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FUNDAMENTAL FLUX TUBES IN THE SOLAR MAGNETIC FIELDS

1. Significance of the flux-amount; possibility of rising and durability

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ABSTRACT

For a satisfactory understanding of the solar magnetic phenomena it may be advantageous first to interrelate phenomenologically observations involving different scales of length and time and in different layers of the solar atmosphere as, for example, in Piddington's model (1975a, 1976 a, b, c).

As a first step towards the construction of such a phenomenological model it is tentatively concluded that a large number of the solar magnetic phenomena may be due to the rise of 'fundamental flux tubes' of magnetic fluxes $\sim 10^{17}-10^{18}$ Mx (FFT's) presumably originating in large ($>10^5$ km) depths in the convection zone and rising across the observable layers of the solar atmosphere through bends of successively larger sizes

Such a conclusion poses several theoretical questions. As plausible answers to three of these questions it is shown that: (i) FFT's may provide the most efficient convection of energy transferred to them by convective flows in the large depths, (ii) the thermal diffusion and the heating associated with the just mentioned energy transfer may be adequate to raise the FFT from an equilibrium in 'large' depths to 'moderate' ($<10^5$ km) depths in a time scale comparable to $\sim$ 1 y, and (iii) in the large depths the diffusion of the magnetic flux in an FFT might take place on time scales $\approx$3 y.

Key Words: solar magnetic fields—magnetohydrodynamics.

1. Introduction

Presently available observational data about the solar magnetic fields and the related phenomena (especially the solar activity) covers a vast range of length-scales and time-scales at and above the photosphere. On the contrary the nature of the subphotospheric fields, through which all observations must be basically related, remains a subject of theoretical extrapolation. Therefore, detailed theories dealing with a single set of length-scales and time-scales (or with individual types of phenomena) are unlikely either to reveal the nature of the subphotospheric fields or to explain the interrelations between various phenomena on different scales. For example, some properties of the solar magnetic fields on scales larger than or comparable to those of the sunspot cycle (e.g. the periodicity, the period and its anticorrelation to the amplitude etc.) have been reproduced by the 'turbulent field' model based on the randomness of the fields on the smaller scales (cf. Krause and Rädler, 1971; Parker, 1970; Stix, 1972; Roberts and Stix, 1972). However, it is difficult to improve upon these models, or to test their assumptions using the observations on smaller scales (e.g. developments of active regions or the structures and phenomena within an active region). The difficulty is that with decreasing scales, the importance of the magnetic forces which are neglected in these models goes on increasing. Babcock's (1961) schematic model and the Leighton's (1969) semi-empirical models do predict some medium scale observations such as the laws of the sunspot polarities, the butterfly diagrams, the formation and the expansion of bipolar magnetic regions etc. However, even with these models, one would face the same difficulty as with the random field models when one tries to incorporate observations on still smaller scales (eg. the structures and phenomena within an active region). Moreover, as pointed out by Piddington (1972a, 1975b) the models of all the forementioned types also suffer from several basic theoretical difficulties.

Finally we note that the assumptions in the theories of the small-scale phenomena such as sunspots, prominences, etc. may not be so realistic and mutually consistent as to yield synthesized models for the large-scale phenomena like the solar cycle.

Under these conditions it may be advantageous to construct *first a tentative phenomenological model* trying to link (at least crudely to begin with) observed properties of *as many phenomena of different kinds and scales* as possible, (temporarily confining theoretical discussion only to the *plausibility considerations*) and then to go in for: (i) a detailed examination of the theoretical questions involved (ii) inclusion of other observations and (iii) inclusion of more details of the phenomena already considered (i e. the construction of detailed models of the individual types of phenomena).

A large number of phenomena with different scales (upto those of the migrations of the large scale photospheric fields) have already been linked phenomenologically by Piddington (1975a, 1976 a, b, c) in his model of the solar magnetic fields as magnetic flux tubes and flux ropes. However, in his model the plausibility of the assumed structures and processes is discussed only in a preliminary (often purely qualitative) way. Moreover the process of the formation of the flux tubes and flux ropes is not discussed, and (probably because of this) the relation of the incorporated phenomena with the overall reversal of the large-scale field every 11 years or so is not clear.

In the present *two* papers we attempt to develop a preliminary phenomenology relating the small and medium-scale phenomena with the reversal of the large-scale field, along with a semi-quantitative (order of magnitude) discussion of the plausibility of the structures and processes assumed.

First, in Section 2 of this paper we emphasize the fundamental role played by the $\sim 10^{17}-10^{18}$ Mx flux tubes in relating the small and large-scale phenomena. There we tentatively conclude from various known observations that the magnetic phenomena at and above the photosphere are due to emergence of 'fundamental' flux tubes of magnetic flux $\sim 10^{17}$-10^{18} Mx from 'large' depths ($> 10^5$ Km) in convection zone. The following questions immediately arise from the foregoing tentative conclusion :

(i) What is the physical significance of the order of magnitude $\sim 10^{17}-10^{18}$ Mx in a typical FFT ?

(ii) How could such flux tubes (FFT's) be generated in the convection zone ?

(iii) Could parts of such flux tubes rise above the photosphere on time scales < 11 y ?

(iv) Could those portions of the FFT's which remain in the convection zone have life time > 1 y ?

(v) What role would their generation play in the reversal of the Sun's general field ?

In Sections 3, 4 and 5 we attempt to give plausible answers to the questions (i), (iii) and (iv). Questions (ii) and (v) will be dealt with in paper II.

2. Observational Background

2.1 *Observed Flux Tube Structures (In General)*

Structures associated with solar magnetic fields indicate that the fields are in the form of flux tubes in all the observable regions of the solar atmosphere, e.g, 'sunspots', 'pores', 'magnetic knots', 'penumbral filaments' and faculae' in the photosphere (of Zwaan, 1967 ; Beckers and Schröter, 1968 ; Sheeley, 1969), 'arch filamentary systems' (AFS), 'fine mottles', 'spicules', 'Hα - fibrils' in the chromosphere (cf. Frazier, 1972; Foukal, 1971; Beckers, 1968), 'loop prominences', 'coronal rays' and 'streamers', 'polar-plumes', 'X-ray arcs' etc. in the corona (e.g. Bruzek, 1967; Newkirk, 1967; Vaiana *et al*, 1973; and Piddington, 1972 b).

2.2 *Speciality of the '$10^{17}-10^{18}$ Mx Flux Tubes'*

Among the above mentioned structures, the cross sections of the thinnest structures so far resolved (viz. the magnetic knots, penumbral filaments faculae, individual filaments in the AFS, fine mottles, spicules, and the coronal rays) and the corresponding observed or expected field intensities indicate that these structures are associated with flux tubes of magnetic fluxes $\sim 10^{17}-10^{18}$ Mx. Features of the same order of magnetic flux have also been observed in the interplanetary space around ~ 1 AU (e.g. cf. Burlaga, 1972).

The structures with larger magnetic fluxes e.g. spots, AFS, prominences, coronal streamers, etc. often show fine structures of dimensions comparable to the thinnest structures in the respective regions. Decay of pores and sunspots is apparently by dispersal of '$\sim 10^{17}-10^{18}$ Mx flux tubes' from their boundaries (Harvey and Harvey, 1972). These observations indicate that in all the observed layers, the thicker structures may be essentially bundles of $\sim 10^{17} - 10^{18}$ Mx flux tubes'.

It is quite likely that unresolved thinner features exist within the presently resolved "thinnest" structures mentioned here. In fact in the interplanetary space, many observed features do have scales much smaller than those of the 'filaments' observed by Burlaga. However, the question arises why structures corresponding to a specific order of magnitude of magnetic flux, viz $\sim 10^{17} - 10^{18}$ Mx should always be present in phenomena involving so widely different scales and in all the layers of the solar atmosphere. A simple plausible explanation is that these structures may be parts of '$10^{17} - 10^{18}$ Mx flux tubes' rising across the solar atmosphere on sucessively larger scales.

2.3 *Interrelations between the Photospheric Field Developments on Small and Large Scales*

2.3.1 *'10^{17}-10^{18} Mx flux tubes' as common constituents of the active region fields and the quiet region fields*

From comparisons of simultaneous recordings in various magnetically sensitive Fe I lines Livingston and Harvey (1969), Howard and Stenflo (1972), Frazier and Stenflo (1972) and Stenflo (1973) have shown that the photospheric magnetic flux *everywhere* outside the pores and spots, even outside the active regions, is concentrated in flux tubes of fluxes $\sim 10^{17} - 10^{18}$ Mx ('Mangnetic filaments'). Chapman (1974) has arrived at a silmilar conclusion. Stenflo (1974) also has arrived at a similar conclusion independently from his observations about the differential rotation of the photospheric magnetic features and the photospheric plasma although these last mentioned observations do not lead to any estimate of the magnetic flux content of the flux tubes.

Thus, '$10^{17} - 10^{18}$ Mx flux tubes' seem to be the *common constituents* of the *active* region fields *and* the *quiet* region fields at the photosphere. Therefore, the corresponding flux tubes may provide the required link (cf. Introduction) between the small scale observations (e.g. structures and phenomena within the active regions) and the large-scale phenomena (e.g. BMR's and the solar cycle).

The simplest way to reconcile the near absence of any magnetic flux outside the magnetic filaments with the dilution and the migrations of the large-scale (BMR) fields (e.g. Babcock, 1961 ; Leighton, 1964 ; and Howard, 1974) is to suppose that the dilution and the migrations of the large-scale fields are due to horizontal movements of the flux tubes constituting the 'magnetic filaments'. (The vertical movements of the horizontal parts of these flux tubes might account for the disappearance of most of the total photospheric magnetic flux appearing in the thousands of active regions created in an '11y' cycle : Cf. Gokhale, 1975).

2.3.2 *Requirements of the proposed relation*

The connection between the small-scale and the large-scale photospheric phenomena as just proposed would also require that : (i) the real life of magnetic filaments be $\gtrsim 1$ y (the time scales of dilution and migration), i.e. very large compared to the time ~ 1 h for which a magnetic filament can be tracked from its thermal identity as a "magnetic knot" (e.g. Beckers and Schröter, 1968) and that (ii) the flux tubes of the magnetic filaments in the high latitudes (e.g. $\gtrsim 40°$) should reach large depths($> 10^5$ km) in the convection zone where the reverse field for the succeeding cycle is presumably formed.

The first requirement seems theoretically plausible (cf. Section 3.3). The second requirement seems to be indeed satisfied as shown by Stenflo's (1974) observations. From his observations about the rotation of photospheric magnetic fields he concluded that the magnetic fields are in the form of thin flux-tubes which are pushed by faster moving deeper layers in the convection zone and that the largest depths are reached in the high latitudes. He pointed out that these depths must be the depths where fields for successive cycles originate.

2.4 *Summary of subsections 2.2 and 2.3*

Summarizing the last two subsections we tentatively conclude that a large number of the presently observed solar magnetic phenomena of different scales may be due to :

(i) flux tubes of fluxes $\sim 10^{17} - 10^{18}$ Mx originating (presumably) from large depths in the convection zone and finally rising across the observable layers with bends of different sizes (eg. the smaller bends merging with one another to form successively larger bends),

(ii) creation, annihilation and movements of the photospheric intersections of these flux tubes as they rise in the above mentioned fashion.

In view of their possibly fundamental role in such a phenomenology, the $10^{17} - 10^{18}$ Mx flux tubes will be called "fundamental flux tubes" or briefly as "FFT"'s.

This tentative conclusion leads to the theoretical questions listed in Section 1. We now turn to some of these questions.

3. The Role of the FFT's in the Energy Transport in the Convection Zone

3.1 Transfer, Transport and Dissipation of Energy near a Magnetic Flux Tube

At some depth in the convection zone, let there be a more or less horizontal flux tube which has stable magnetic structure and which is long and thin compared to the dimensions of the surrounding flows.

Let us suppose that initially the average density $<\varrho_i>$ of the plasma in it is the same as the density ϱ of the surrounding plasma i. e.

$$<\varrho_i> = \varrho \quad . \tag{1}$$

The lateral pressure equilibrium requires that the temperature T_i of the plasma inside it must be lower than the temperature T of the outside plasma i. e.

$$T_i < T \quad . \tag{2}$$

The magnetic field intensity in such flux tube will be held around a value

$$B \approx \sqrt{4\pi\varrho}\; V \quad , \tag{3}$$

where V is the typical velocity of the flows at that depth (eg. Weiss, 1971). Therefore portions of such a flux tube will act as non-rigid obstacles in the paths of the surrounding flows (excepting for those rare flows, if any, which happen to be parallel to the flux tube).

Any such non-parallel flow will first push the flux tube portion on its way to an extreme position beyond which further displacement is not possible eg. due to the magnetic tension in the same flux tube or due to the pressure of other flux tubes downstream (as in the case of flux tubes near the surface of a cluster of flux tubes). Further transfer of energy from the flow to the flux tubes will be through the turbulence with eddies of dimensions $\sim$d which will have meanwhile developed in the neighbourhood of the flux tube, where d is the mean dimension of the cross section of the flux tube.

If ε_{turb} is the energy input in this turbulence per unit mass per unit time in the steady state, then the mean velocity V_d of these d – size eddies will be given by

$$V_d^3/2d \approx \varepsilon_{turb} \quad . \tag{4}$$

This energy will dissipate at the same rate on the "smallest" scales λ such that

$$W^3\lambda^{-1} \approx 2\varepsilon_{turb} \quad , \tag{5}$$

and

$$\tfrac{1}{2}\lambda W \approx D \quad , \tag{6}$$

where W is the velocity of the "smallest" eddies and D is the largest among the thermal (radiative), the ohmic and the viscous diffusion coefficients.
The last two equations yield:

$$\lambda \approx 2^{3/4}D^{3/4}V_d^{-3/4}d^{1/4} \quad , \tag{7}$$

and

$$W \approx 2^{1/4}D^{1/4}V_d^{3/4}d^{-1/4} \quad . \tag{8}$$

Across a thin layer of thickness $\sim\lambda$ at the boundary of the flux tube the energy will diffuse into the flux tube and thereby heat the plasma inside the flux tube. The power P_h going in such heating will be approximately equal to the power dissipated in the boundary layer, and therefore

$$\begin{aligned} P_{mhd} &\approx \varepsilon_{turb}\varrho\lambda ds \approx \tfrac{1}{2}\varrho W^3 ds \\ &\approx 2^{-1/4}\varrho D^{3/4}V_d^{9/4}d^{1/4}s \end{aligned} \tag{9}$$

where s is the lateral dimension of the flow.

At the same time the d-size eddies will keep on shaking the flux tube transversely and thereby creating slow mhd waves which will longitudinally carry away part of the energy input into the turbulence. The power so escaping will be

$$P_{mhd} \approx \varrho V_d^2 V_s d^2 \quad ,$$

where

$$V_s \approx B/\sqrt{4\pi\varrho}$$

is the velocity of slow mhd waves and waves going in both the directions along the flux tube are taken into account. Since $V_s \approx V$ according to Equation (3), we have

$$P_{mhd} \approx \varrho\, V_d^2\, V d^2 \tag{10}$$

3.2 A "Convection Mechanism" Provided by the Magnetic Flux Tubes

The slow mhd waves generated during the action of the convective flows on a magnetic flux tube will keep on imparting wave energy to the flux tube. This energy will remain associated with the flux tube (This is true also for flux tubes which are only partially below the photosphere since at the photoshphere the slow mhd waves are substantially reflected downward : cf. Savage, 1969; Wilson, 1975).

The heating by P_h will reduce the plasma density within the flux tube and thereby tend to build up the buoyancy forces.

Through these two processes the action of the flows on the magnetic flux tube provides a "convective mechanism" for the upward transport of energy.

3.3 *"The Most Efficient Convection"*

In the foregoing action of a flow on a flux tube, P_h can never exceed P_{mhd} since $P_h > P_{mhd}$ would imply a *preference for local heating* over an *easier escape of energy* (Here we have assumed that the energy exits from the system as fast as it can). Thus

$$P_h \lesssim P_{mhd} \qquad (11)$$

The *most efficient heating* of the flux tube will occur when

$$P_h = P_{mhd} \quad .$$

Further, since the turbulence is generated and maintained by the flow, we shall have :

$$V_d \leqq V \quad .$$

The rate $\mathcal{E}_{turb}$ of *energy extraction* from the flow will be *maximum* when :

$$V_d = V \quad . \qquad (12)$$

It follows from Section 3.2 that the most efficient convective transport from the flows at any given depth will be given by those flux tubes which satisfy equations (11) and (12) *simultaneously*. From equations (9) and (10) it follows that such flux tubes must have mean thickness $d \approx d_c$, where d_c is given by :

$$d_c = 2^{-1/7} D^{3/7} V^{-3/7} s^{4/7} \quad . \qquad (13)$$

The magnetic flux of such a flux tube would be :

$$\phi_c = B d_c^2 = 2^{5/7} (\pi \varrho)^{1/2} D^{6/7} V_d^{1/7} s^{8/7} \quad . \qquad (14)$$

Thus, at any given depth the flux tubes of flux $\sim \phi_c$ provide the most efficient convection of energy from the flows at that depth.

In Table I we give the values of ϕ_c for various plausible values of s and V at different depths in the convection zone. The values of D and ϱ correspond to the values of the thermodynamical quantities in Spruit's (1974) model (According to Spruit, the values of the thermodynamical quantities are more reliable than the values of s and V).

We note that near the base of the C.Z. the value of ϕ_c is of the order $\sim 10^{17} - 10^{18}$ Mx.

Thus we can conclude that flux tubes of flux $\sim 10^{17} - 10^{18}$ Mx may provide *"the most efficient convection"* (i.e. the most efficient heating *and* the maximum extraction) of energy from the flows near the base of the C.Z.

3.4 *Values of* (V_d/V) *and* (P_h/P_{mhd}) *for flux tubes with* $\phi \neq \phi_c$

For a flux tube with $\phi > \phi_c$ we cannot have $P_h = P_{mhd}$ since this would require $V_d > V$. For such flux tubes the fastest disposal of energy would imply $V_d = V$ and this would correspond to $P_h/P_{mhd} \approx (d_c/d)^{7/4}$.

For a flux tube with $\phi < \phi_c$, we *cannot* have $V_d = V$, since this would require $P_h > P_{mhd}$. In this case the fastest disposal of energy would imply $P_h \approx P_{mhd}$ and this would correspond to $V_d/V \approx (d/d_c)^7$.

4. The Rise of the FFT's

4.1 *The necessity of assuming an initial equilibrium for the FFT's*

If the FFT's were formed with the same temperature inside as that outside, then they would rise rapidly above the photosphere by magnetic buoyancy (cf. Parker, 1955). In that case to account for the appearance (and disappearance) of the total photospheric magnetic flux $\sim 3 \times 10^{24}$ Mx within an 11 y cycle from an initial poloidal field of $\sim 10^{22}$ Mx, the FFT's would have to undergo rapid reconnections, *eg. on time scales much less than a few years.* Such reconnections are rather unlikely (cf. Section 5; see also Piddington, 1975 b) Alternatively, the large value of the total photospheric magnetic flux appearing in an 11 y cycle may be accounted for by *elongation of the FFT's* e.g. as in Babcock's (1961) model. For this the FFT's will have to remain within the convection zone for at least a few years or so. *For this reason we assume that the FFT's are either created with the equilibrium conditions (1) and (2) or somehow achieve that state.* The buoyancy forces *will have to be built up* for raising them from such an equilibrium.

4.2 *Role of the heating process of section 3.1 in raising the FFT's.*

In all except the smallest ($\lesssim 10^3$ km) depths the magnetic energy density in an FFT will generally not exceed the kinetic energy density in the surrounding flows and therefore the transverse (bending) forces $F_{flow} \approx \frac{1}{2}\rho V^2 ds$ exerted by the flows will be strong compared to the restoring forces $F_{mag} \approx B^2d^2/8\pi$ exerted by the magnetic tensions (For example, if $B \approx \sqrt{4\pi\rho}\,V$, $F_{flow}/F_{mag} \approx s/d$). Hence the flows will generally impart to the FFT's curvatures $\sim s^{-1}$. These curvatures will be further accentuated by the buoyancy (/gravity) forces on the density-inhomogeneities which would be produced by the longitudinal mass motions resulting from the bending. However due to these processes an FFT will simply keep on fluctuating around a mean horizontal position. Even if some upward bends could occasionally reach the photosphere, their magnetic tensions would not be adequate to pull up the intermediate segments from the large depths. (Such a pulling is necessary for the ultimate removal of most of the total $\sim 3 \times 10^{24}$ Mx or so – magnetic flux formed during each '11 y' cycle.) Thus in the large depths, some independent mechanism will be needed to produce an overall rise of the FFT's.

The heating process of Section 3.1 can provide such a mechanism by keeping on reducing the overall density in lengths (of FFT's) even larger than s and thereby keep on building up the buoyancy forces on length-scales > s.

4.3 *Time scales for "Triggering" the rise of an 'FFT' by heating*

For an FFT portion to start rising as a result of the buoyancy force

$$f_b \approx (\Delta\rho)\, g\, d^2$$

(per unit length), this force must overcome the restoring force of the magnetic tension, which is

$$f_{mag} \approx (B^2d^2/8\pi)\, s^{-1}$$

(per unit length,) owing to the curvatures s^{-1} which exist at any instant of time (cf. Section 4.2). Here,

$$\Delta\rho = \rho - \rho_i \quad ,$$

where ρ_i and ρ are the mass densities within and outside the FFT portion. (Of course, the buoyancy force F_b ($\approx s f_b$) *may not* and *need not* overcome the forces F_{flow} exerted by the *downward* flows which, at any instant, would *temporarily hold down* some portions of the FFTs).

The condition

$$f_b > f_{mag}$$

will be satisfied when $\Delta\rho$ reaches a critical value $(\Delta\rho)_c$ given by:

$$(\Delta\rho)_c \approx \frac{B^2}{8\pi g s} \quad . \tag{15}$$

Starting from an equilibrium satisfying equations (1) and (2) the density reduction $(\Delta\rho)_c$ inside the flux tube may be achieved by raising the internal temperature T_i by an amount

$$(\Delta T)_c \approx \frac{T}{\rho}\,(\Delta\rho)_c \approx \frac{B^2 T}{8\pi\rho g s} \quad .$$

Here, for estimating the order of magnitude of $(\Delta T)_c$ we have ignored the changes δ (Δ P) in the pressure difference $\Delta P = P - P_i$ in comparison to the pressure P_i inside the flux tube and the pressure P outside.

The forementioned rise in temperature would require the following amount of heat input per unit mass of the plasma inside the flux tube;

$$\Delta Q = C_p\,(\Delta T)_c = \frac{C_p B^2 T}{8\pi\rho g s} \quad , \tag{16}$$

where C_p is the specific heat at constant pressure.

During the life of each external flow acting on the FFT, the heating process described in Section 3.2 provides heat energy at a rate:

$$\dot{Q} \approx P_h/(\rho d^2 s) \tag{17}$$

per unit mass per unit time. Hence in a duration

$$t_o = \frac{(\Delta Q)}{\dot{Q}} \approx \frac{C_p B^2 T d^2}{8\pi g P_h} \quad , \tag{18}$$

starting from the equilibrium satisfying conditions (1) and (2), a flow would heat the FFT portion across its path to a stage where the upward thrust of the buoyancy force in that portion would overcome the restoring force of the magnetic tension.

Taking P_h from (9), writing $d = \phi_o^{1/2}B^{-1/2}$ where $\phi_o \approx 10^{18}$Mx or 10^{17}Mx and using Equation (3), we obtain the following expression for the t_o corresponding to the FFT's.

$$t_o \approx \frac{C_p\, T\, \phi_o^{7/8}\, V^{9/4}}{2^{13/8}\, g\pi^{7/16}\, \rho^{7/16}\, D^{3/4}\, V_d^{9/4}\, s}$$

Finally, since $\phi_o \approx \phi_o$ near the base of the C.Z. and $\phi_o > \phi_o$ in the upper layers, we take $V = V_d$ (Cf. eg. Section 3.3).

Thus

$$t_c \approx \frac{C_p\, T\, \phi_*^{7/8}}{2^{13/8} g\, \pi^{7/16}\, \varrho^{7/16}\, D^{5/4}\, V^{9/8} a_3} . \qquad (19)$$

This gives the values t_c corresponding to the different depths as given in Table 1. These values should be compared with $t_{rd} \approx d^2 D_{rad}$ which are the time scales for direct heating by radiative diffusion.

When t_c or t_{rd} is less than the life t_{flow} of a typical flow, it represents the duration required for triggering the rise of a typical FFT segment of length $\sim s$. However since each FFT segment in the given depth will be acted upon by some typical flow in that depth, t_r or t_{rd} would also represent the duration for triggering the rise of the *whole length of the FFT in that depth.* From Table 1 we find that in most cases t_{rd}, $t_c > t_{flow}$. In such cases, the longitudinal mass motions during the rearrangements of the flows will tend to average out the density deficiencies $\Delta\varrho$ in different segments. However, in order of magnitude the average rate of heating on length scales $> s$ and and time scales $> t_{flow}$ will be the same as that given by equation (17), and all other equations will still be valid except that $\Delta\varrho$, ΔQ, etc. will have to be replaced by their averages $<\Delta\varrho>$, $<\Delta Q>$ etc. for lengths $> s$. Therefore *even when* t_{rd} or $t_c > t_{flow}$, t_{rd} (or t_c) can be taken as a *crude estimate* of the duration required by the heating process of Section 3.1 to *"trigger"* the overall rise of an FFT starting from an "equilibrium" at a given depth.

4.4 Time Scales of Rise

The condition (15) would impart the rising FFT segments accelerations of the order :

$$a \approx \frac{(\Delta\varrho)_c\, g}{\varrho} \approx \frac{B^2}{8\pi\varrho s} ,$$

when $B^2 \approx 4\pi\varrho\, V^2$, we have :

$$a \approx V^2/2s .$$

Hence the time scale of actual rise across a scale-height (assumed to be $\approx s$) will be

$$t_{rise} \approx 2s/V.$$

The total time required by a flux tube to rise through a scaleheight will be :

$T_{tot} \approx t_c + t_{rise}$ or $T_{tot} \approx t_{rd} + t_{rise}$ whichever is smaller.

4.4.1 *The rising from 'large' (>10^5 km) depths to 'moderate' (<10^5 km) depths*

The estimates t_c near the base of the convection zone and near the middle of the convection zone are comparable *to* t_{rd}. This indicates that in these depths the heating mechanism of Section 3.1 will contribute substantially to the heating of the FFT's. The total heating will raise the FFTs from the base of the convection zone to well above the middle of the convection zone in a period $\sim T_{tot}$ which is less than a *few years.*

4.4.2 *The subsequent rising*

Tho observed length scales of the arch filamentary systems (eg. Frazier, 1972) indicate that the mechanism just discussed may not be raising an FFT *all the way above the photosphere in one single step.*

This suggests that after being raised to well above the middle of the convection zone the FFT's must be reaching *another 'quasiequilibrium'* before reaching the photosphere.

The values of t_c and t_{rd} in depths $\approx 10^4$ km indicate that the heating mechanism discussed here will be much more efficient than the heating by direct diffusion. However even this more efficient *mechanism* may or may not be adequate to raise the FFT's from the quasi equilibrium in such depths to the photosheric layers on time scales <11 y. [This will allow the FFTs to remain in tho convection zone for long enough to be highly elongated by the differential rotation of the convection zone as required for producing hundreds of active regions with total magnetic flux two orders of magnitude larger than the poloidal flux (cf. eg. Babcock, 1961)]. However, the forces F_{flow} exerted by the *upward supergranulation flows* may eventually bring segments of comparable lengths right upto the photosphere where the local buoyancy forces in such segments may become dominant and lead to their further rise (eg. Parker, 1955). Finally the magnetic tensions, which dominate at small depths and above the photosphore, may pull up the intermediate segments above the photosphere when these segments are *released from the withholding forces exerted by the downward supergranulation flows* (eg. Gokhale, 1975).

5. The life of an FFT-Portion Near the Base of the Convection Zone

For the decay of the electric currents in a portion of an FFT, the maximum magnetic diffusivity will be

Table 1. Crude estimates for ϕ_c, t_c, t_{rd}, t_{rise} and T_{tot} at different depths in the C.Z. (Values in brackets are for fluxtubes with $\phi_* \approx 10^{17}$Mx.)

depth (cm)	The model of the C.Z.	V (cm s^{-1})	s (cm)	t_{flow} (s)	ϕ_c (Mx)	t_c (s)	t_{rd} (s)	t_{rise} (s)	T_{tot} (s)
~ 10^8 (near the top of the C.Z.)	Spruit	4.4×10^4	3×10^7	7×10^2	2×10^{13}	10^8 (10^7)	10^8 (10^7)	1.5×10^5	10^8 (10^7)
~ 10^9 (Comparable to supergranulation scales)	Spruit	1.2×10^4	3×10^8	2.5×10^4	4×10^{12}	3×10^9 (3×10^8)	10^{11} (10^{10})	5×10^4	3×10^9 (3×10^8)
~ 10^{10} (middle of the C.Z.)	Spruit	3.4×10^3	3×10^9	10^7	5×10^{16}	3×10^7 (3×10^6)	10^8 (10^7)	2×10^6	3×10^7 (5×10^6)
~ 2×10^{10} (near the base of the C.Z.)	Spruit	4×10^2	5×10^9	1.2×10^7	4×10^{17}	7×10^7 (7×10^6)	10^7 (10^6)	2.5×10^7	3.5×10^7 (2.5×10^7)
	Simon & Weiss (1968)	7×10^2	10^{10}	1.5×10^7	6×10^{17}	2×10^7 (2×10^6)	"	3×10^7	4×10^7 (3×10^7)
	arbitrary	10^4	5×10^9	5×10^5	6×10^{17}	2×10^6 (2×10^5)	"	10^6	3×10^6 (10^6)
	arbitrary	10^4	10^{10}	10^6	1.3×10^{18}	10^6 (10^5)	"	2×10^6	3×10^6 (2×10^6)

operative during the presence of the turbulence maintained by the external flows at its boundaries (Cf. section 3.1). The magnitude ν_{turb}^{max} of this diffusivity will be given by

$$\nu_{turb}^{max} \approx (1/2)\, W\delta \qquad (20)$$

where

$$\delta \approx (4\pi\varrho W^2/B^2)\lambda \qquad (21)$$

is the distance through which an eddy in the turbulence displaces the magnetic flux across its path against the magnetic tension, and λ and W are respectively the mean eddy size and the mean eddy velocity given by equations (7) and (8). Equation (21) follows from the comparison of the magnetic and the eddy forces.

A crude estimate T_1 of the "life" of an FFT portion (diameter d_c) will be d_c^2/ν_{turb}^{max} which together with equations (7), (8), (13), gives:

$$T_1 \approx 2\, D^{-3/7}\, V^{-4/7}\, s^{10/7} \quad . \qquad (22)$$

For 'FFT-portions' in depths > 10^5 km this gives, with values of D, V and s as before,

$$T_1 \approx 10^{10.5}\, s \;(\approx 10^3\, y) \quad .$$

The actual "life" T_2 may be

$$T_2 \approx (\Delta/d_c)\, T_1 \;, \qquad (23)$$

if the eletric currents are concentrared in a *thin sheet of thickness* Δ *at the boundary* of the FFT.

However, if the eletric currents are generated from the interaction of the flows of velocity $V \approx 10^4$ cm s^{-1} and an initial weak field of intensity $B_i \approx 3$ G then the current density J may not exceed $\sigma B_i V \approx 10^{-9.5}$ emu (where $\sigma \approx 10^{-5}$ emu is the electrical conductivity in the depths under consideration). The product $J\Delta$ (which is the current per unit length) must be $\approx 10^4$ emu for yielding an average field $\approx 10^4$ G in the flux tube (eg. as in the case of a current carrying solenoid), and hence the current sheet thickness Δ must be $\approx 10^{4.3}$ cm. (This is also just sufficiently large for the applicability of the turbulent diffusivity with eddies of sizes $\lambda \sim 10^4$ cm which is used here.) Thus, if the electric currents in an FFT portion in large depths happen to be concentrated in a thinnest possible layer at the boundary, the "life" T_2 of the portion will be $\approx 10^8$ s (i.e. 3 y). The "life" will be larger if the current sheets are thicker.

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