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Boundary to the solar system set by the Galaxy according to King-Innanen's formula

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Abstract. Supposing the whole mass of the Galaxy to be concentrated at its centre (within its core radius), a system of a planet/comet-sun-Galaxy is considered in order to find the limiting direct and retrograde orbits around the sun using King-Innanen's formula (King 1962; Innanen 1979). This provides a boundary to the solar system.

Key words : Galaxy—solar system—orbits

1. Introduction

Smoluchowski & Torbett (1984) have considered a system consisting of a comet and the sun rotating around galactic centre and periodically traversing the nearly harmonic field of the galactic plane and described the results of a three-dimensional stability study. They have also determined the shape of the boundary of the solar system, defined as the surface within which the gravitational attraction of the sun, rather than that of the rest of the Galaxy, controls the orbital motion of the bodies such as planets and comets. A two-dimensional model of a greatly simplified version of this problem was previously studied by Chebotarev (1965, 1966) and the corresponding three-dimensional Hill surface has been discussed by Antonov & Latyshev (1972). Using King-Innanen's formula (King 1962; Innanen 1979), Rawal (1985a) has discussed the boundary to the solar system considering a planet/comet-sun-solar companion system in place of moon-planet-sun system considered by Innanen (1979) (for solar companion see Davis, Hut & Muller 1984; Whitmire & Jackson 1984). Rawal (1985b) has also applied King-Innanen's work along with the concepts of synchronous orbits around the planets and the tidal drags acting within them to investigate whether or not Mercury and Venus have at least one retrograde satellite each around them. Supposing the whole mass of the Galaxy to be concentrated at its centre (within its core radius), we consider here a system of a planet/comet-sun-Galaxy to calculate the limiting direct and retrograde orbits around the sun using King-Innanen's formula discussed below. This provides a boundary to the solar system set by the Galaxy.

2. Work of King and Innanen

King (1962) in his study of clusters estimated the tidal limit of a cluster by defining it to be a point, on the line connecting the centre of the cluster with the galactic centre, at which a star can remain on the line of centres with an acceleration along that line that is zero with respect to the centre of the cluster. That is, at the moment of perigalactic passage, the star is pulled neither toward nor away from the cluster. King obtained the expression for the limiting tidal radius, r_{lim}, of the cluster to be

$$r_{\text{lim}} = \left[\frac{Gm}{\Omega^2 - d^2v/dR^2}\right]^{1/3}, \qquad \ldots(1)$$

where r is the radial distance of a star from the centre of the cluster; m the mass of the cluster; Ω the angular velocity of the cluster around the galactic centre; V the gravitational potential energy of the galaxy; and R the radial distance of the cluster from the galactic centre. If we represent the force field of the galaxy by an inverse-square law due to mass M, then

$$dV^2/dR^2 = -2GM/R^3; \qquad \ldots(2)$$

and hence (when the cluster is in circular motion about the centre)

$$r_{\text{lim}} = \left(\frac{m}{3M}\right)^{1/3} R. \qquad \ldots(3)$$

If the cluster's orbit about the galactic centre is an ellipse, then the angular velocity at any point is given by

$$\Omega^2 = GMa(1 - e^2)/R^4, \qquad \ldots(4)$$

where a is the semi-major axis of the ellipse; and e its eccentricity. At the perigalactic point, R takes the value

$$R_p = a(1 - e), \qquad \ldots(5)$$

and equation (3) in this case becomes

$$r_{\text{lim}} = \left[\frac{m}{(3 + e)M}\right]^{1/3} R_p. \qquad \ldots(6)$$

Innanen (1979) applied King's (1962) formula (for a system of a star-cluster-galaxy) to systems of a moon-planet-sun and a star-dwarf galaxy-galaxy. A moon revolving about a planet that in turn is revolving in the same sense about the sun will, at some limiting distance from the planet, become unstable because of the action of the sun's tidal force. If the same moon at the same distance were to revolve opposite to the sense of the planet's revolution, it could better resist the sun's tidal force. At greater limiting distance from the planet, this retrograde moon would eventually succumb to the sun's tidal force. This limiting retrograde radius should properly define the true gravitational sphere of influence of a planet. Innanen (1979) used the equation for acceleration in a rotating coordinate frame with an additional Coriolis term of magnitude $2\Omega v_r$, where v_r is the velocity of the moon relative to the planet. The familiar right-hand rule immediately shows that the Coriolis term is always directed radially between the moon and the planet. It counteracts the planet's gravity for direct motion of the moon, but effectively supplements the planet's gravity for retrograde motion. For the limiting direct and retrograde radii, r_d and r_r respectively, of a moon around a planet Innanen gets

$$r_r/r_d = 3^{2/3} \qquad \ldots(7)$$

and

$$r_d = \left(\frac{m}{3^2 M}\right)^{1/3} R. \qquad \ldots(8)$$

3. Boundary to the solar system set by the Galaxy

Innanen (1979) considered a moon-planet-sun system and found the limiting direct and retrograde orbits around all planets, thus setting boundaries to all satellite systems. In order to find the boundary to the solar system set by the Galaxy, we suppose the whole mass of the Galaxy to be concentrated at its centre (within its core radius), and consider a planet/comet-sun-Galaxy system to calculate the limiting direct and retrograde orbits, r_d and r_r respectively, of a planet/comet around the sun. We assume the orbit of the sun around the galactic centre to be circular. For the radius of the solar orbit, we take the value 8.5 kpc and calculate the limiting direct and retrograde radii of a planet/comet around the sun for the following four different values of the mass of the Galaxy : $2 \times 10^{11} M_\odot$, $3 \times 10^{11} M_\odot$, $5 \times 10^{11} M_\odot$, and $10^{12} M_\odot$ (see Rawal 1982 and references therein; and also *IAU Symposium No. 84*). They are set out in table 1. r_r in the boundary to the solar system set by the Galaxy.

Table 1

Mass of the Galaxy, $M_\odot$	Limiting direct radius of a planet/ comet around the sun, r_d, in		Limiting retrograde radius of a planet/ comet around the sun, r_r, in	
	AU	pc	AU	pc
2×10^{11}	1,44,100	0.6981	2,99,500	1.4520
3×10^{11}	1,25,800	0.6099	2,61,700	1.2690
5×10^{11}	1,06,100	0.5144	2,20,700	1.0700
10^{12}	84,240	0.4084	1,75,200	0.8494

If we take for the mass of the Galaxy a value from the range 2×10^{11}–$10^{12} M_\odot$, we find the boundary to the solar system (table 1) which is in conformity with the observed initial dimensions of the star-forming clouds (Bok & Reilly 1947; Larson 1969; Hoyle 1978; Rawal 1985c). Our solar system is thought to have formed from such a cloud. This makes us confident about the results that we have obtained here.

As is mentioned, if the solar companion exists it sets a boundary to the solar system (Rawal 1985a). This boundary to the solar system is its boundary with respect to the solar companion, that is, when a perturber is the solar companion. As the solar companion itself is bounded to the sun, and as the concept of the boundary to the solar system set by the Galaxy includes all objects bounded to the sun, the orbit of the solar companion should also lie within this boundary. In other words, with respect to the concept of the boundary to the solar system set by the Galaxy, the status of the solar companion is like that of a planet to the sun, and hence, this boundary is also the boundary to the binary system consisting of the sun and its companion star, and hence, even the solar companion is not expected to cross this boundary. In order that this should be so, the aphelion distance of the solar companion should also lie within this boundary. Depending upon its mass 0.005 $M_\odot$, 0.05 $M_\odot$, 0.1 $M_\odot$, 0.2 $M_\odot$ or 0.3 $M_\odot$, the solar companion has its corresponding aphelion distance 149, 396 AU, 151, 623 AU, 153, 969 AU, 158, 508 AU, or 162, 758 AU. Of course, these distances depend upon the eccentricity of the orbit of the solar companion which we have taken it to be 0.7 (Rawal 1985a). If the solar companion exists and has for its mass, a value in the range 0.005–0.3 $M_\odot$ and for the eccentricity e of its orbit having the value 0.7, table 1 shows that the orbit of the solar companion even for its mass 0.005 $M_\odot$ must be retrograde, as its aphelion distance exceeds the limiting direct orbit even for the lowest mass assumed for the Galaxy. We, therefore, conclude that if the solar companion exists and has its mass in the range 0.005–0.03 $M_\odot$, and eccentricity 0.7, then it necessarily has a retrograde orbit. The question, whether the orbit of the solar companion is retrograde or direct, is settled at least for its mass in the range 0.005 $M_\odot$–0.3 $M_\odot$. We are inclined to believe that probably the solar companion is the result of the condensation of the material in the outermost ring of tenuous material of the solar nebula.

One of the views of the formation of the solar system is due to Laplace (1796), Several authors (Kuiper 1951; Prentice 1978; Rawal 1984) have studied the formation of the solar system according to Laplacian view and arrived at the conclusion that most probably the solar system has formed through the successive formation and detachment of the rings of material of the solar nebula and the planets resulted from the condensation of the solar material in those rings. It, therefore, leads us to believe that the very wide ring of tenuous solar material beyond 50 AU which got detached from the solar nebula during the course of its free fall from its initial dimensions to the dimensions of the planetary system is what corresponds to the comet cloud hypothesized by Oort (1950). According to this concept the existence of the Oort comet cloud is the outcome of the formation of the solar system and is thus bound to the sun. From the literature, one can also see that the Oort cloud may be taken to begin at about 50 AU and extend as far as 150,000 AU or more. According to this scenario, therefore, the solar companion, if it exists, belongs to the Oort comet belt and, as said earlier, could be taken to be the condensation of the solar material in the wide ring of the tenuous material of the solar nebula.

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