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Different distribution functions of solar x-ray flares

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Abstract. A statistical study has been done on soft x-ray (SXR) flares, the data of which were collected by Geostationary Operational Environmental Satellites (GOES) during the period July 1993–June 1994. From the analysis different distribution functions were obtained empirically which give the number of SXR flares per unit interval of intensity and asymmetry in duration and angular interval of longitude. These distribution functions have been used for evaluating the center to limb variation of optical thickness for SXR flares.

1. Introduction

In one of our earlier papers on Solar x-rays (Das et al. 1994) we made studies on x-ray flares in relation to H_α - flares classified according to their visual indications. In another paper (Das et al. 1999) the variation of North-South (N-S) as well as East-West (E-W) asymmetries with the x-ray intensity was studied. It was examined that N-S asymmetry of x-ray flares remains always positive when their latitudinal position and intensity are considered and it increases with the increase of intensity. The E-W asymmetry is found to be zero, when longitudinal distribution is considered, but it remains positive upto a certain intensity value above which it becomes negative. In the present paper we have studied the energy distribution as well as longitude and latitude distributions of solar X-rays. These distribution functions have been used to find out the directivity and optical thickness of the absorbing layer.

Directivity studies are of two types, statistical and stereoscopic. Datlowe et al. (1977) statistically examined a sample of several hundred hard x-ray flares and searched for a systematic limb brightening (or darkening) of events. Their results regarding brightness variation are too noisy to be conclusive at a 2σ confidence level. On the other hand, Kane et al. (1980) looked for the anisotropy of x-ray emission in a series of events observed simultaneously with ISEE 3 and PVO satellites. Their results show no systematic trend toward limb brightnening or darkening. Directivity and its energy dependence in solar flare energetic emission was also investigated by Peng Li (1995) who studied 72 solar flares simultaneously observed by Gamma Ray Spectrometer (GRS, 0.3-1 Mev and the Hard x-ray burst spectrometer (HXRBS, 30-500

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Kev) on the Solar Maximum Mission (SMM). The GRS observations show significant centre to limb variations in both spectral and size distributions, while HXRBS observations show insignificant variations. Bogovalov (1989) examined the directivity of hard x-ray emission of solar flares based on data SNEG-2M2 instrument. He obtained the systematic softening of the x-ray emission spectra of flares towards the centre of solar disk in the region of photon energy ≥ 50 Kev.

2. Method of analysis

The x-ray flare data which were used in the present analysis were obtained from the Solar Geophysical Data bulletins published by U.S. Department of Commerce, Boulder, Colorado. Nearly two thousand soft x-ray (SXR) flare data observed by the Geostationary Operational Environmental Satellites (GOES) during the period from July, 1993 to June, 1994 which cover a part of the post maximum phase of the 22nd solar cycle, were collected from these data books.

The GOES included ionization chambers with pass bands in the ranges 0.5 – 4Å and 1-8 Å provided by the Space Environment Laboratory of the National Oceanic and Atmospheric Administration (NOAA). The ionization chambers record an electric current which depends on a wavelength transfer function of the detector and the incident solar x-ray flux. To convert the measured current to a corresponding flux value, an averaged transfer function is calculated, based on an assumed incident x-ray spectrum.

In order to find out the flux distribution of x-ray flares the data were grouped in different convenient ranges of x-ray flux and the number of events in those adopted intervals were counted. After dividing the number of such events by the respective flux intervals we get the flux distribution of SXR flares as defined below :

$$f(E) = \frac{\Delta N_E}{\Delta E} \quad (1)$$

where ΔN_E gives the number of SXR flares in the flux interval ΔE. This function has been determined after grouping the data in different ranges of longitudes and latitudes. We have defined another parameter Asymmetry in duration (A) as follows :

$$A = \frac{\text{Rise time}}{\text{Duration}} \quad (2)$$

This 'A' gives an idea about the degree of fastness or sluggishness in the process of evolution of energy. The function f (A) which gives the distribution in the Asymmetry in duration is defined as :

$$f(A) = \frac{\Delta N_A}{\Delta A} \quad (3)$$

Another function f(θ) which gives the number of x-ray flares in unit regular interval of longitude is defined as :

$$f(\theta) = \frac{\Delta N_\theta}{\Delta \theta}$$

3. Results

The flux distribution f (E) was found out for the SXR flares in the three ranges of longitudes 0-30°, 30-60° and 60-90°. This was done for the events occurring both in the eastern and western hemispheres. The plots of the function f (E) against E are shown in Figures 1(a) and 1(b) in which logarithmic scales have been used in both the axes of co-ordinates. Each of the lines as shown in Figure can be fitted with the equation as stated below :

$$f\,(E) = 10^{m}\,E^{-n} \tag{1}$$

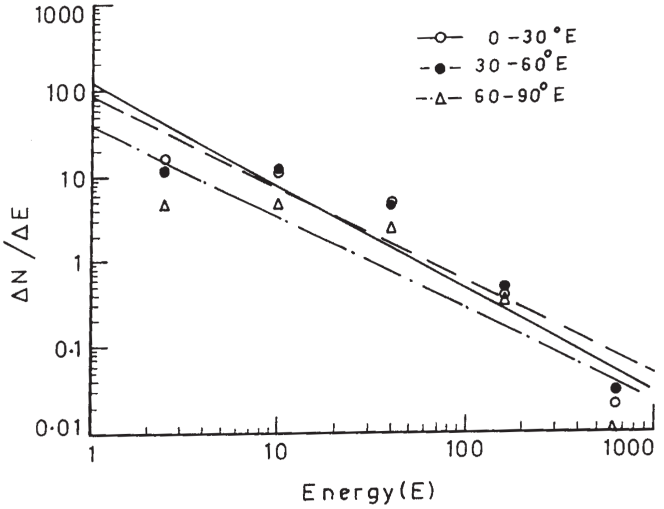

Figure 1(a). Flux distribution of solar x-ray flares occurring in the longitude ranges 0-30°E, 30-60°E and 60-90°E.

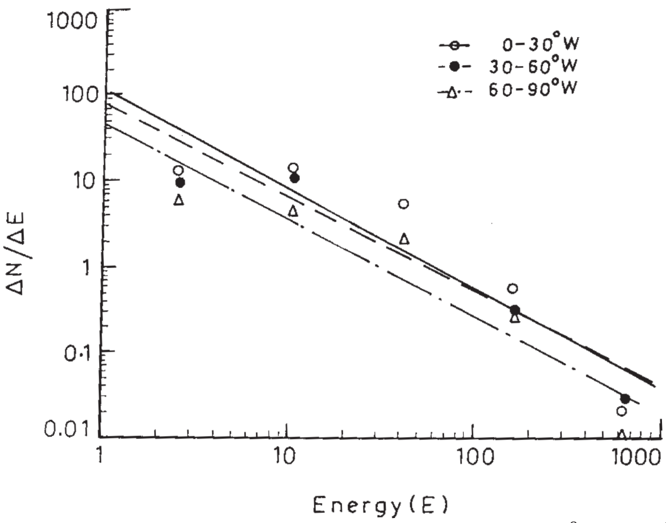

Figure 1(b). Flux distribution of solar x-ray flares for the longitude ranges 0-30°W, 30-60°W and 60-90°W.

The values of the constant ‘m’ and ‘n’ were determined separately for each of the lines and are tabulated in Table 1.

Table 1. Values of the constant ‘m’ and ‘n’.

Range of Longitude	Value of ‘m’	Value of ‘n’
0-30^o E	2.0885	1.2063
30-60^o E	1.9715	1.1000
60-90^o E	1.6111	1.0817
0-30^o W	2.048	1.1625
30-60^o W	1.8823	1.0884
60-90^o W	1.6361	1.1330

It appears from the table that the value of the constant ‘m’ decreases as one moves from the central region towards the limb region on the solar disc. This indicates that for a particular value of E the function f (E) decreases in the limb region and this holds true for SXR flares occurring both in the eastern and western solar hemispheres. The value of ‘n’ also decreases towards the limb region excepting in the range 60-90^o W.

The flux distribution of SXR flares was also evaluated after grouping the data according to their latitudinal positions on the solar disk. As the active regions are not developed beyond 50^o (Bray and Loughhead, 1964), the SXR flares are found to be concentrated within latitude range 0-40^o, which holds good for both the northern and southern hemispheres. The results of this type of distribution are shown in Figure 2 and the respective constants are tabulated in Table 2.

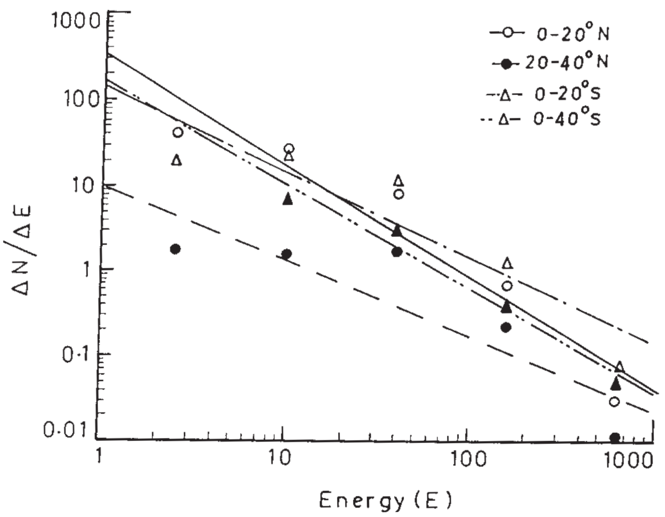

Figure 2. Flux distribution for latitude ranges 0-20^{o}N, 20-40^{o}N, 0-20^{o}S and 20-40^{o}S.

Table 2. Values of constants m_1 and n_1.

Latitude ranges	Value of m_1	Value of n_1
0-20^o N	2.5221	1.2990
20-40^o N	0.9952	0.8802
0-20^o S	2.1766	1.0095
20-40^o S	2.5632	1.5783

The impulsiveness distribution f (I) of SXR flares was also found out for the same set of data in a similar technique as stated above. The results obtained from such studies are shown in Figures 3(a) and 3(b). The equation giving the variation of f (I) with the impulsiveness 'I' can be represented as follows :

$$f\,(I) = 10^{Þ}\ I^{-q} \qquad (2)$$

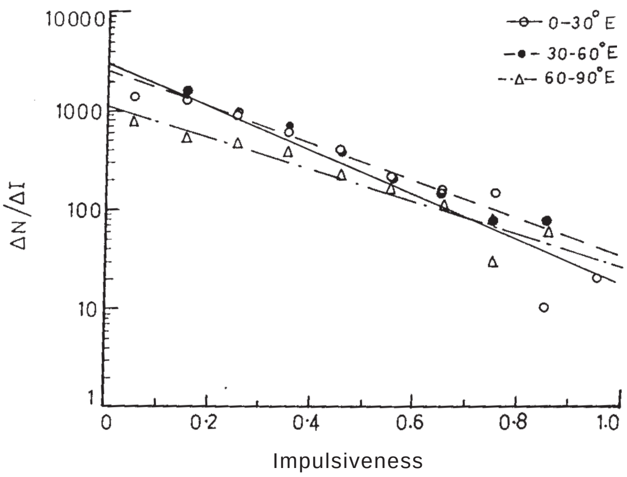

Figure 3(a). Distribution of Impulsiveness of solar x-ray flares occurring in the ranges of longitude 0-30^{o}E, 30-60^{o}E and 60-90^{o}E.

Table 3. Values of the constants 'Þ' and 'q'.

Range of Longitude	Value of 'Þ'	Value of 'q'
0-30^o E	3.4725	2.2505
30-60^o E	3.3782	1.8803
60-90^o E	3.0297	1.6258
0-30^o W	3.5099	2.2675
30-60^o W	3.4328	2.1666
60-90^o W	2.9917	1.8014

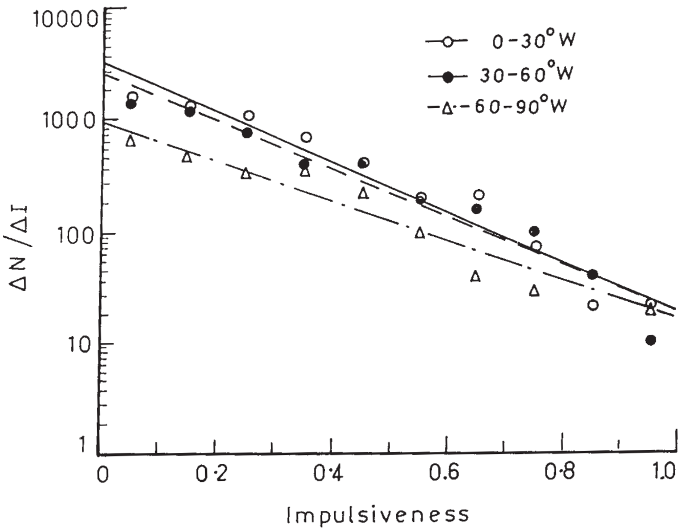

Figure 3(b). Distribution of Impulsiveness of solar x-ray flares in the longitude ranges 0-30°W, 30-60°W and 60-90°W.

The values of the constants Þ and q were calculated separately for each of the lines and are given in Table 3.

The north-south latitudinal distributions of the impulsiveness of SXR flares were also determined for the same set of data. The similar power law distributions are also found to hold good in this case, differing only in the values of constants which are designated as $Þ_1$ and q_1.

Table 4. Values of constants $Þ_1$ and q_1.

Range of latitude	Value of $Þ_1$	Value of q_1
0-20° N	3.7100	2.1249
20-40° N	2.7938	1.7622
0-20° S	3.9340	2.4115
20-40° S	2.9764	1.8536

The longitude and latitude distributions of SXR flares were also found out by counting the number of events in different ranges of longitude and latitude respectively. This sort of study was made separately for Northern and Southern latitudes as well as for Eastern and Western longitudes as shown in Figure 4. The distribution is found to follow the relationship as noted below :

$$\text{Log} \frac{\Delta N}{\Delta\theta} = a - b \log \theta \qquad (3)$$

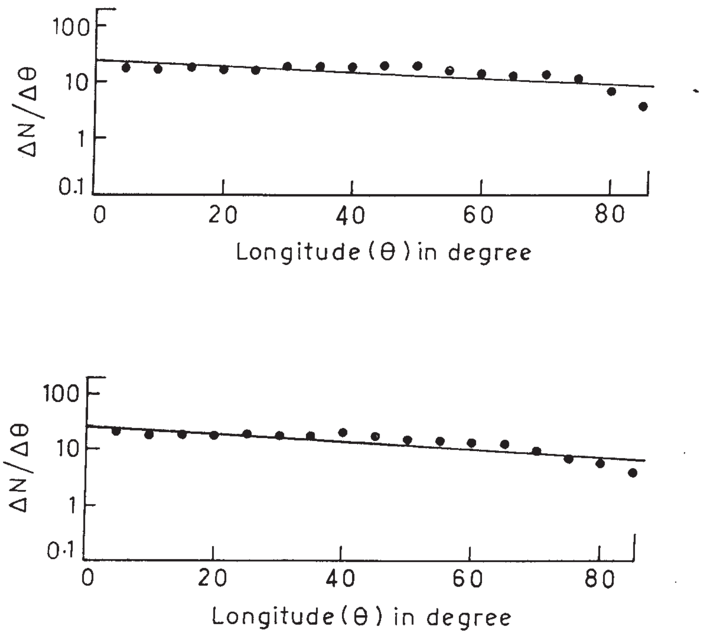

Figure 4. Longitude distribution of solar x-ray flares plotted in semilog scale. Upper panel graph shows the distribution of bursts occurring in eastern longitude and the lower one for the Western longitude.

where $\frac{\Delta N}{\Delta \theta}$ is the number of events per unit angular interval of longitude or latitude, 'a' and 'b' are constants the values of which are given in Table 5.

Table 5. Values of constants 'a' and 'b'.

Position	Value of 'a'	Value of 'b'
North latitude	2.3227	0.05778
South latitude	2.5798	0.0675
East longitude	1.3712	0.0052
West longitude	1.4130	0.0067

After deriving the empirical relationship for the flux distribution and the longitude distribution of SXR flares, we have evaluated the optical thickness of the absorbing layer. The absorption in the plasma is usually expressed through the optical thickness τ. The intensity of radiation, E, is thus attenuated as $E_0 \exp^{-\tau}$. Generally, the optical thickness is expressed as an integral of the absorption along the emission-ray path :

$$\tau = \int_0^R \mu dr \tag{4}$$

where 'μ' is the absorption coefficient, the x-ray source is at zero position, and the observer is at R.

From the flux distribution of SXR flares we get the number of x-ray events in the flux range E to E + dE, from which it follows :

$$N(\theta) = \int_{a(\theta)E_o}^{\infty} 10^m E^{-n} dE = \beta [a(\theta)]^{-n+1} E_o^{-n+1} \quad (5)$$

where β is an arbitrary constant and the reciprocal of a (θ), i.e., $a(\theta)^{-1}$ gives the directivity of SXR bursts. Assuming the curvature of layer above the source region to be negligible, the relation between the directivity and the optical thickness can be determined.

$$\text{Now, } a^{-1}(\theta) = \frac{E_o e^{-\tau \sec\theta}}{E_o e^{-\tau}} = e^{-\tau(\sec\theta - 1)} \quad (6)$$

$$\text{Hence, } \tau = \frac{1}{(n-1)(1-\sec\theta)} \log_e \frac{N(\theta)}{N(O)} \quad (7)$$

In evaluating $\frac{N(\theta)}{N(O)}$ we have used the equation (3) after putting some boundary conditions. We have assumed that N (o) is the number of bursts actually observed in the longitude range 0-5^o and N (π) = 0 which means that no burst is observed at an angle θ = 180^o. Under this consideration the optical thickness of the absorbing layer has been calculated. The values of the optical thickness as derived from the aforesaid formulae are cited in the Table 6.

Table 6. Optical thickness in soft x-ray band.

Longitude ranges	0-30^o	31-60^o	61-90^o
in eastern side	28.47	15.47	4.75
in western side	43.90	21.07	3.47

From the Table 6 it is observed that the optical thickness is maximum for the SXR bursts occurring in the central part of the solar disc and it decreases as one moves from the central region towards the limb region of the disc.

3. Discussion

From the previous analyses it appears that the flux distribution of SXR bursts follows a power law with exponent in the range 1.08-1.20. Similarly, the impulsiveness distribution also follows a power law with exponent in the range 1.62-2.26. In both the cases the value of the exponent decreases slowly with the increase of longitude, although in case of latitudinal dependence no such variation is observed.

By applying the flux distribution and the longitude distribution of SXR bursts, the optical thickness of the absorbing layer is calculated out. The optical thickness of SXR bursts is found to decrease with the increase of longitude which holds good for the bursts occurring both in the eastern and western solar hemispheres.

Recent observations by RATAN-600 yield information about the Brightness distribution over the solar disk of the quiet sun at centimeter and decimeter wavelengths. At longer wavelengths limb brightening has been reported, the results are in good agreement with the theoretical prediction based on a homogeneous model of the solar atmosphere (Gelfreik, 1992). But in the x-ray wavelengths no conclusive report is available as discussed earlier in the present paper. But the present study on SXR flares yields significant centre to limb variation in x-ray intensity, giving rise to pronounced limb darkening.

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