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The Solar Cycle and the Associated Behaviours of Sunspots and Prominences

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ABSTRACT

The paper deals principally with two specific problems of solar physics, *viz.* (a) the equatorward drift of sunspots according to the Carrington-Spoerer law and (b) the general poleward movement of prominences recently established by M. and Mme L. d' Azambuja. On the basis of his theory of magneto-hydrodynamical waves Alfvén has attempted to explain the above phenomena, but his explanations are much too complex and, in any case, not really satisfactory. In fact, there exists no altogether convincing theory capable of accounting for these two and a number of other related solar phenomena; in the present state of affairs, therefore, it is permissible to construct new models and new theories which seem promising.

The present paper aims to show that the two afore-mentioned phenomena admit of a fairly simple explanation on a theory based upon straightforward classical dynamics. The purely dynamical considerations here presented lead to the conclusion that on the photosphere there ought to exist a resultant acceleration directed from either pole towards the equator, while on the chromosphere a resultant acceleration directed from the equator towards the poles should be expected to occur. The magnitudes of these accelerations at different latitudes on the sun are calculated by using the well-established values of angular velocity at the corresponding heliographic latitudes according to measurements made at the Greenwich Observatory; it is shown that the observed rates of equatorward drift of sunspots and of poleward motion of prominences at different heliographic latitudes are not inconsistent with the corresponding velocities derived from the theory here proposed. With the help of the two oppositely directed accelerations at the two levels and the general dynamical mechanism advocated by the author in earlier papers dealing with a variety of solar problems a broad qualitative explanation is also suggested of the formation of bipolar sunspots, of the simultaneous occurrence of opposite magnetic polarities in sunspots of the northern and southern hemispheres, and of the 22-year cycle of reversal of the magnetic polarities of sunspots. From the same dynamical mechanism it appears quite natural that sunspots should be associated with photospheric faculae and chromospheric flocculi. The peculiar motion of gases, known as the Evershed-effect, from the umbral region to the periphery of a sunspot parallel to the photospheric surface also becomes intelligible on the basis of this mechanism.

INTRODUCTION

Although as early as 1610 Fabricius recognised the sunspots to be truly solar phenomana, their systematic visual observation with sufficient precision began only during the earliest years of the nineteenth century; and it was from 1858 that John Herschel & Warren de La Rue commenced regular photography of the solar disk at the Kew Observatory in England. All these and other later observations carried out at various international observatories over a period of 100 to 150 years have established a number of important statistical conclusions concerning the behaviour of sunspots; but despite the several attempts made during the last fifty years or more there exists to this day no theory which can explain really satisfactorily how sunspots are formed and why they behave as they do.

Of all the existing theories of sunspots the one that attempts to encompass the largest number of observational results is due to V. Bjerknes (1926). Bjerknes's theory is based upon classical dynamics or more specially that branch of it which deals with the motion of fluids. This very ingenious theory postulates the existence of circulatory currents between latitude about 40° N or S and the equator, the flow being directed from the higher latitudes towards the equator near the upper boundary of the photosphere and in the opposite direction deeper down. This certainly offers a fairly plausible explanation of the drift of sunspots towards the equator in the course of the solar cycle, but Bjerknes's postulate does not explain why all prominences, regardless of the latitudes of their origin, move towards the poles, as has been clearly established recently by M and Mme L.d' Azambuja (1948); also, on Bjerknes's theory it is not easy to see why the equatorward velocity of the sunspot belts should

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decrease with the decrease of latitude. In agreement with Hale's (1908) mechanism of the origin of sunspot magnetism Bjerknes's theory tacitly assumes that the magnetic field in sunspots arises from the rotation of electrostatic charges in the sunspot whirl. In order to explain the occurrence of bipolar spots, of sunspots with opposite polarities in the northern and southern hemispheres and the observed 22-year cycle of reversal of magnetic polarity in sunspots Bjerknes has however to make new postulates which seem rather artificial and are, moreover, incapable of ever being subjected to observational test.

In Bjerknes's hydrodynamical theory the existence of a rotatory motion in a sunspot is an inescapable necessity. It is however to be noted that uptill now no absolutely sure evidence of the existence of such whirling motion has been detected in all sunspots; this has led some recent theorists, notably Alfvén, to question the occurrence of such motion and to regard the magnetic field of a sunspot to be its most fundamental characteristic which must therefore arise from some entirely different mechanism. While it must be admitted that the most important observed property of a sunspot is its magnetic field, it does not seem to the present writer that the fact that rotary motion has so far not been observed as an invariable concomitant of sunspots is sufficient proof that sunspots are indeed not whirls even in the deeper layers completely inaccessible to observation*. In fact, the methods of observation usually employed are incapable of detecting rotatory motion even in the observable parts of a sunspot; for, at the time when the deepest parts of a spot are accessible to observation, *viz.* when the spot is at or near the central meridian, the rotary motion is practically at right angles to the line of sight and therefore undetectable by Doppler shift, while at the times when the spot is seen edgewise, *viz.* when it is sufficiently far away from the central meridian the extremely superficial parts alone of a spot are accessible to observation. It therefore appears still quite justified to proceed on the belief that sunspots may well be likened to hydrodynamic circular vortices in which the magnetic field arises from some mechanism similar to that imagined by Hale; at any rate, there is no less reason for this belief than there is in support of a mechanism which requires the magnetism of sunspots to be brought up from a supposed highly magnetised central core of the sun.

Apart from Bjerknes's theory, there is at present another theory of sunspots originated by H. Alfvén (1943) which enjoys a considerable amount of popularity among some theorists. This theory, which attributes the magnetism of sunspots to magneto-hydrodynamical waves reaching the sun's surface from an unstable and highly magnetised central core, attempts to explain the equatorward drift of sunspots and the poleward motion of prominences of latitudes outside the sunspot belts (Alfvén 1954) as the progression of magneto-hydrodynamic waves along the lines of force of the general surface magnetic field of the sun. Alfvén's theory however meets with even greater difficulty than Bjerknes's in explaining why prominences of the equatorial regions also move from the equator towards the poles. It is, furthermore, extremely difficult to understand how the hydromagnetic waves travel along the lines of force of the surface general magnetic field when observations of very high precision made during the last several years have progressively reduced the general magnetic field on the surface of the sun to practically zero. It therefore seems that, in spite of the unquestionable importance of the new concept of magneto-hydrodynamical waves, we have to admit that the equatorward motion of sunspots and the general poleward motion of prominences are by no means satisfactorily explained by Alfvén's theory of magneto-hydrodynamic waves. In fact, many peculiarities of the motions and constitutions of sunspots and prominences seem to point to some underlying mechanism quite distinct from that which the properties of electric and magnetic particles in an electro-magnetic field would require.

Equatorward Motion of Sunspots

Let us consider a large (astronomically speaking) mass of incompressible fluid in equilibrium under its own gravitation. Such a mass will have a spherical shape. Now let this spherical fluid mass be given a constant angular velocity around a fixed central axis. It is well-known in hydrodynamics that as a result of the rotation the form of the free surface of this fluid mass will become an oblate spheroid with an equatorial bulge and flattening at the poles. The equilibrium figure of the fluid mass will be such that, in each hemisphere, along the surface the component of gravity towards the pole is exactly balanced by the equatorward component of centrifugal force at every point. But if, due to some additional mechanism whose nature is yet unknown†, the mass retains its spherical shape inspite of the rotation, then evidently there will be an uncompensated component of the centrifugal force along the surface of the sphere which will be directed from the pole towards the equator. This is easily seen from Fig. 1. At any point M on the surface, the magnitude of this equatorward acceleration (linear) will be $\omega^2 R \sin\varphi . \cos\varphi$ where φ=latitude of the point M, OM=R, MB=r, ω=angular velocity, PP′=axis of rotation and EQ=equator.

*Mr. Evershed did indeed observe a mean rotational velocity of 0·35 km/sec. in the lower chromosphere above sunspots, and this rotational velocity, according to his observations, changed sign in passing from the northern to the southern hemisphere.

†The flattening at the poles might conceivably be counteracted by a higher temperature at the poles than at the equator (see A. K. Das and K. D. Abhyankar, 1955).

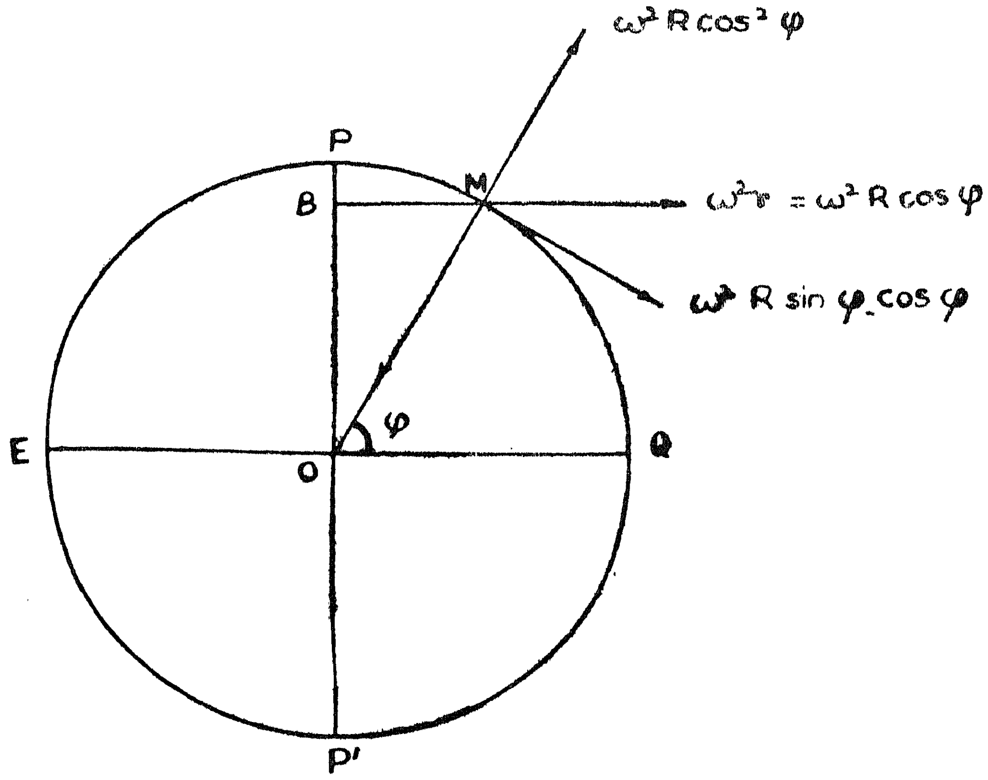

Fig. 1. (Photosphere)

The existence of the uncompensated equatorward force is therefore inevitable so long as the figure of the fluid mass is spherical; and it will be so even if the angular velocity ω varies from latitude to latitude, although the magnitude ω^2 R $\sin\varphi$ $\cos\varphi$ of the force will in that case vary with latitude in a slightly different manner.

Now we may identify our rotating spherical fluid mass with the sun, provided the variation of angular velocity with latitude is taken to be precisely the same on its surface as is observed on the sun and provided it is permissible to regard the sun as a sphere. So far as the sun's photosphere is concerned, there are very many measurements of its radius in different directions; and in spite of the difficulties inherent in the measurements at present available there exists no serious reason for suspecting that the photosphere is not perfectly spherical. It is likely that the method of photographing the sun from sounding balloons at great heights recently developed both in Europe and in America will eventually yield more accurate measures of the photospheric radius than are currently available; but there is sufficient justification for taking the radical view that even such measures will also confirm the spherical shape of the photosphere. We therefore accept the spherical shape of the photosphere as an observational result, although the cause of this strange phenomenon is admittedly obscure and perhaps as ill-understood as the cause of the curious law of variation of angular velocity with latitude which the sun's surface exhibits.

We suggest that the equatorward acceleration ω^2 R $\sin\varphi$ $\cos\varphi$ is responsible for the equatorward drift of sunspot belts in the course of the solar cycle. The numerical values of the acceleration at various latitudes on the photosphere of radius R can easily be computed by using Carrington's formula $\xi = 14^\circ.37 - 2^\circ.60 \sin^2\varphi$, in which ξ= distance in degrees traversed in one day$=\omega \times \frac{180}{\pi} \times 24 \times 60 \times 60$. These values are given in column 3 of Table I. It will be noticed that the equatorward acceleration has a maximum value at latitude 40° and

TABLE I

Latitude (in degrees)	ξ (in degrees)	ω^2 R sin φ cos φ (cm/sec.^{2})
0	14·37	0
10	14·29	0·099
20	14·06	0·180
30	13·70	0·231
40	13·27	0·246
50	12·80	0·229
60	12·37	0·188
70	12·01	0·131
80	11·78	0·067
90	11·70	0

is zero at the pole and at the equator. For comparison with observational quantities it is however convenient to derive the theoretical velocities for the various latitudes from the relation

$$\frac{d^2\varphi}{dt^2} = \omega^2 \sin\varphi \cos\varphi = \tfrac{1}{2}\,\omega^2 \sin 2\varphi$$
$$= \tfrac{1}{2}\,(a - b\sin^2\varphi)^2 \sin 2\varphi$$
$$= \tfrac{1}{2}\,(a^2 + b^2\sin^4\varphi - 2ab\sin^2\varphi)\sin 2\varphi$$

Multiplying both sides of this relation by $2\frac{d\varphi}{dt}$ and integrating we obtain

$$\int 2\frac{d\varphi}{dt}\cdot\frac{d^2\varphi}{dt^2}\,dt = \int\left(a^2\sin 2\varphi.\frac{d\varphi}{dt} + b^2\sin^4\varphi.\,\sin 2\varphi.\frac{d\varphi}{dt} - 2\,ab\sin^2\varphi.\,\sin 2\varphi.\frac{d\varphi}{dt}\right)dt$$

or $$\left(\frac{d\varphi}{dt}\right)^2 = -\frac{a^2}{2}\cos 2\varphi + \frac{b^2}{3}\sin^6\varphi - ab\sin^4\varphi + \text{Const.}$$

It is known from observations that sunspots do not cross the equator; therefore we put $d\varphi/dt=0$ at $\varphi=0$ and obtain the const. of integration$=a^2/2$. We finally have

$$\left(\frac{d\varphi}{dt}\right)^2 = \frac{a^2}{2}(1-\cos 2\varphi) - b\sin^4\varphi(\omega + \frac{2b}{3}\sin^2\varphi), \qquad \dots\dots\dots(1)$$

where $$a = \frac{14^\circ.37\times\pi}{180\times24\times60\times60} = 2.901\times10^{-6}$$

and $$b = \frac{2^\circ.60\times\pi}{180\times24\times60\times60} = 5.25\times10^{-7}$$

We have used the relation (1) for computing the theoretical values of the velocities corresponding to a number of latitudes for which the velocities of drift of sunspot belts are available from observation. Table II gives these theoretical velocities alongside the velocities derived by Gleissberg (1944) from Waldmeier's diagram which gives the average heliographic latitudes of sunspots against the time reckoned from the epoch of the maximum for a cycle whose maximum is of medium height.

TABLE II

Latitude (degrees)	Velocity (cm/sec.) $\times 10^{-4}$ (Theoretical)	Velocity cm./(sec) $\times 10^{-2}$ (Observed)	Velocity (Theoretical) / Velocity (Observed)
26	8·841	1·475	599·2
24	8·206	1·291	635·6
22	7·560	1·291	585·6
20	6·902	1·148	601·5
18	6·236	1·033	603·9
16	5·563	0·939	592·6
14	4·883	0·795	614·6
12	4·197	0·689	609·5
10	3·504	0·574	610·6

It is evident that the theoretical velocity varies with latitude in pretty closely the same manner as the observed velocity of drift of the sunspot belts. But the theoretical velocity at every latitude is approximately 600 times larger than the corresponding observed velocity, which is not surprising since the theoretical velocity has been computed without taking any account of the effects of frictional resistance. It is to be noted that the observed velocities are not the actual velocities of translation of individual spots which are always short-lived compared to the duration of the solar cycle, but are, in a sense, 'local terminal' velocities of the zones in which spots appear. They are, like the theoretical equatorward velocities and equatorward accelerations for the various latitudes, characteristic of the parallels of latitudes. These velocities are to be considered as equilibrium velocities under conditions in which all forces are balanced: for instance, suppose (see Fig. 2) that an acceleration $G(=\omega^2 R \sin\varphi . \cos\varphi)$ acting in the direction $\overrightarrow{MG}$ on a particle at M (lat. φ) is responsible for producing a velocity towards the equator of the sun. Then because of Coriolis force due to the sun's rotation the particle will actually

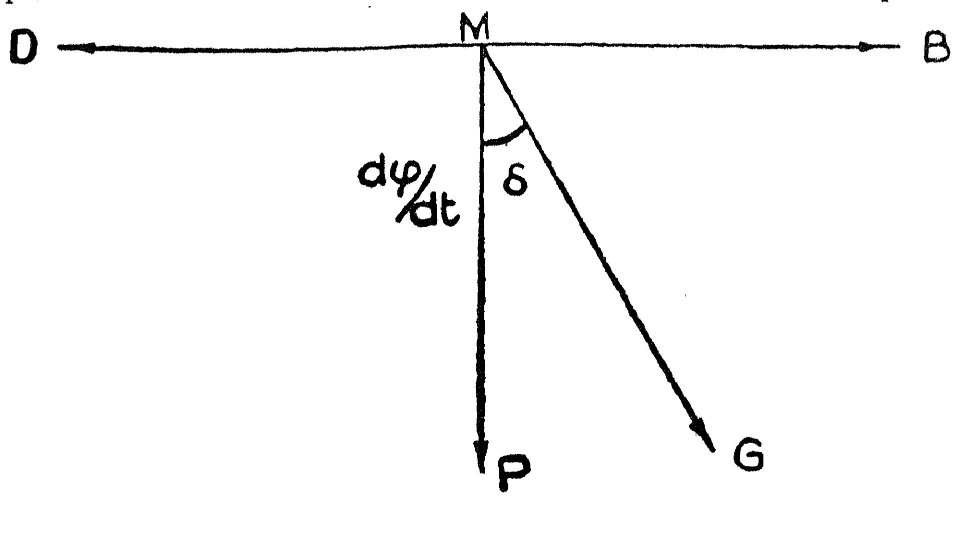

Fig. 2.

move in the direction $\overrightarrow{MP}$ with the velocity $\frac{d\varphi}{dt}$ making an angle δ with $\overrightarrow{MG}$. Therefore under equilibrium conditions when all forces are balanced we shall have

$$\omega^2 R \sin\varphi \cos\varphi . \sin\delta = 2\omega \sin\varphi . \frac{d\varphi}{dt}$$

and $$\omega^2 R \sin\varphi . \cos\varphi . \cos\delta = f . \frac{d\varphi}{dt},$$

where $2\omega \sin\varphi . \frac{d\varphi}{dt}$ = Coriolis force in the direction $\overrightarrow{MD}$, which is balanced by the component of G in the direction $\overrightarrow{MB}$, $f.\frac{d\varphi}{dt}$ is the frictional resistance which balances the component of G in the direction $\overrightarrow{MP}$, f being the coefficient of friction, and $d\varphi/dt$ is the equilibrium velocity. The above realtions, when φ varies over a small range, can be written also in the following forms:

$$\frac{dt}{d\varphi} . \sin 2\varphi = \frac{4}{R} . \frac{\sin\varphi}{\omega \sin\delta} = \text{Const. (approx.)}$$

and $$\frac{dt}{d\varphi} . \sin 2\varphi = \frac{2f}{\omega^2 R \cos\delta} = \text{Const. (approx).}$$

It will be noticed that the above relations are precisely of the same form as what Gleissberg (1944), in the note referred to earlier, found empirically. Our method of derivation of the same relation however shows that the approximate constancy of $\frac{dt}{d\varphi} \cdot \sin 2\varphi$ is not "merely accidental", but indeed has a "real significance"; our approach to this ill-understood problem of solar physics also brings out what may be the true nature of the interdependence of the equatorward drift of sunspot zones and the so-called "polar retardation" of solar rotation.

Poleward Drift of Prominences

An examination of the statistics of prominence observations, such as those that have so far been published for more than half a century by the Kodaikanal Observatory, shows that the distribution of prominence activity in latitude is not quite the same as that of sunspot activity. Instead of one belt of activity on either side of the equator usually found in the case of spots there are most often two regions of prominence activity in each hemisphere; one of these corresponds closely (though slightly higher up in latitude) with the zone of spot activity at the time, while the other occurs at a high latitude approximately between 50° and 80°, the belt appearing at the highest latitude slightly earlier than the maximum phase of the solar cycle. It has therefore become usual to classify the prominences (—these naturally include also the absorption markings or filaments on the disk—) into two groups, *viz.*, equatorial and polar. Until some ten years ago it was generally believed that the so-called equatorial prominences moved from about latitude 40° towards the equator in the course of the solar cycle in practically the same way as the spots do, whereas the polar prominences moved in the opposite sense towards the poles. But recently (1948) M and Mme L.d' Azambuja have, from their analysis of a fairly large amount of observational data on filaments, concluded that there is no real distinction between the equatorial and the polar filaments (or prominences);—both move in the same direction, namely from the equator towards the pole, during the progress of the 11-year solar cycle. The same conclusion was reached, though with less certitude, earlier by W. Moss (1929) from an analysis of limb prominences. Although the general movement of all prominences in the direction opposite to that of the drift of sunspots is well-established by the work of the d' Azambujas, the rates of movement of prominences at different latitudes do not appear to be determined so reliably. To our knowledge, there is at present no theory which attempts to explain this systematic poleward drift of prominences.

In the following parts of this section we discuss a possible mechanism, based entirely upon dynamical (or rather hydrodynamical) considerations, which appears to be capable of accounting for the afore-mentioned behaviour of prominences. It is fundamentally the same process as discussed in the previous section; it has the merit of being very simple and capable, in principle, of being subjected to observational test at least in some of its salient aspects. We again consider the sun as a rotating fluid mass, the dense outermost part of which, namely the photosphere, for some unknown reason retains its spherical shape; but we suppose that the thin mantle of gas *i.e.*, the chromosphere which surrounds the photosphere behaves normally according to the laws of hydrodynamics and assumes the shape appropriate to the sun's rotation. This assumption is permissible according to Clairaut's Theorem; also, there is nothing to contradict this hypothesis* so far as observations, at present available,

*A sufficiently long series of photographs of the chromosphere taken, especially from great altitudes above the earth's surface, with instruments like the Lyot monochromatic heliograph seems capable of giving a clear verdict for or against this hypothesis.

are concerned. Referring back to Fig. 1 we see that the gravitational potential at M is given by $V = -\frac{\text{Const.}}{R}$

so that the gravitational attraction per unit mass (*i.e* acceleration due to gravity) at M is $g = \frac{dV}{dR} = -\frac{C}{R^2}$.

For the evaluation of the constant let us assume that at some distance R_o from the centre of the mass the acceleration due to gravity is g_o, so that $g_o = -\frac{C}{R_o^2}$. Then the gravitational potential at any point M can, in general, be written $V = -\frac{g_o R_o^2}{R}$.

From this, the form of the free surface of the fluid mass is given by the equation

$$\tfrac{1}{2}\,\omega^2 r^2 + \frac{g_o R_o^2}{R} = \text{Const.}$$

$$\text{or} \quad \tfrac{1}{2}\,\omega^2 r^2 = g_o R_o^2 \left(\frac{1}{R_o} - \frac{1}{R}\right),$$

if we put $R = R_o$ at that point of the surface where $r = 0$, that is, at the pole. Now since $r = R\cos\varphi$, the equation of the free surface becomes

$$\frac{1}{R} = \frac{1}{R_o} - \frac{\omega^2}{2g_o R_o^2}\,.\,R^2 \cos^2\varphi,$$

which for small values of ω can be written as follows :

$$R = R_o \left(1 + \frac{\omega^2 R_o}{2g_o}\,.\cos^2\varphi\right). \quad \text{......................}(2)$$

This represents, to a first approximation, a pole-flattened spheroid when ω is constant over the whole surface. On this equilibrium surface the component of gravity towards the pole is exactly balanced by the component of centrifugal force towards the equator, so that there is no resultant acceleration either polewards or equatorwards. But on the photosphere of the sun, as we know it, the angular velocity varies with latitude, and in fact, decreases with increasing latitude. Recent observations have also shown that all solar layers accessible to direct observation, including the photosphere which carries the sunspots and the chromosphere which carries the prominences, have practically the same angular velocity at the equator and the same polar retardation. Therefore we conclude

TABLE III

Latitude (degrees)	$(R-R_o) \times 10^{-4}$ cms.	
	† ω = constant	ω varying with φ
0	4·989	7·526
10	4·840	7·221
20	4·406	6·362
30	3·741	5·130
40	2·927	3·767
50	2·061	2·467
60	1·248	1·394
70	0·583	0·615
80	0·150	0·153
90	0	0

†The constant value of ω is that at the pole.

that the chromosphere cannot have a figure of equilibrium for which the poleward and equatorward accelerations are exactly balanced. In fact, the actual chromosphere with its polar retardation of angular velocity ought to depart from the figure of equilibrium in such a way that there is an increased pole-flattening; this can be clearly seen from Table III which gives the values, at various latitudes, of $R-R_0$ computed from equation (2) both for ω=constant and for ω varying with latitude according to Carrington's formula. The result of the increased flattening at the poles is of course the occurrence of a net acceleration directed towards the poles at all latitudes. The magnitude of the resultant poleward acceleration and its variation with latitude can be determined by the following simple procedure. In Fig. 3 let O be the centre of the sun and OP the axis of rotation. Then at the point M of the chromosphere the centrifugal force acts in the direction $\overrightarrow{BMF}$ and gravity in the direction

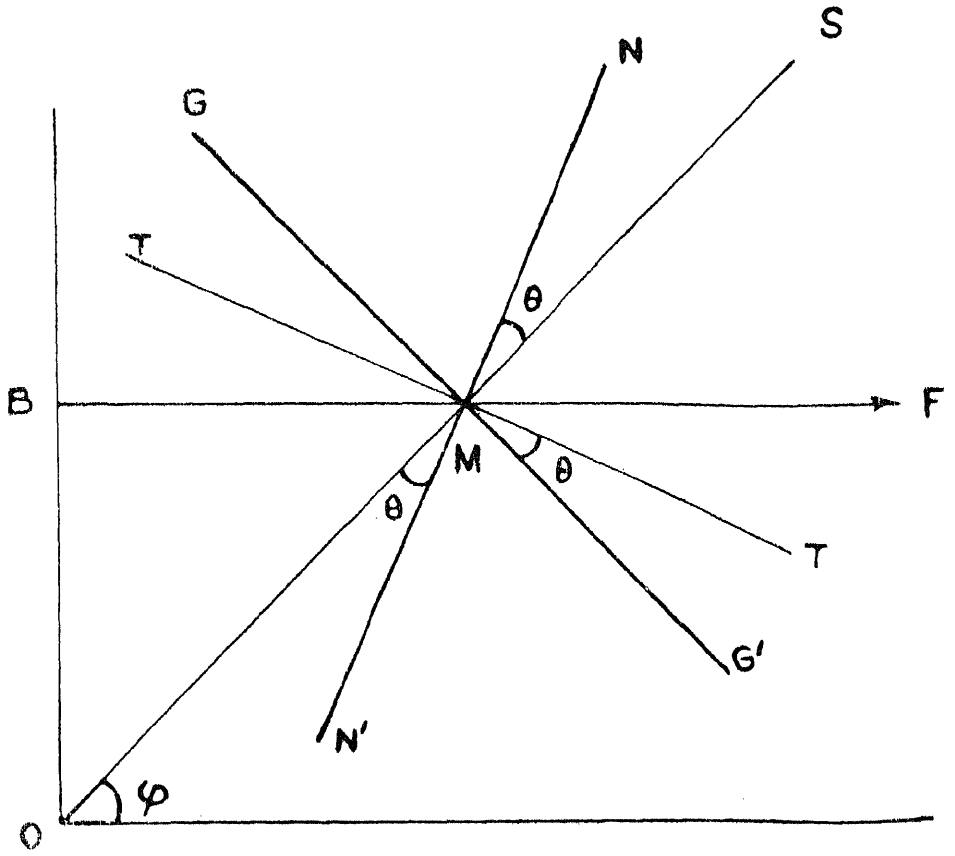

Fig. 3. (Chromosphere)

$\overrightarrow{SMO}$. Now, if TT' is the tangent to the surface of the actual chromosphere at M, NN' is perpendicular to TT' and GG' is perpendicular to SMO, then the component of gravity (g) along TT' towards the pole is g sin θ and the component of the centrifugal force ($\omega^2 R \cos\varphi$) along TT' towards the equator is $\omega^2 R. \cos\varphi. \sin(\varphi+\theta)$. Therefore, the net force towards the pole is given by

$$
\begin{aligned}
F_{pole} &= g \sin\theta - \omega^2 R \cos\varphi. \sin(\varphi+\theta) \\
&= g \sin\theta - \omega^2 R \sin\varphi. \cos\varphi. \cos\theta - \omega^2 R \cos^2\varphi. \sin\theta \\
&= g\,\theta - \omega^2 R \sin\varphi. \cos\varphi - \omega^2 R \cos^2\varphi. \theta,
\end{aligned}
$$

as θ is a small angle. θ can be estimated as follows: Let A and B be two points at latitudes φ and $\varphi+d\varphi$ on the chromosphere and let R and R+dR be the corresponding radii (Fig. 4). Join AB and produce it to J. and draw AX perpendicular to OA. Then, to a first approximation

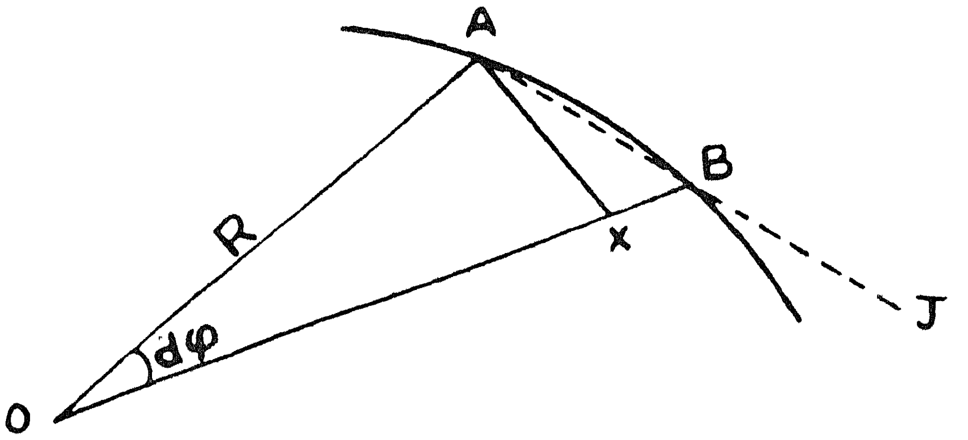

Fig. 4.

BX=dR and AB=R.dφ. The small angle between AX and ABJ can therefore be taken to be dR/Rdφ; when A approaches B indefinitely, this angle becomes, in the limit, equal to θ(See Fig. 3). Therefore $\theta = \frac{dR}{Rd\varphi}$ and accordingly

$$F_{pole} = g.\frac{dR}{Rd\varphi} - \omega^2 R \sin\varphi.\cos\varphi - \omega^2 R \cos^2\varphi.\ \frac{dR}{R.d\varphi}$$

$$= \frac{1}{R}.\frac{dR}{d\varphi}.g - \omega^2 R \sin\varphi.\cos\varphi - \omega^2 \cos^2\varphi.\frac{dR}{d\varphi}\cdots\cdots\cdots(3)$$

For the numerical evaluation of the poleward acceleration for different latitude ranges from equation (3) we may take g and R to have, to a first approximation, constant values over the whole chromosphere, namely $g=2.745\times10^4$ cm/sec.^{2} and $R=7.028\times10^{10}$ cms; and dR/dφ can be obtained either graphically from a curve

TABLE IV

Latitude Range (degrees)	$\frac{dR}{R\varphi}\times10^{-5}$ (cm/degree)	$\omega^2 R \sin\varphi.\cos\varphi$	$\omega^2\cos^2\varphi.\frac{dR}{d\varphi}\times10^5$	$\frac{g}{R}\times\frac{dR}{d\varphi}$	F_{Pole} (cm/sec.^{2})
0—10	0·0305	0·05	0·622	0·069	0·019
10—20	0859	14	1·613	194	0·056
20—30	1222	21	1·939	276	0·066
30—40	1363	·24	1·677	307	0·067
40—50	1300	24	1·121	293	0·053
50—60	1073	20	0·427	242	0·042
60—70	0779	·16	0·215	176	0·016
70—80	0462	10	0·049	104	0·004
80—90	0152	03	0·003	034	0·004

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of $R-R_0$ against φ prepared with the help of columns 1 and 3 of Table III, or by calculation from the equation (2) of the free surface of the chromosphere by using values of ω according to Carrington's formula. The results of these computations are collected in Table IV. It will be seen that the poleward acceleration has a maximum* value at about latitude 35°. We identify this poleward acceleration on the chromosphere as the cause of the general movement of all prominences from the equator towards the poles in the course of the 11-year solar cycle. In order to check this theory against observation it is necessary to derive from the equation (3) the theoretical velocities for different latitudes and to compare them with the observationally available velocities of prominences.

For obtaining the theoretical velocities we first deduce $dR/d\varphi$ from the equation

$$R=R_0\left(1+\frac{\omega^2 R_0}{2g_0}\cos^2\varphi\right)=R_0\left\{1+\frac{(a-b\sin^2\varphi)^2}{2g_0}.\ R_0\cos^2\varphi\right\}$$

$$=R_0+\frac{R_0^2}{2g_0}(a-b\sin^2\varphi)^2.\cos^2\varphi$$

We have $\frac{dR}{d\varphi}=-\frac{R_0^2}{2g_0}\sin 2\varphi\,(a^2+2ab\cos 2\varphi+b^2\sin^4\varphi-2b^2\sin^2\varphi.\cos^2\varphi)$.

Now, $dR/d\varphi$ is negative for an oblate spheroid and the resultant acceleration on the chromosphere is directed towards the pole. Hence we have from (3), as the term $\omega^2\cos^2\varphi.\ dR/d\varphi$ is negligibly small compared to the other terms,

$$F_{pole}/R=\frac{g}{R^2}.\left\{\frac{R_0^2}{2g_0}.\sin 2\varphi\,(a^2+2ab\cos 2\varphi+b^2\sin^4\varphi-2b^2\sin^2\varphi.\cos^2\varphi)\right\}$$

$$-(a-b\sin^2\varphi)^2.\sin\varphi.\cos\varphi$$

$$=ab\sin 2\varphi.\cos 2\varphi-b^2\sin^2\varphi.\cos^2\varphi.\sin 2\varphi+ab\sin^2\varphi.\sin 2\varphi.$$

$$=d^2\varphi/dt^2$$

Multiplying both sides of this equation by $2\,d\varphi/dt$ and integrating we get

$$\int 2.\frac{d\varphi}{dt}.\frac{d^2\varphi}{dt^2}.dt=\int 2\frac{d\varphi}{dt}\,ab\sin 2\varphi.\cos 2\varphi.dt$$

$$-\int 2\frac{d\varphi}{dt}.b^2\sin^2\varphi.\cos^2\varphi.\sin 2\varphi.dt$$

$$+\int 2\frac{d\varphi}{dt}.ab\sin^2\varphi.\sin 2\varphi.dt$$

or

$$\left(\frac{d\varphi}{dt}\right)^2=\int 2ab\sin 2\varphi.\cos 2\varphi.d\varphi-\int 2b^2\sin^2\varphi.\cos^2\varphi.\sin 2\varphi.d\varphi$$

$$+\int 2ab\sin^2\varphi.\sin 2\varphi.d\varphi$$

$$=\tfrac{1}{2}ab\sin^2 2\varphi+b\sin^4\varphi\,(a-b\sin^2\varphi)+\tfrac{2}{3}b^2\sin^6\varphi;\ \ldots\ldots\ldots(4)$$

here the integration constant=0, on putting $d\varphi/dt=0$ for $\varphi=0$. The values of the theoretical poleward velocity for different latitudes computed from equation (4) are given in Table V, which also includes the values derived from observation by M and Mme L. d'Azambuja. These theoretical poleward velocities, like the

*We recall that the equatorward acceleration on the photosphere also has a maximum at about latitude 40°. One may wonder if the occurrence of a maximum both in the equatorward acceleration on the photosphere and the poleward acceleration on the chromosphere at a latitude of 35°-40° is merely accidental or has a real significance in relation to the well-established fact that the first sunspots of every new solar cycle appear at about the same latitude.

equatorward velocities of belts of sunspot activity considered in the previous section, are to be regarded as characteristic of the parallels of latitude; they are not identical with the actual poleward velocities of individual limb prominences and disk filaments, but represent rather the average rates of displacement in latitude of the zones of prominence activity.

It is clear from Table V that the poleward velocity of prominences, according to the present theory, ought to increase with increasing latitude up to about latitude 70° and thereafter decrease towards the pole. On the other hand, although the theory is capable of accounting for the general poleward movement of all prominences, the variation of poleward velocity with latitude according to the observational results of

TABLE V

Latitude (degrees)	Theor. velocity $\times \overline{10}^{4}$ (in cm/sec.)	Latitude Range (degrees)	Theor. velocity $\times \overline{10}^{4}$ (in cm/sec.)	Observed velocity (in cm./sec.)
0	0			
		0—10	1·046	1184
10	2·091			
		10—20	3·059	823·9
20	4·027			
		20—30	4·849	669·4
30	5·671			
		30—40	6·299	618·0
40	6·926			
		40—50	7·340	463·4
50	7·754			
		50—70	8·087	412·0
60	8·177			
70	8·330			
80	8·326			
90	8·303			

M. and Mme d' Azambuja appears to be contrary to what this theory indicates. However, it seems from their memoir that M and Mme d' Azambuja consider that their work has established beyond all doubt that the direction of movement of all prominences in the course of the 11-year cycle is from the equator towards the poles; but they do not appear to claim that their determination of the magnitudes of the poleward velocity at different latitudes is very precise. In fact, it is only between latitude 11° and 40° that they have been able to use a fair, though not really large, number of filaments for the determination of the velocities for different latitude ranges. They also mention particularly that if one divides the 11-year cycle into three periods, namely the ascending phase, the maximum phase and the declining phase of sunspot activity, then one finds that, for the polar filaments the poleward displacement during the ascending phase is at least double that during the descending phase. There are, moreover, other difficulties and uncertainties inherent in the measurement of the displacement in latitude of both equatorial and polar filaments which are after all short-lived and changeable phenomena; one cannot therefore suppose that the magnitudes of the poleward movement of filaments at various latitudes are yet determined with a very high degree of accuracy. Furthermore, our assumption that the polar retardation of angular rotation at the upper boundary of the chromosphere is correctly given by Carrington's formula may not be strictly true; in fact, there are indications that the polar retardation as determined from filaments is somewhat less than that obtained from spots. In view of all these incertitudes one should not perhaps place too much reliance on numerical agreements or disagreements; the important consideration is that the theoretical mechanism here proposed appears to offer a simple and plausible explanation of the general poleward movement of all prominences, regardless of the latitudes at which they form. At any rate, it seems not only desirable but essential to extend the measurements to a much larger amount of observational material covering a substantially longer period before the variation of the poleward velocity of filaments with latitude can be considered to be known with sufficient precision.

Magnetic Polarity of Sunspots

As has been mentioned earlier in this paper, there is ample justification for the hypothesis that sunspots are similar to circular hydrodynamical vortices. We therefore assume that inside a sunspot some effective mechanism exists for a separation of electric charges, and that the magnetic field of a sunspot arises from the rotation of electrostatic charges around the axis of the spot. Starting from this hypothesis, it seems possible to evolve a plausible mechanism capable, at least qualitatively, of accounting for bipolar spots, the simultaneous occurrence of spots of opposite polarities on opposite sides of the equator and for the reversal of sunspot polarity every 11-years.

In a paper written nearly 20 years ago (Das, 1940) a purely dynamical mechanism had been worked out for the formation of quiescent prominences and absorption markings, and for their change of orientation with latitude; the same mechanism was, in later papers, shown to be capable of explaining several other solar phenomena. The basis of this mechanism is that the central core, which the sun is believed to possess, is highly convective and gives off matter through some eruptive process. The matter thus ejected eventually reaches the photosphere and beyond, its motion outside the photosphere being determined by purely dynamical laws. If the gaseous matter is supposed to issue radially from the photosphere with a small velocity (about 0.2 km/Sec.—see Das, 1941), it does not rise much above the photosphere; while emerging out of the photosphere it is also acted upon by the equatorward force, which exists on the photosphere as already shown. Due to the combined effect of the radial velocity and the equatorward force at the photospheric boundary the ejected mass of gas may be expected to acquire a rotation around an axis approximately parallel to the parallels of latitude, the sense of rotation, as seen from the west limb, being counter-clockwise in the northern and clockwise in the southern hemisphere. We should thus have, just above the photosphere, two cylindrical vortex pipes with opposite rotations in the two hemispheres. Now, as is well known in hydrodynamics, the ends of each such vortex pipe must bend round either to close upon each other or to meet the surface of the photosphere. If the ends of each vortex pipe bend down to the photosphere, we have in each hemisphere a pair of circular vortices rotating in opposite senses; the rotation will of course also be opposite in the two hemispheres. Thus, the mechanism here conceived would give rise to bipolar spots in each hemisphere and to spots of opposite polarities in the two hemispheres. Although sunspots most frequently appear in pairs, single spots are also fairly common. Such single spots could be regarded as having been caused by the bending down of one end of the cylindrical vortex pipe much faster than the other. In the view here advocated the centrifugal force due to rotation is considered to be responsible for the separation of electric charges. Whether or not this rotation will be fast enough to account quantitatively for the observed magnetic field of a sunspot cannot be decided without very careful consideration. However, it is an observed fact that a sunspot usually takes several days to develop its full magnetic field which, after remaining constant for a longer period, gradually disappears again (along with the dissolution of the spot) in about the same time as it has, in the beginning, taken to build up. This indicates, on our view, that the growth or decay of the magnetic field is merely a reflection of the increase or decrease of the unobservable speed of rotation in the sub-photospheric levels of the sunspot whirl. Although the physical process for the formation of sunspots here contemplated is very different from the mechanism which gives rise to terrestrial tornadoes, yet our view of regarding sunspots as revolving fluid columns implies that the structure of a sunspot should be very similar to that of a terrestrial tornado which has a small diameter at the end nearest to the ground and a much larger diameter at its upper end attached to the cumulo-nimbus cloud; thus the lower end of the sunspot vortex below the surface of the photosphere should be considerably smaller in diameter than the part of the sunspot visible on the photosphere. Now if, as is normally to be expected, the principle of conservation of angular momentum is to be satisfied in a sunspot, then the speed of rotation at its sub-photospheric end ought to be very considerably greater than at its upper end. In terrestrial tornadoes the maximum wind speed has been observationally estimated to increase 100-fold over normal wind velocity. In sunspots the conditions may well be analogous. There may therefore be no real difficulty as regards the adequacy of the rotational velocity to produce the required strength of magnetic field. However, there are gaps in the vortex theory of viscous fluids, and the deficiencies of the observational data on sunspots are numerous; at the present time, it does not seem feasible to derive from the theory a truly reliable quantitative estimate of the magnetic field of sunspots.

The observations of sunspots made between 1755 and 1928 have been analysed by Waldmeier (1934) who has concluded that the maxima of consecutive sunspot cycles are alternately large and small; in other words, if the maximum of any cycle is large, the next cycle will have a smaller maximum and the cycle following will again be characterised by a large maximum. Now, according to the mechanism we have been considering both sunspots and prominences arise from essentially the same process, namely the ejection of matter from the central core, the fundamental difference between the two phenomena being that the radial velocities with which the gases emerge from the photosphere are different in the two cases. It appears reasonable to suppose that radial velocity will be greater during a solar cycle whose maximum is larger than during a cycle with a smaller maximum. Consequently, we may expect that every 11 years the velocity of ejection of matter will be alternately larger and smaller. When the velocity of emission of gases from the photospheric boundary is such that the gases do not rise much above the photosphere, sunspots of a certain polarity (*i.e.*, a certain sense of rotation) will be formed in the way contemplated in the preceding paragraph; but when the velocity is larger (of the order of 2 km/sec., see Das *loc. cit*) and such that the gases rise just above the chromosphere, sunspots of opposite polarity (*i.e.*, opposite sense of rotation) will be formed through the joint action of the radial velocity and the poleward force, which exists on the surface of the chromosphere.

On the present theory, therefore, the reversal of sunspot polarity every 11 years is to be considered as a direct consequence of the existence of an equatorward force on the photosphere and a poleward force on the chromosphere. Thus the observed equatorward drift of the sunspot belts and general poleward motion of the zones of prominence activity, the occurrence of bipolar spots with opposite polarities in the northern and southern hemispheres and the so-called 22-year cycle of sunspot polarity appear to be intimately inter-connected and can therefore be regarded as diverse manifestations of one underlying common internal mechanism which is indeed very simple, the diversity of phenomena being fundamentally due to the different radial velocities with which matter is thrown out from the internal convective core. Also, it may well be that the bright photospheric faculae and chromospheric plages, which always accompany sunspots, are nothing other than small vortex pipes whose ends have closed upon each other, and therefore they may have considerably longer lives than the sunspots with which they are associated. They may be formed by small amounts of material ejected from the sun's interior by minor eruptions prior to the major eruption that produces the sunspot. The curious phenomenon of "invisible sunspots" may be due to the magnetic field of these small vortex pipes.

Evershed-effect

The origin of the Evershed-effect may also be pictured in the following way: The rapid rotation inside a sunspot will produce a lowering of gas pressure and of temperature, and the appearance of a magnetic field as a consequence of the rotation will perhaps help to lower the pressure further (Biermann 1941). Thus, the sunspot whirl will be an area of decidedly low pressure which will draw gaseous matter both from above and below the level where it exists. The result of this suction both downwards and upwards will be that at some "neutral level" inside the spot whirl but close to the photosphere the gases must flow laterally from the umbra outwards in all directions. This would provide a qualitative picture of the Evershed-effect and also of the circulatory system in the neighbourhood of sunspots as indicated by the observations of St. John and others. The "photospheric" faculae and the "chromospheric" plages which form above the photosphere, but evidently well inside the chromosphere, ought to remain under the influence of the circulatory system around sunspots, so that their equatorward motion during the progress of the 11-year solar cycle would be expected to follow practically the same course as that of the sunspots. From the picture here presented it will also be easily understood why spots with higher magnetic field strengths also show greater Evershed velocities (Michard 1951).

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