Kodaikánal Observatory.

BULLETIN No. XVI.

ON THE CURVATURE OF LINES IN THE SPECTRUM FORMED BY A PLANE GRATING.

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IN connection with some measurements of spectra recently made at Kodaikánal it was found that the lines were slightly curved. The phenomenon had been pointed out by Hale, but no formula for its amount appeared to be available; the following theoretical examination may therefore have some use.

2. Let the origin of reference be taken in the centre of the grating and OZ along one of the lines ruled upon it, and let OX be drawn normally outwards Let the incident light consist of parallel waves propagated in the direction $(-l, -m, -n)$ so that (l, m, n) is the normal to the waves drawn in the direction to meet the light. and let the diffracted plane waves be propagated in the direction (l', m', n').

We shall consider the spectrum of the p^{th} order and shall take a representative point P on the r^{th} line of the grating reckoned from OZ towards the side on which y is positive. If then d be the distance between consecutive lines on the grating, P may be taken as $(0, rd, z)$; and as the projection of a vector in any direction is equal to the sum of the projections of its component vectors in that direction, the projections of OP along the normals to the incident and diffracted waves are $(mrd + nz)$ and $(m'rd + n'z)$ respectively. Thus the total length of the path traversed by the incident and diffracted rays when incidence occurs at O will exceed the length when incidence occurs at P by the amount

$$mrd + nz + m'rd + n'z,$$

and as the spectrum is of the p^{th} order this must be equal to $pr\lambda$. Thus

$$(m+m')\, rd + (n+n')\, z = pr\lambda.$$

This will hold for all values of r and z if

$$(m+m')\, d = p\lambda \text{..........................(1).}$$
$$n+n' = 0 \text{..........................(2).}$$

which are the equations giving m' and n'.

3. We shall now suppose that the incident light passes through a straight slit parallel to the lines of the grating and that the centres of the slit and of the collimating lens lie on the line

$$\frac{x}{\cos \alpha} = \frac{y}{\sin \alpha}, \; z = 0.$$

The incident ray (l, m, n) will thus be parallel to the line joining the centre of the lens to some point on the slit and we shall have

$$\frac{l}{\cos \alpha} = \frac{m}{\sin \alpha} \text{..........................(3)}$$

Now corresponding to $n = 0$ we shall have

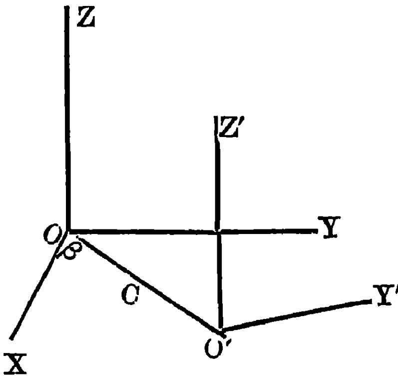
$$l = \cos \alpha,\; m = \sin \alpha,\; (\sin \alpha + m')\, d = p\lambda,\; n' = 0$$

Thus if the angle of diffraction in this case of symmetrical incidence be β,

$$l' = \cos \beta,\; m' = \sin \beta,$$

where $(\sin \alpha + \sin \beta)\, d = p\lambda$ .... (4), the usual formula.

4. Now let us consider the image formed by a camera lens whose centre C lies on the line $\frac{x}{\cos\beta}=\frac{y}{\sin\beta}\cancel{=\frac{z}{0}}$. $z=0$



We shall suppose that the photographic plate on which the image is formed cuts this line at right angles at O', and that O' is at a distance f from C, this being the focal length for the wave length λ. Let $O'Y'$, $O'Z'$ be axes of reference in the plane of the plate, $O'Z'$ being parallel to OZ; then if (y', z') be the co-ordinates, referred to these axes, of a point Q in the image on the plate, a line from C to Q will be parallel to the diffracted ray (l', m', n'). Now the projections of CQ parallel to OX, OY, OZ will be $f\cos\beta - y'\sin\beta$, $f\sin\beta+y'\cos\beta$, z' : and hence

$$\frac{f\cos\beta-y'\sin\beta}{l'}=\frac{f\sin\beta+y'\cos\beta}{m'}=\frac{z'}{n'}=(f^2+y'^2+z'^2)^{\frac{1}{2}}\text{ ......(5).}$$

5. Now if the ratio of the length of the slit to the focal length of the collimating lens be treated as a small quantity of the first order, n will be a small quantity of that order; and as far as squares of n, by (3),

$$\frac{l}{\cos a}=\frac{m}{\sin a}=(1-n^2)^{\frac{1}{2}}=1-\tfrac{1}{2}n^2$$

Thus by (1)—

$$m'=\frac{p\lambda}{d}-(1-\tfrac{1}{2}n^2)\sin a$$
$$=\sin\beta+\tfrac{1}{2}n^2\sin a,\text{ by (4);}$$

and by (2)—

$$n'=-n.$$

Thus substituting for m', n', in (5)

$$\frac{f\sin\beta+y'\cos\beta}{\sin\beta+\frac{1}{2}n^2\sin a}=\frac{z'}{-n}=(f^2+y'^2+z'^2)^{\frac{1}{2}}\text{ ......(6)}$$

Hence z' is of the first order of small quantities and

$$z'=-nf\left\{1+\frac{y'^2+z'^2}{2f^2}\right\}\text{ ......(7)}$$

Also by (6)—

$$f\sin\beta+y'\cos\beta=(\sin\beta+\tfrac{1}{2}n^2\sin a)f\left\{1+\frac{y'^2+z'^2}{2f^2}\right\},$$

so that y' is of the second order of small quantities, being given by

$$y'\cos\beta=\tfrac{1}{2}n^2f\sin a+\tfrac{1}{2}\frac{y'^2+z'^2}{f}\sin\beta.$$

Omitting the term in y'^2 which is of the fourth order of small quantities we obtain

$$2fy'\cos\beta=n^2f^2\sin a+z'^2\sin\beta$$
$$=z'^2(\sin a+\sin\beta)\text{ ......(8).}$$

by (7), omitting fourth powers of small quantities.

Hence the light of wave-length λ passing through the slit will come to a focus on the curve (8), a parabola of semi-latus-rectum

$$\frac{f\cos\beta}{\sin a+\sin\beta}$$

or, to the same approximation, a circle of this radius.

6. Let us now suppose that the normal to the photographic plate instead of being parallel to the diffracted ray lies in the plane XOY at an angle γ with the diffracted ray; the plate will now be inclined γ to its former position and the new radius of curvature ρ' of the inclined section of the cone of rays through C will be connected with the old radius ρ of the normal section by the relation.

$$\rho'=\rho\cos\gamma$$

Hence

$$\rho'=\frac{f\cos\beta\cos\gamma}{\sin a+\sin\beta}.$$

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