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Optical depth towards the nucleus of Halley's comet

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Abstract. A new relation between the nuclear radius and the optical depth towards the nucleus of comet Halley has been derived on the basis of the radial variation of the effective area of the grains along the Giotto trajectory in the coma. It is found that the expected optical depth towards the nucleus varies from 0.089 to 0.036 corresponding to the nuclear radius in the range 8 to 16 km.

Key words : comet Halley—dust grains—nucleus—optical depth in coma

1. Introduction

The radial variation of the effective area A_r of the dust grains per unit volume along the Giotto trajectory in Halley's coma has been reported by Green & Hughes (1986) and by McDonnell *et al.* (1987) on the basis of *in situ* measurements of the grain masses in the dust impact detection system (DIDSY) (see, for example, McDonnell *et al.* 1987). These results are used here to derive a relationship between the nuclear radius and the optical depth towards the nucleus of comet Halley.

2. Description of the model, discussions and the results

We shall consider here, for sake of illustration, the post-encounter data (solid line) given in figure 8 of Green & Hughes (1986) who have also shown the error bars. Figure 1 shows the smoothed out plot of the same data on log-log scale. The straight line fit is represented by equation

$$A_r = 9.567 \times 10^{-11} r^{-2.308}, \qquad \ldots(1)$$

where A_r is expressed in the unit of m^2/m^3 and r, in the unit of 10^3 km, is the radial distance within the coma.

The construction of the model of optical depth towards the nucleus of a comet requires, among other things, *a priori* knowledge about the geometry, size distribution function and composition of the dust grains in the coma. Each of these features is discussed below briefly at appropriate juncture.

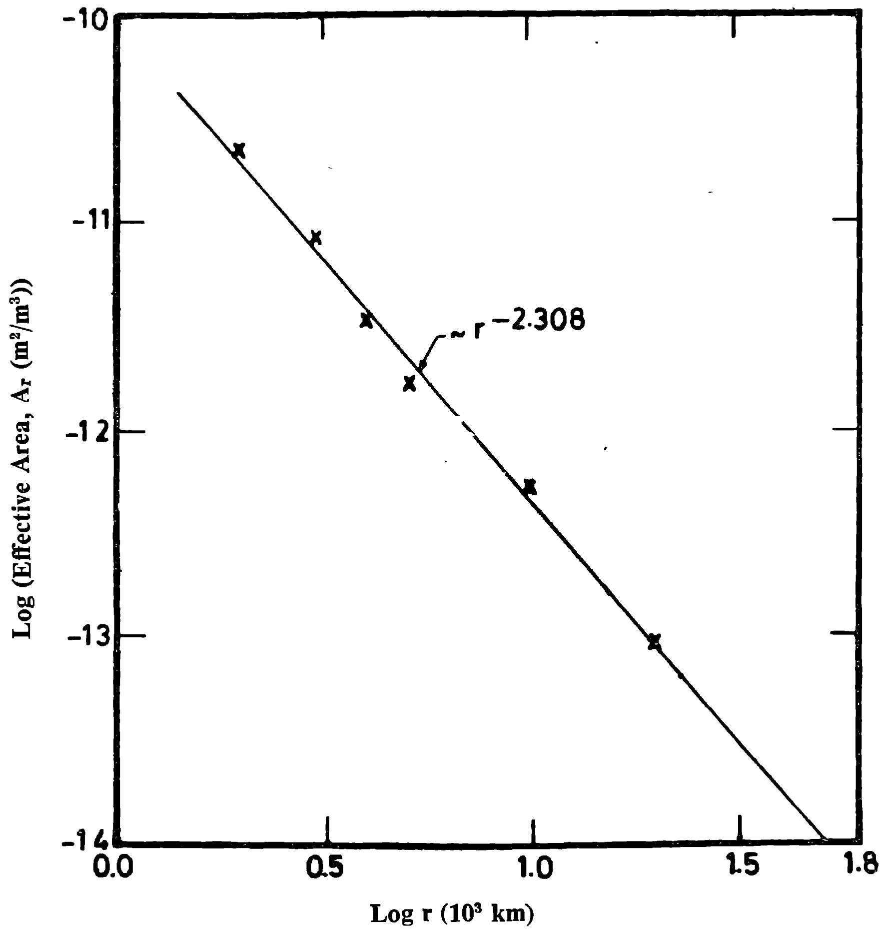

Figure 1. Effective area (m^2) of the dust grains per m^3 along the Giotto trajectory as calculated from DIDSY data assuming a density of 0.8 g cm^{-3} for the grain material. Crosses represent post-encounter data adopted from Green & Hughes (1986) wherein error bars are also shown.

The dust grains in coma or, for that matter, in interstellar, circumstellar and other cosmic environments, could have irregular shapes, structure and surface roughness. They could be fluffy, large conglomerates of very fine particles which may gradually disentangle.

We shall however assume that the dust grains are in the form of smooth, homogeneous and isotropic spheres which are amenable to detailed calculations by using the rigorous electromagnetic scattering theory originally formulated by Mie (1908).

The size distribution of the dust grains in comet comae can vary with radius, azimuth, latitude, time, and epoch (see, for example, Mazets *et al.* 1986 and references therein). The representative size distribution function (i.e. size spectrum) of the dust grains, based on *in situ* measurements in Vega mission, has been adopted here in the form (Mazets *et al.* 1986; Mukai *et al.* 1987) :

$$\begin{aligned} n(a) &\sim a^{-2},\ a \leqslant 0.62\ \mu\text{m}, \\ &\sim a^{-2.75},\ 0.62\ \mu\text{m} < a \leqslant 6.2\ \mu\text{m}, \\ &\sim a^{-3.4},\ a > 6.2\ \mu\text{m}, \end{aligned} \qquad \ldots(2)$$

where a is the radius of the grains, and $n(a)\,da$ represents relative number of the grains per unit volume in the size interval $(a, a + da)$. In order to make these expressions useful, they have been properly normalized to unity and adjusted for appropriate relative number density of the grains in each size range (Shah 1989). It is assumed that the size distribution function given in equation (2) holds uniformly throughout the volume of the coma. The average geometrical cross section under this size distribution is given by

$$\langle \pi a^2 \rangle = 1.360 \times 10^{-9} \text{ cm}^2, \qquad ...(3)$$

where we have considered size in the range $0.01\ \mu\text{m} \leqslant a \leqslant 100.2\ \mu\text{m}$. Let N_r be the number density of the grains at a distance r from the centre of the coma/nucleus. So we can set

$$A_r = N_r \langle \pi a^2 \rangle. \qquad ...(4)$$

Solving equations (1), (3) and (4) for N_r, one obtains

$$N_r = 2.035 \times 10^{15} \{r(\text{cm})\}^{-2.308} \text{ cm}^{-3}. \qquad ...(5)$$

This may be treated purely as an empirical relation. It is clear from equations (1) and (5) that the DIDSY data indicate a radial variation of area or number density of the grains steeper vis-a-vis the inverse square law.

If R_n and R_c denote the radii of the nucleus and coma, respectively, the optical depth τ_n towards the nucleus can be expressed as

$$\tau_n = \int_{R_n}^{R_c} N_r \left[\int_{a_{min}}^{a_{max}} \pi a^2 n(a)\, Q_{ext}\, da \right] dr, \qquad ...(6)$$

where $Q_{ext} = Q(a, \lambda)$ is the extinction efficiency of a grain of size a at wavelength λ and a_{min} and a_{max} represent the lower and upper bounds on a, respectively, in the size spectrum. The integral in the bracket defines an average extinction cross section, $\bar{\sigma}_{ext}$, of the grains with specific composition.

An important input to the numerical calculations of scattering parameters from Mie theory is the index of refraction, i.e. the composition of the grain material. The analyses of the space probe experiments in comet Halley missions have revealed possible classes of composition of the dust grains in coma (see, for example, Clark *et al.* 1987; Wallis *et al.* 1987; Langevin *et al.* 1987, Krishna Swamy and Shah 1988). The latest results indicate that about 30% of the grains consist of light elements (H, C, O and N or what is called "CHON", organic refractories, etc.), about 35% of the grains are silicate-like minerals similar to CI carbonaceous chondrite with composition matching within a factor of two. The balance 35% is composed of a mixture of these two primary components in varying amounts. These relative proportions vary with time during the transit of the space probe within the coma. For the present purpose of the optical depth studies, the composition is not a crucial parameter. For instance, with $a_{min} = 0.01\ \mu$m and $a_{max} = 100.2\ \mu$m, we get almost the same results for the optical depth by choosing the index of refraction, $m(\lambda) = m'(\lambda) - im''(\lambda)$, for a variety of materials such as (i) dirty ice, (ii) olivine, (iii) $m(\lambda)$ quoted by Mukai *et al.*, (iv) hypothetical materials with $m'(\lambda) = 1.4$, 1.45, 1.5, 1.55 or 1.6 and imaginary part $m''(\lambda)$ in each case linearly decreasing from 0.2 to 0.05 in the wavelength range 0.4 μm $\lesssim \lambda \lesssim$ 0.62 μm, (v) $m'(\lambda)$ for olivine with $m''(\lambda) =$ 0.1, (vi) Tholin with real part modified to $m' = 1.4$, etc. Thus, in the relevant visual wavelength range, the calculated average extinction cross-section of the grains, $\bar{\sigma}_{ext}$, based on the size distribution given in equation (2) and Mie theory (see, for example, van de Hulst 1957; Shah 1977) is approximately given by

$$\bar{\sigma}_{ext} = \int_{a_{min}}^{a_{max}} \pi a^2 n(a)\, Q_{ext}\, da \simeq 3.0 \times 10^{-9}\ \text{cm}^2, \qquad \ldots(7)$$

where we have considered $a_{min} = 0.01$ μm and $a_{max} = 100.2$ μm.

The column density of the grains towards the nucleus can be evaluated from equation (5) as follows:

$$\int_{R_n}^{R_c} N_r dr = 1.556 \times 10^{15}\, [R_n^{-1.308} - R_c^{-1.308}]. \qquad \ldots(8)$$

Since $R_c \gg R_n$, the second term in the bracket can be neglected. Therefore, from equations (6), (7) and (8), one derives the following relation for the optical depth (τ_n) towards the nucleus having radius R_n :

$$\tau_n = 4.670 \times 10^6\, R_n^{-1.308}, \qquad \ldots(9)$$

where R_n is in cm. Thus R_n can be found if τ_n is known or vice versa. This is a numerical, unit specific equation of type II in the sense defined by Kiang (1987). Figure 2 shows the calculated optical depth τ_n as function of the nucleus radius R_n.

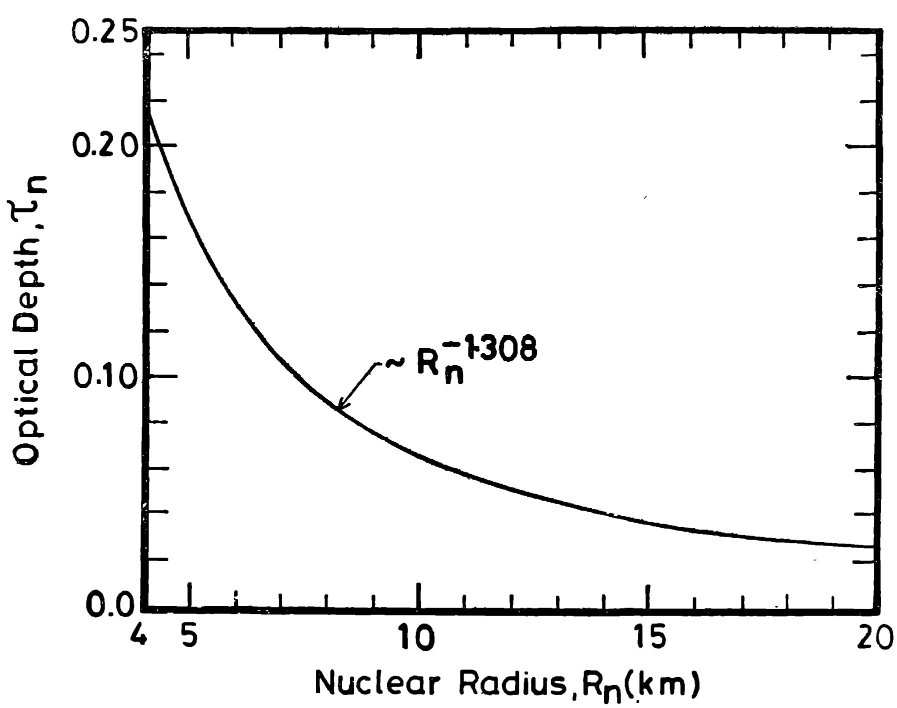

Figure 2. Calculated optical depth versus nuclear radius of comet Halley. The size distribution function covers the range $a_{min} = 0.01$ μm to $a_{max} = 100.2$ μm. See the text for particulars.

As mentioned by Herman & Salo (1987), the opacity of the coma quoted in the literature required certain *a priori* assumptions regarding the scattering properties of the dust grains. For example, Keller *et al.* (1987) have derived optical depth in Halley's coma, $\tau_{max} = 0.28$ by assuming that (a) the single scattering albedo $\simeq 0.03$, (b) the optical extinction cross section is equal to the geometrical cross section, and (c) the sizes of the grains are in the range of 1 μm. Somewhat realistic values of coma opacity of $\simeq 0.1$ at 1 AU and 0.6 at perihelion have been estimated by Weisman & Keiffer (1981) in connection with thermal modelling of comet Halley's nucleus.

However, we are concerned here mainly with the optical depth towards the nucleus which, in quiet time, is equal to 0.03 at wavelength $\lambda = 0.62$ μm (Mukai *et al.* 1985). This corresponds to $R_n \simeq 18$ km according to equation (9) and figure 2. This is close to the largest dimension of Halley's nucleus. For nuclear radius of Halley's comet in the range 8 km $\leqslant R_n \leqslant$ 16 km (see, for example, Mendis 1988), the optical depth towards the nucleus varies in the range $0{\cdot}089 \geqslant \tau_n \geqslant 0.036$ according to equation (9).

3. Conclusion

It is shown that a relation exists between the coma optical depth towards the nucleus and the radius of the nucleus of comet Halley at a given epoch. *A priori* knowledge about the radial variation of (i) the effective area, (ii) the size spectrum, and (iii) the composition of the grains is a prerequisite. If the data on spatial variations of these quantities are available, a more exact relation can be derived. Now the question is : can there be a generic relation of this type for other comets ? A sort of general expression can be written in the form

$$\tau_n = \alpha R_n^{-\beta}, \qquad ...(10)$$

where the constants α and β would depend on various factors such as heliocentric distance and the physics of a comet. Only a detailed study of comae and nuclei of other comets in future can reveal the trends in the variation of the constants α and β.

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