2)^{1/2}}{c\left(1 - \frac{2GM}{R_1 c^2}\right)} \qquad \textbf{(4.21)}$$

for the white hole surface (where in the comoving frame of reference, $c\frac{dt}{ds} = 1$ **). The quantities** $\frac{dR}{d\lambda}$, $\frac{d\varphi}{d\lambda}$ **and** $\frac{dT}{d\lambda}$ **are given by eqs. (4.11-13). Hence, a little algebraic manipulation leads to**

$$\frac{\nu_0}{\nu} = \frac{(1-\alpha r_b^2)^{1/2} - \frac{r_b \dot{S}(t_1)}{c} F(R_1, q)}{\left(1 - \frac{2GM}{R_1 c^2}\right)} \qquad \textbf{(4.22)}$$

The approach from first principles gives the same answer. It can be noted that the equation is well behaved for all $R_1 > 0$ **including** $R_1 = 2GM/c^2$**. In order to show this for** $R_1 = 2GM/c^2$**, let us multiply the numerator and the denominator by** $\left[(1-\alpha r_b^2)^{1/2} + \frac{r_b \dot{S}(t_1)}{c} F(R_1, q)\right]$ **. The result is**

$$\frac{\nu_0}{\nu} = \frac{1 + \frac{q^2 r_b^2 \dot{S}^2(t_1)}{c^2 R_1^2}}{(1-\alpha r_b^2)^{1/2} + \frac{r_b \dot{S}(t_1)}{c} F(R_1, q)} \qquad \textbf{(4.23)}$$

At $R_1 = 2GM/c^2$, this reduces to

$$\left.\frac{\nu'}{\nu_0}\right|_{R_1 = \frac{2GM}{c^2}} = \frac{1 + \frac{q^2}{R_1^2}(1-\alpha r_b^2)}{2(1-\alpha r_b^2)^{1/2}} \tag{4.24}$$

For the case of a radially emitted photon ($q = 0$) the frequency blueshift is larger, as expected.

We now ask what is the possible upper limit on the impact parameter q so that the null geodesic it specifies shows a blueshift ($\nu > \nu_0$). In order to see this, we set $\nu = \nu_0$ and solve for q:

$$q^2 < \frac{R_1^2\left(2\sqrt{1-\alpha r_b^2} - 1\right)}{(1-\alpha r_b^2)}$$

Since at $R_1 = 2GM/c^2$, $1 - \alpha r_b^2 = r_b^2 \dot{S}^2/c^2$,

$$q^2 < \frac{r_b^2 S_1^2\left[2\sqrt{1-\alpha r_b^2} - 2 + \frac{\alpha r_b^3}{r_b S_1}\right]}{r_b^2 \dot{S}^2/c^2}$$

or

$$q^2 < \frac{\left[2\sqrt{1-\alpha r_b^2} - 2 + \frac{\alpha r_b^2}{S_1}\right]}{\alpha(1-S_1)/S_1^3} \tag{4.25}$$

For small $S_1 \; (<< \alpha r_b^2 < 1)$, we therefore have

$$q^2 \lesssim r_b^2 S_1^2 = R_1^2 \tag{4.26}$$

For $S_1 > \alpha r_b^2$ (i.e, $R_1 > 2GM/c^2$), there is another requirement on q: that F should be real, so that

$$\frac{1}{q^2} \geqslant \left(1 - \frac{2GM}{R_1 c^2}\right) \frac{1}{R_1^2} \qquad (4.27)$$

For $R_1 = 3GM/c^2$ the right hand side of this equation has a maximum. Therefore, for $q \leqslant 3\sqrt{3}GM/c^2$, F is always real. One can notice that eq. (4.26) is automatically satisfied for $R_1 < 2GM/c^2$, i.e., for $S_1 < \alpha r_b^2$. We shall, however, be more interested in eq. (4.26) which applies in the very early stages of the white hole- explosion.

4.5 THE RATE OF GROWTH OF ANGULAR SIZE:

Now we shall study the angular appearance of a white hole seen by a remote abserver. We have already noted that the angle subtended by the white hole is proportional to the impact parameter (q) of nonradially emitted photon from the white hole surface. We now ask what the maximum value of q is at any given time T_2 and how it changes with T_2. To answer the question we will assume that only the blueshifted photons are easily detected by the remote observer. The redshifted photons would be too faint to be detected. This is only a crude approximation to reality where the actually measured angular size of a white hole would depend on a number of parameters related to the measuring instrument, such as its sensitivity, its response to specific wavelengths, the signal to noise ratio, etc. All that is implied in the following analysis is that the photons which are blueshifted are relatively easier to detect than those which are redshifted.

So, let us consider a typical blueshifted photon whose impact parameter is q and which leaves the white hole surface at a comoving time $t = t_q$ in such a way that it arrives at the distant detector at $R = R_2$ at the observer's proper time $T = T_2$. The answer to our question is then obtained by calculating the largest value of q for a given T_2, subject to eq. (4.26), i.e.,

$$q \lesssim r_b S(t_q) = R_q \tag{4.28}$$

For this photon, eq. (4.16) reads as

$$T_2 = T(t_q) + \frac{1}{c}\int_{R_q}^{R_2} \frac{dR}{F(R,q)\left(1-\frac{2GM}{Rc^2}\right)} \tag{4.29}$$

As we change q, t_q also changes. Thus for a fixed T_2 the last equation gives

$$\Delta t_q = -\frac{1}{c}\left(1-\frac{2GM}{R_q c^2}\right) \frac{\int_{R_q}^{R_2} \frac{q}{R^2}\left[F(R,q)\right]^{-3} dR}{\left(1-\alpha r_b^2\right)^{1/2} - \frac{r_b \dot{S}(t_q)}{c\, F(R_q,q)}}\, \Delta q \tag{4.30}$$

It can be verified that in the early stages of the expansion of the white hole ($R_q << 2GM/c^2$) and for the photon with impact parameter

$$q < \frac{R}{(1-\alpha\gamma_b^2)^{1/2}} \qquad (4.31)$$

t_q decreases as q increases. This in fact means that a photon with a larger impact parameter should start earlier than its counterpart with a smaller q, in order to arrive simulatenously at $R = R_2$. Obviously the radially emitted photon ($q = o$) starts last. As q increase from q = o, photons have to start at earlier epochs with smaller S. Clearly, q can increase only upto the limit set by eq. (4.18) which is consistent with eq. (4.31). Hence the largest angular size is obtained by setting $q = R_q$ in the specification of the null geodesic. This gives us:

$$T_2 = T(t_q) + \frac{1}{c}\int_q^{R_2} \frac{dR}{\left(1-\frac{2GM}{Rc^2}\right) F(R,q)} \qquad (4.32)$$

This is an implicit relation between q and T_2. We differentiate this equation with respect to q and use eq. (4.9) so that

$$\frac{dT_2}{dq} = \frac{1}{c}\,\frac{(1-\alpha\gamma_b^2)^{1/2}}{\left(1-\frac{2GM}{qc^2}\right)} - \frac{1}{c}\,\frac{(2GM/qc^2)^{1/2}}{\left(1-\frac{2GM}{qc^2}\right)}$$

$$+\frac{1}{c}\int_q^{R_2} \frac{(q/R^2)\,dR}{\left[1-\frac{q^2}{R^2}\left(1-\frac{2GM}{Rc^2}\right)\right]^{3/2}} \qquad (4.33)$$

Since $q = r_b\, S(t_q)$,

$$dq = r_b\, \dot{S}(t_q)\, dt_q \tag{4.34}$$

Hence, eq. (4.33) becomes

$$\frac{dT_2}{dq} = \frac{1}{c}\left(1-\frac{2GM}{qc^2}\right)^{-1}\left[\frac{(1-\alpha r_b^2)^{1/2}}{r_b\,\dot{S}(t_q)} - \left(\frac{2GM}{qc^2}\right)^{1/2}\right]$$

$$+\frac{1}{c}\int_q^{R_2}\frac{q}{R^2}\left[1-\frac{q^2}{R^2}\left(1-\frac{2GM}{Rc^2}\right)\right]^{-3/2} dR\,.$$

For $q \ll 2GM/c^2$, $\dot{S}^2 \approx \alpha c^2/S$. **Hence**

$$\frac{dT_2}{dq} = \frac{1}{c}\left[1-(1-\alpha r_b^2)^{1/2}\left(\frac{qc^2}{2GM}\right)^{3/2}\right] + \frac{1}{c}\int_q^{R_2}\frac{q}{R^2}\left[F(R,q)\right]^{-3} dR. \tag{4.35}$$

For small q, we approximate the value of the integral in eq. (4.35) in the following manner:

$$\frac{1}{c}\int_q^{R_2}\frac{q}{R^2}\left[1-\frac{q^2}{R^2}\left(1-\frac{2GM}{Rc^2}\right)\right]^{-3/2} dR \quad = I \text{ (say)}$$

$$\approx \frac{q}{c}\int_0^{1/q}\left(1-q^2x^2+\frac{2GM}{c^2}q^2x^3\right)^{-3/2} dx \tag{4.36}$$

where $x = R^{-1}$ **. Using the transformation**

$$y = \left(\frac{2GM}{c^2}q^2\right)^{1/3} x$$

this further leads to

$$I \approx \frac{1}{c}\left(\frac{qc^2}{GM}\right)^{1/3}\int_0^\infty \left[1-\left(\frac{qc^2}{GM}\right)^{2/3} y^2 + 2y^3\right]^{-3/2} dy$$

$$\approx \left(\frac{q}{GMc}\right)^{1/3}\int_0^\infty (1+2y^3)^{-3/2}\, dy$$

$$\approx \quad A\left(\frac{q}{GMc}\right)^{1/3} \tag{4.37}$$

where A is a constant of the order of unity. Thus for small values of q, the behavior of q with respect to T_2 can be written down as

$$\frac{dT_2}{dq} \sim A\left(\frac{q}{GMc}\right)^{1/3} \tag{4.38}$$

which gives

$$T_2 \sim \frac{3A}{4(GMc)^{1/3}}\, q^{4/3} \tag{4.39}$$

Finally we get

$$\frac{dq}{dT_2} \sim \frac{k}{T_2^{1/4}} \quad , \; k = \left(\frac{3}{4}\right)^{1/4} A^{5/4} (GMc)^{-5/12} \tag{4.40}$$

K being a constant. The corresponding angular radius of the white hole therefore increases at a rate given by

$$\frac{d\epsilon}{dT_2} = \frac{k}{R_2\, T_2^{1/4}} \tag{4.41}$$

For small values of T_2, eq. (4.40) suggests the white hole expansion taking place at a superluminal ($v > c$) speed. It is interesting to note that in the case of radio components of many quasistellar sources superluminal speeds of separation have been observed [64]. In a typical example, two components appear to be separating from each other at speeds v exceeding that of light. This feature has puzzled astronomers to a great extent. For, if the speeds are real, they either contradict the special theory of relativity or the cosmological interpretation of the QSO redshifts. The example of superluminal expansion that we have cited here although refers to a spherical white hole, it points out a likely explanation of the above phenomenon. In order to get quantitative estimates for comparison with observations, it is however necessary to consider linearly expanding white holes. Eqs. (4.39) and (4.40) give the observed expansion speed as

$$v = \frac{dq}{dT_2} \sim \frac{c}{(2\eta)^{1/3}} \tag{4.42}$$

where

$$q = R_1 = \eta \left(\frac{2GM}{c^2} \right)$$

Thus, a white hole superluminally expands when $\eta \lesssim 1/2$.

Lastly a few words about the ρ-integral (4.17) may be mentioned. In this equation, ρ_1, the gravitational bending suffered by a nonradially emitted photon, is finite except when

$$F^2(R, q) = 0 \tag{4.43}$$

has a double root in R. This of course happens when

$$q = \frac{3\sqrt{3}\,GM}{c^2} \tag{4.44}$$

As $q \to 3\sqrt{3}GM/c^2$, φ_1 diverges, implying that such a photon keeps going round the object at $R = 3GM/c^2$. In the case of a collapsing mass or a highly collapsed object, photons with $q \simeq 3\sqrt{3}GM/c^2$ give rise to rings and multiple images [62, 63]. However, this is not expected to occur in the case of a white hole for the following reasons:

(1) The interesting values of q for white holes are far smaller than $3\sqrt{3}GM/c^2$. For these photons, φ_1 is not very large.

(2) The expanding white hole cannot permit even the photons with $q \gtrsim 3\sqrt{3}GM/c^2$ to circulate for ever because it soon encompasses the region $R = 3GM/c^2$.

CHAPTER 5

MOTION OF A PARTICLE IN THE BACKGROUND OF A WHITE HOLE

5.1 INTRODUCTION

In this Chapter, we shall study the motion of a particle in the background of a white hole, particularly from the viewpoint of its visibility to an observer at infinity, with the hope that the analysis may find application in a number of situations in astrophysics. In what follows, we shall confine our attention to radial timelike geodesics and the photons emitted by particles following these geodesics.

5.2 RADIAL TIMELIKE TRAJECTORIES:

We consider the canonical white hole model of Section (4.2). In its interior, the Friedmann metric applies:

$$ds^2 = c^2 dt^2 - S^2(t)\left[\frac{dr^2}{1-\alpha r^2} + r^2(d\theta^2 + \sin^2\theta\, d\phi^2)\right] \quad (5.1)$$

$$r \leq r_b$$

while in the exterior the Schwarzschild one applies:

$$ds^2 = c^2 dT^2\left(1 - \frac{2GM}{Rc^2}\right) - \frac{dR^2}{\left(1 - \frac{2GM}{Rc^2}\right)} - R^2(d\theta^2 + \sin^2\theta\, d\phi^2) \quad (5.2)$$

The motion of a particle can be studied by solving equations of geodesic motion for the interior (5.1) and for the exterior metric (5.2) matching them at the boundary of the white hole, r_b.

According to Hoyle and Narlikar [81], the Schwarzschild coordinates R and T are related to the comoving r and t through

$$R = r\,S(t)\,,\quad T = \Phi(A)\,,\quad A = \left[\int_{t_b}^{t} \frac{c^2 dt}{S\dot{S}} + \int_{r}^{r_b} \frac{r\,dr}{1-\alpha r^2}\right] \qquad (5.3)$$

such that eq. (5.1) is

$$ds^2 = e^{\nu} dT^2 - e^{\lambda} dR^2 - R^2 (d\theta^2 + \sin^2\theta\, d\varphi^2) \qquad (5.4)$$

where

$$e^{\nu} = \frac{S^2 \dot{S}^2 (1-\alpha r^2)}{\Phi_1^2 c^4 \left(1-\alpha r^2 - \frac{r^2 \dot{S}^2}{c^2}\right)}\,,\quad e^{\lambda} = \frac{1}{\left(1-\alpha r^2 - \frac{r^2\dot{S}^2}{c^2}\right)} \qquad (5.5)$$

$$\Phi_1 = \frac{d\Phi}{dA}.$$

In this Section, we restrict ourselves to a study of radial timelike trajectories. The equation of motion

$$c^2 \frac{d^2 t}{ds^2} + \frac{S\dot{S}}{1-\alpha r^2}\left(\frac{dr}{ds}\right)^2 = 0 \qquad (5.6)$$

solves in view of the metric (5.1) to give

$$c\,\frac{dt}{ds} = \left(1 + \frac{K^2}{S^2}\right)^{1/2} \tag{5.7}$$

K being a constant, a measure of the energy of the particle with respect to the comoving frame of reference, and $S \geq S_1$, the epoch of the commencement of the peculiar motion. Hence, eq. (5.1) suggests

$$\frac{dr}{ds} = \pm\, \frac{K(1-\alpha r^2)^{1/2}}{S^2} \tag{5.8}$$

The three-velocity of a particle, as according to an observer whose instanteneous position coincides with that of the particle at (r, t), is

$$v(t) = (-g_{11})^{1/2}\,\frac{dr/ds}{dt/ds} = \pm\,\frac{Kc}{(K^2+S^2)^{1/2}} \tag{5.9}$$

This shows that v decreases from c at S=0 to $Kc(1 + K^2)^{-1/2}$ at S = 1. For very small K (and for K $\ll S_1$ also), $v(t) \simeq$ Kc/S. Solving eq. (5.9) for K, we get

$$K = \frac{S(v/c)}{(1 - v^2/c^2)^{1/2}} \tag{5.10}$$

For K to be > 1, $v(1 + S^2)^{\frac{1}{2}} > c$. The linear momentum of the particle per unit mass is given as

$$P(S) = \frac{v}{(1 - v^2/c^2)^{1/2}} = \frac{Kc}{S} \tag{5.11}$$

which decreases with expansion. The equation for scale factor, S, describing its evolution with time according to

$$\frac{dS}{dt} = c\left[\frac{\alpha(1-S)}{S}\right]^{1/2}, \tag{5.12}$$

and eq. (5.8) together enable one to study the radial motion of a particle while inside the white hole. As long as the expansion lasts, eq. (5.8) can be used in the form

$$\Sigma_1 = \int_{r_1}^{r_2} \frac{\alpha^{1/2}\,dr}{(1-\alpha r^2)^{1/2}} = k\int_{S_1}^{S_2} \frac{dS}{\left[S(1-S)(k^2+S^2)\right]^{1/2}} \tag{5.13}$$

while the particle is in motion between the epochs S_1 and S_2. For the sake of convenience, one may choose $r_1 = o$ and $r_2 = r_b$ so that

$$\Sigma_1 = \Sigma = \sin^{-1}(\alpha r_b^2)^{1/2} = \text{const} \tag{5.14}$$

The upper limit on the integral on the right hand side of eq. (5.13) would then refer to the epoch of emergence, i.e., when the test particle reaches the white hole boundary. By fixing constant values for S_1, Σ and K, one can find S_2 by iteration. If $S_2 \simeq 1$, the particle reaches the white hole boundary when the expansion is just complete. It can be seen from eq. (5.11) that a particle continues to move along a geodesic, even after expansion is complete once it is set into a peculiar motiön inside the white hole, unless brought to rest by irregular forces like gravitational scattering etc. Eq. (5.13) can be rewritted in terms of the transformation:

$$S = \sin^2\eta \quad , \qquad 0 \leq \eta \leq \pi/2 \tag{5.15}$$

as

$$\Sigma = 2k \int_{\eta_1}^{\eta_2} \frac{d\eta}{(K^2 + \sin^4\eta)^{1/2}} \tag{5.16}$$

In the limits K $>>$ 1 and K $<<$ S_1, its approximations read as

$$\Sigma \simeq \left[2\eta - \frac{1}{K^2}\left(\frac{3\eta}{8} - \frac{3}{8}\sin\eta\cos\eta - \frac{1}{4}\sin^3\eta\cos\eta \right) \right] \Bigg|_{\eta_1}^{\eta_2} \tag{5.17}$$

and

$$\Sigma \simeq \frac{2k \, \sin(\eta_2 - \eta_1)}{\sin\eta_1 \, \sin\eta_2} \tag{5.18}$$

respectively. As K $\to$ (i.e. $v \to c$), $\Sigma \to 2(\eta_2 - \eta_1)$, or

$$\eta_2 \to \frac{\Sigma + 2\eta_1}{2} \tag{5.19}$$

Thus when $\eta_1 < \frac{\Sigma}{2}$, the particle reaches the boundary of the white hole before the latter crosses the event horizon at the epoch $S(t_s) = \sin^2\Sigma$. In the extreme case of $\Sigma = \pi/2$ (expansion complete at the event horizon, the white hole is gray), the particle and the white hole boundary would reach the event horizon simultaneously if $\eta_1 = \pi/4$.

For K $<<$ $\sin^2 \eta_1$, i.e., for a slowly moving particle, η_2 would be quite larger than η_1 and from eq. (5.18), we have

$$\Sigma \simeq 2k \cot \eta_1 \;, \quad \eta_2 \simeq \pi/2 \qquad (5.20)$$

For instance, if $\Sigma = \pi/100$ and K $\simeq$ 0.005, $\eta_1 \simeq$ 18°. In case a particle, set into peculiar motion at an instant when the white hole expansion is nearing completion, or, a slowly moving particle, reaches the boundary of the white hole at epochs later than $\eta_2 = \pi/2$, eq. (5.16) should rather be replaced by

$$\Sigma = \int_{\eta_1}^{\pi/2} \frac{2k\, d\eta}{(k^2 + \sin^4\eta)^{1/2}} + kc\left(\frac{\alpha}{1+k^2}\right)^{1/2} \int_{t_0}^{t'} dt \qquad (5.21)$$

where t_0 refers to the time of explosion from the epoch S = 0 to S = 1 (eq. 4.7). The second part in eq. (5.21) arises by setting $\eta = \pi/2$ in eq. (5.16) and it is assumed that the white hole stays stable after the completion of the expansion, at least for a period t', such that $t_0 < t' \lesssim t_1$ where t_1 is not arbitarily large. In the limit K $\gg$ 1 (which means that $\eta_1 \simeq \eta_2 = \pi/2$),

$$\Sigma \rightarrow 2\left(\frac{\pi}{2} - \eta_1\right) + c\,\alpha^{1/2}\,(t' - t_0) \qquad (5.22)$$

i.e.,

$$t' \rightarrow \frac{2t_0}{\pi}\left(\Sigma + 2\eta_1 - \frac{\pi}{2}\right) \simeq \frac{2t_0}{\pi}\left(\Sigma + \frac{\pi}{2}\right) \qquad (5.23)$$

For the case of a slowly moving particle ($K \ll \sin^2 \eta_1$),

$$t' \simeq \frac{2t_0}{\pi}\left(\Sigma - 2k \cot \eta_1 + \frac{\pi}{2}\right) \qquad (5.24)$$

5.3 THE FREQUENCY SHIFT OF RADIATION

Faulkner et al [9] and Narlikar and Apparao [10] have shown that leakage of radially emitted photons from an exploding surface through the event horizon is possible. The same has been shown in the case of nonradially emitted photons in the previous Chapter. We now ask: can a radiating particle ejected from the boundary of the white hole also be visible before it bursts out of the event horizon?

To answer the question let us consider the situation when a particle ejected with a peculiar speed v in the frame of reference of an observer comoving with the outgoing white hole matter instantaneously happens to be at the boundary of the white hole. Using the Schroedinger formula, we can calculate the frequency shift of a photon radiated by the particle in the radial direction. For this, we focus our attention on an ejection along the line of sight through R = o towards the observer at infinity. Eqs. (5.3) then lead one to

$$\left.\frac{dR}{ds}\right|_{r=r_b} = \left.\left(r\dot{S}\frac{dt}{ds} + S\frac{dr}{ds}\right)\right|_{r=r_b} = \left.\frac{1}{S}\left[(k^2+S^2)^{1/2}\frac{r\dot{S}}{c} + k(1-\alpha r^2)^{1/2}\right]\right|_{r=r_b} \quad (5.25)$$

and

$$\left.\frac{dT}{ds}\right|_{r=r_b} = \left.\left(\frac{\partial T}{\partial t}\frac{dt}{ds} + \frac{\partial T}{\partial r}\frac{dr}{ds}\right)\right|_{r=r_b} = \left.\left[\frac{\Phi_{,t}c}{S\dot{S}}\left(1+\frac{k^2}{S^2}\right)^{1/2} + \frac{\Phi_{,1}kr}{S^2(1-\alpha r^2)}\right]\right|_{r=r_b} \quad (5.26)$$

where $\frac{dR}{ds} = \frac{r_b\dot{S}}{c}$ refers to the white hole boundary. Therefore

$$u^\alpha|_{source} = \left(\frac{dT}{ds}, \frac{dR}{ds}, 0, 0\right), \quad u^\alpha|_{obs} = (1, 0, 0, 0)$$

$$p^\alpha = \left(\frac{dT}{d\lambda}, \frac{dR}{d\lambda}, 0, 0\right), \quad c\frac{dT}{d\lambda} = \frac{\gamma'}{1 - R_S/R} \quad (5.27)$$

$$\frac{dR}{d\lambda} = \gamma', \qquad R_S = \frac{2GM}{c^2}$$

Here λ is an affine parameter, p^α refers to the 4 - momentum of the photon. The Schroedinger's formula

$$1+z = \frac{\nu_0}{\nu} = \frac{\vec{u}\cdot\vec{p}\,|_{source}}{\vec{u}\cdot\vec{p}\,|_{observer}}$$

gives

$$1+z = \frac{\Delta \sin \eta_2}{\sin(\eta_2 + \Sigma)} \quad , \quad \Delta = \frac{\left(k^2 + \sin^4 \eta_2\right)^{1/2} - k}{\sin^2 \eta_2} \qquad (5.28)$$

For K=o, this equation refers to the frequency shift of a photon radially emitted from the white hole surface [10]. For $K << \sin^2 \eta_2$,

$$1+z \simeq \left(1 - k \operatorname{cosec}^2 \eta_2\right) \frac{\sin \eta_2}{\sin(\eta_2 + \Sigma)} \qquad (5.29)$$

whereas for $K >> \sin^2 \eta_2$, $\Delta \simeq \sin^2 \eta_2 / 2k$ and

$$1+z = \frac{\sin^3 \eta_2}{2k \sin(\eta_2 + \Sigma)} \qquad (5.30)$$

This suggests a highly severe frequency blueshift $\left[(1+z)^{-1} \right]$ compared to that for radiation from the white hole surface. The expression (5.28), is well behaved at $R_b = R_s = 2GM/c^2$, i.e., $(1+z)^{-1}$ is finite and larger than that for the white hole radiation.

Ournext step is to consider the ejection of the particle in an arbitary direction. There would always be photons with a certain value of impact parameter q to be able to reach infinity. For a nonradially emitted photon, the Schwarzschild metric and the equations of motion suggest that

$$\frac{dR}{d\lambda} = \gamma' e^{-(\lambda+\nu)/2} F(R,q) \;, \quad F(R,q) = \left(1 - \frac{e^{\nu} q^2}{R^2}\right)^{1/2}$$

$$\frac{d\varphi}{d\lambda} = \frac{h}{R^2} \;, \qquad q = \frac{h}{\gamma}$$

$$p^{\mu} = \left[\frac{dT}{d\lambda} \,,\, \frac{dR}{d\lambda} \,,\, 0 \,,\, \frac{d\varphi}{d\lambda} \right] \qquad (5.31)$$

Here, h is the orbital angular momentum parameter, F = 1 for a radial and F = 0 for a tangential photon. The Schroedinger formula now gives

$$1+z = \left[\left(1+\frac{k^2}{S^2}\right)^{1/2} \frac{\left\{ (1-\alpha r^2)^{1/2} - \frac{r\dot{S}}{c} F \right\}}{1 - R_s/R} + \frac{k}{S} \frac{\left\{ \frac{r\dot{S}}{c} - F\,(1-\alpha r^2)^{1/2} \right\}}{1 - R_s/R} \right]\Bigg|_{r=r_b}$$

$$(5.32)$$

The frequency shift is less severe than that suggested by eq. (5.28). For K = 0 this equation reduces to eq. (4.22) describingthe frequency shift of a nonradial photon emanating from the white hole surface. Eq. (5.32) also is well behaved at $R_b = R_s$. To show this, we let $R_b \rightarrow R_s$, i.e., $F \rightarrow 1 - e^{\nu} q^2/2R_s^{\,2}$, so that

$$1+z\Big|_{R_b \to R_s} \longrightarrow \frac{\Delta}{2\cot\Sigma} + \frac{q^2 \cos\Sigma}{2R_s^2}\left(\Delta + \frac{2k}{\sin^2\eta_2}\right) \qquad (5.33)$$

which is finite. If K $\gg \sin^2\eta_2$,

$$1+z\Big|_{R_b \to R_s} \longrightarrow \frac{k \cot\Sigma \operatorname{cosec}\Sigma \, q^2}{R_s^2} \qquad (5.34)$$

In order that a photon suffer a blueshift, the corresponding limiting impact parameter can be found out by the following considerations: We multiply eq. (5.32) by a factor

$$\left[\left(1+\frac{k^2}{S^2}\right)^{1/2}(1-\alpha r_b^2)^{1/2}+\frac{k}{S}\frac{r_b\dot{S}}{c}+F\left\{\left(1+\frac{k^2}{S^2}\right)^{1/2}\frac{r_b\dot{S}}{c}+(1-\alpha r_b^2)^{1/2}\frac{k}{S}\right\}\right]$$

in the numerator and denominator. The factor $(1-R_S/R_b)$ drops out and consequently

$$1+z=\frac{1+\left[\frac{q}{R_b}\left\{\left(1+\frac{k^2}{S^2}\right)^{1/2}\frac{r_b\dot{S}}{c}+(1-\alpha r_b^2)^{1/2}\frac{k}{S}\right\}\right]^2}{\left[\left(1+\frac{k^2}{S^2}\right)^{1/2}(1-\alpha r_b^2)^{1/2}+\frac{k}{S}\frac{r_b\dot{S}}{c}+F\left\{\left(1+\frac{k^2}{S^2}\right)^{1/2}\frac{r_b\dot{S}}{c}+(1-\alpha r_b^2)^{1/2}\frac{k}{S}\right\}\right]}$$

Now let $R_b \to R_s$. Then $(1-\alpha r_b^2)^{\frac{1}{2}} \to r_b\,\dot{S}/c$ and $F \to 1$ so that

$$1+z=\frac{1+\left[\frac{q}{R_b}(1-\alpha r_b^2)^{1/2}\left\{\left(1+\frac{k^2}{S^2}\right)^{1/2}+\frac{k}{S}\right\}\right]^2}{2\,(1-\alpha r_b^2)^{1/2}\left[\left(1+\frac{k^2}{S^2}\right)^{1/2}+\frac{k}{S}\right]}$$

For $S \ll 1$, $1-\alpha r_b^2 \simeq \alpha\, r_b^2/S$. Hence requiring $(1+Z)^{-1} \geqslant 1$, we are led to

$$q^2 \lesssim \frac{S_2}{k}\, r_b^2\, S_2^2 \qquad (5.35)$$

This is considerably smaller than that for a nonradial photon emitted from the white hole surface ($K = 0$). When $K/S_2 << 1$, we have the limiting impact parameter as

$$q^2 \lesssim \left(1 - \frac{2K}{S_2}\right) r_b^2 S_2^2 \qquad (5.36)$$

This is about the same as that for a nonradial photon emitted from the white hole surface ($q \lesssim r_b S$). Eqs. (5.35 - 5.36) apply only to the epoch when the particle just leaves the white hole boundary. Once the particle has left the surface $r = r_b$, Schwarzschild geometry governs its propagation.

If K is positive definite, the frequency shift in the radiation from the white hole and the particle would be different. In the exterior of the white hole, the Schwarzschild metric suggests the frequency shift of radiation from the particle to be

$$1+z \simeq \left(\gamma + \frac{dR}{ds}\cos\theta\right)^{-1}, \quad \frac{dR}{ds} = \left[\gamma^2 - \left(1 - \frac{R_s}{R}\right)\right]^{1/2} \qquad (5.37)$$

for ejection at an angle θ with respect to the line of sight through $R = 0$. Here γ, the energy of the particle per unit rest mass as measured as infinity, can be evaluated with reference to its position when instanteneously coincident with the white hole surface. If we compare the last equation with eq. (5.25), we get

$$\gamma(\eta_2) = \frac{1}{\sin^2\eta_2}\left[(k^2+\sin^4\eta_2)^{1/2}\cos\Sigma + k\sin\Sigma\cot\eta_2\right] \tag{5.38}$$

For K $\gg$ Sin^2 η_2,

$$\gamma(\eta_2) \simeq \frac{k}{\sin^2\eta_2} \quad \frac{\sin(\eta_2+\Sigma)}{\sin\eta_2} \gg 1 \tag{5.39}$$

Eq. (5.37) would then suggest

$$(1+z)^{-1} \simeq \gamma(1+\cos\theta) = \text{const} \tag{5.40}$$

and from Liouville's theorem, it can be seen that intensity enhancement takes place when

$$\cos\theta > \left(\frac{1}{\gamma} - 1\right) \tag{5.41}$$

In case the particle is ejected at an epoch $S_2 \simeq 1$, or after the completion of the expansion of the white hole,

$$\gamma\left(\frac{\pi}{2}\right) \simeq (1+k^2)^{1/2}\cos\Sigma \tag{5.42}$$

Hence, in general,

$$\gamma(\eta_2) > \gamma(\pi/2) \tag{5.43}$$

If one considers a simple geometry, the speed of separation between the particle and the white hole can be shown to be superluminal for $\gamma \gtrsim 1$ [67]. In principle, the velocity of ejection of a particle from a white hole should be $0 \lesssim v \lesssim c$; for ejection during the expansion phase of the white hole, this condition can be easily achieved.

PART III

PART III

CHAPTER 6

CONCLUSIONS

In conclusion, we summarize the main results and implications of our investigations on some astrophysical aspects of black holes and white holes, especially from the view point of their ability to serve as possible models for the bizarre activities going on in quasars, extragalactic radio sources and related objects.

The justifications for invoking supermassive black holes are:

(1) The formation of a black hole could be a natural phenomenon in galactic nuclei and quasars;

(2) Small size ensures rapid brightness variations, a feature common to many quasars and related objects;

(3) The strong gravitational field of a black hole ensures tremendous amount of energy release ($0.06 - 0.42\ mc^2$).

However, there are some difficulties also. First of all, mass loss from galactic nuclei ($\sim 10 - 100\ M_\odot\ yr^{-1}$) is not explained by a black hole model. The radiation must fluctuate with a period $\sim R_s/c$ ($\sim 10^{-5}$ $M/M_\odot$ sec). In some quasars, such as 3C 273, the fluctuations are not seen. Most of the radiation in black hole models is thermal and produced by gas and dust whereas the radiation from most of the quasars is nonthermal (synchrotron and / or inverse Compton) in nature. On the other hand, the white hole models have got difficulties of their own; for example, as a source of high energy radiation a white hole is too short lived.

The study of the properties of the gravitational searchlight, arising from the non-Euclidean nature of the geometry of spacetime around a highly collapsed object has revealed some interesting features, an important one among them being the ν^{-1} form of the spectrum which is purely of geometrical origin. The emission from the ring of particles around the black hole could possibly have a thermal origin in a narrow frequency range which progressively spreads over a wider frequency band as ϵ decreases. The usual objection that the thermal emission spectrum does not give the observed $\nu^{-\alpha}$ ($\alpha \approx 1$) dependence clearly is not applicable to the model that we have studied. But it is not quite free from difficulties. The orbits about a black hole between $6GM/c^2$ and $3GM/c^2$ are unstable. In the astrophysical situations of interest, unless the particle is injected at relativistic speeds into the unstable circular orbits near $3GM/c^2$, such a mechanism would not be plausible [78].

However, the searchlight effect exists even for a source radiating for an infinitesimally small duration of the coordinate time. One such possible scenario is offered by the black hole model of the quasars ejected from the galaxies as discussed in the Chapter 3. Here, the gas to be injected into high energy orbits is supplied by the stellar collisions and tidal interactions.

We have proposed a model of quasars based on gravitational recoil of a supermassive black hole from a galactic nucleus. The black hole is rendered luminous by the gaseous material provided by stars captured on its way out in violent physical processes. This model is consistent with similar quasar models recently proposed by others [84], and with Arp's observations of quasar-galaxy pairings. Arp's hypothesis of the ejection of quasars from galaxies is supported by the presence of disturbances in the inner regions of such galaxies pointing roughly in the direction of the quasar and the photographic evidence of the tidal interactions between the quasar and the galaxy. In our model, the recoiling supermassive black hole can give rise to such disturbances and bridges as a consequence of its tidal interaction with the galaxy. The only major drawback of the hypothesis remains in resolving satisfactorily the discordant redshifts seen in the spectra of the quasars associated with galaxies. The anomaly can be only partly accounted for by considering the contributions from the Doppler effect and the gravitational redshift to the usual cosmological redshift, common to both the quasar and the galaxy.

The failed recoil of the supermassive black hole can produce some interesting observable features such as large velocity dispersions and disturbances in the inner regions of the galaxy. Therefore, a detailed study of the structural changes in the galaxy would itself be an interesting problem.

In Part II of the thesis, we have presented a study of the white hole. In Chapter 4, employing the geometrical optics approximation we have shown that, like the radial photons, the nonradial ones emitted from the white hole surface can also emerge from within the Schwarzschild barrier and reach a distant observer, although the amount of the frequency blueshift in this case is relatively smaller. However, it is still possible to have photons blueshifted in the early stages of the white hole expansion, provided their impact parameter is smaller than the Schwarzschild radial coordinate of the radiating surface at the time of emission. Assuming that only blueshifted photons are seen by a remote observer, the apparent angular size of the white hole is found to grow so fast in the early stages that it gives the impression of a superluminal expansion.

Although this example appears to point out a likely explanation of the phenomenon of superluminal separation observed in some radio sources, it suffers from a major drawback, viz., this happens only for an insignificant part of the early stages of a white hole's life. In order that a white hole (one that would be observable today) be stable, it has to come into existence with essentially zero delay, i.e., practically simultaneously with the Big Bang [66]. It would therefore not be possible to observe such a superluminal expansion today [92].

In Chapter 5, we have studied the motion of a particle in the background of a white hole, especially from the viewpoint of its visibility while still inside the Schwarzschild barrier. We have shown that photons, radial as well as nonradial, emitted by a particle ejected from the white hole surface can also leak through the event horizon and reach a distant observer. The frequency blueshifts are more severe compared to those of radial and nonradial photons from the white hole surface itself. Therefore, the emergence of an ejectum carrying an insignificant fraction of the white hole mass can produce an apparent intensity enhancement. Using a simple geometry, the speed of separation of the particle from the white hole can be shown to be superluminal if γ (energy per unit rest mass as measured at infinity) $\gtrsim 1$. Although, in the case of a particle ejection during the expansion phase of a white hole this condition can be easily achieved, for large ejecta masses, our analysis would be inadequate; an N-body treatment of the problem is called for. The astrophysical significance of these possibilities is therefore ambiguous at present.

On the basis of the present investigations, we feel that the models for quasars and extragalactic radio sources involving black holes merit serious consideration. The viability of white holes to serve as possible models for certain high energy phenomena in the universe is rather controversial at present; although our results concerning white holes do not make the situation any better, one need not consider the subject of white holes dead. The subject of white holes remains largely unexplored and only the idealizations of a white hole explosion have been studied to some extent. The studies should be pursued, particularly in view of the sufficient evidence for exploding phenomena in the universe, irrespective of the various implications in the white hole theory seen at present. We hope that a detailed study of the astrophysical aspects of black holes and white holes will in the near future shed light on hitherto unexplained phenomena in the universe.

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