Bull. Astr. Soc. India (1985) 13, 339–345

Einstein gravity as a low-energy effective theory: Comparison with weak and strong interactions*

C. Sivaram *Indian Institute of Astrophysics, Bangalore 560 034*

Received 1985 June 27

Abstract. Einstein's theory which correctly describes gravity at long distances (low energies) is first compared with Fermi's theory, which describes weak interactions at low energies. Analogous to strong interactions at high energies being described by the gauge invariant finite theory of quantum chromodaynamics (QCD), gravity at high energies would be described by an asymptotically free-scale invariant theory. Then just as an effective theory of pions, describing low energy strong interactions, emerges from QCD at low energies, Einstein's theory would be the low energy effective counterpart of this gauge invariant high energy theory, ironically arising from the scale invariance being broken by quantum fluctuations. Analogies between QCD and scale invariant gravity are discussed and the cosmological constant problem is also considered in this context.

key words: Gravitation—weak interactions—strong interactions

1. Introduction

Einstein's general theory of relativity provides within experimental errors a very good description of the behaviour of gravity at large distances, providing a basis for the understanding of astronomical systems and the universe at large. However, when one tries to combine this elegant theory with the laws of quantum mechanics one runs into problems. For instance the cross-sections and amplitudes for processes involving the interaction of the quanta of the gravitational field with particles of other fields diverge at high energies; a dimensionless amplitude of order G^n diverging as $G^n E^{2n}$. This 'nonrenormalizability' of the theory may be traced to the coupling constant $(16\pi G)^{-1}$ which appears in the Hilbert action for the Einstein field equations, i.e. $I_{\rm grav} = (1/16\pi G)\, R$, R being the curvature scalar, having the dimensions of $({\rm mass})^2$.

*Received 'honourable mention' at the 1985 Gravity Research foundation essay competition.

Similar behaviour appears in the case of the Fermi weak interaction theory which provides a good description of beta-decay processes, like the decay of neutrons and muons, at low energies. However, the Fermi interaction is also characterized by a dimensional constant G_F so that the cross-sections for processes like $e^+ + e^- \rightarrow$ neutrinos diverge as $G_F^2 E^2$ with energy. Thus the Fermi theory is another example of a non-renormalizable theory which, however, provides a good description of the low energy; *i.e.* $E < \approx 1/\sqrt{G_F} \sim 100$ GeV phenomenon. But now we know that the Fermi theory is only a long-wavelength (*i.e.* low-energy) effective theory for the weak interactions. The correct fundamental theory describing the weak interactions (manifesting at high energies) is a renormalizable gauge theory with SU(2) $\times$ U(1) symmetry and characterized by a *dimensionless* coupling constant which, at energies $\sim$ 100 GeV, becomes identical with the electromagnetic coupling constant thus uniting the two interactions above this energy. This gives a relation between the electric charge e and G_F as $G_F \approx e^2/m_W^2$, where $m_W \approx 100$ GeV is the intermediate boson mass. Perhaps Einstein's gravity is also similarly only an effective long-wavelength theory, the correct theory at high energies (now characterized by $E \sim 1/\sqrt{G} \sim 10^{19}$ GeV) being a renormalizable gauge theory with a dimensionless coupling constant.

To give another example, we know that the theory of strong interactions at low energies ($\sim$ 1 GeV) between pions and nucleons is not a renormalizable theory; it is based on a chiral SU(2) $\times$ SU(2) symmetry. The coupling constant is > 1 at these energies and all 'perturbative' amplitudes are divergent. There is no renormalizable theory of the pion-nucleon (for *e.g.* 3-3) resonances. But we know that this chiral theory like the Fermi theory is only an effective long-wavelength theory. The underlying theory of strong interactions manifesting at high energies is quantum chromodynamics (QCD) which describes the fundamental colour interactions between quarks and gluons. This is a renormalizable gauge theory with local SU(3) colour as the underlying symmetry. The effective low energy nonrenormalizable theory of pions (which are bound states of quarks) then emerges from this fundamental theory of QCD. So one does not worry about the low energy effective theory being problematic. This analogy with strong and weak interactions strongly suggests that Einstein's theory of gravity which is badly behaved at high energies is again an effective long-wavelength theory. The fundamental theory of gravity at high energies would then not be described by the Einstein-Hilbert action. It may be based on an asymptotically free (coupling tending to zero at high energies) scale invariant action like that of Weyl. The quadratic form of the scale invariant action would also bring gravity in conformity with the gauge-invariant Yang-Mills action characterizing other interactions as these are also of quadratic form.

In section 2 we point out similarities and differences between the Fermi and Einstein theories and in section 3 develop the analogy with QCD to get a theory of gravity at high energies. Section 4 further explores these ideas and also deals with the cosmological constant problem.

2. Weak and gravitational interactions

Many authors (for *e.g.* Sivaram 1975, 1977; Zee 1979; De Sabbata & Gasperini 1979) have used the fact that both the Newtonian and Fermi constants have dimensions of $(\text{mass})^{-2}$ to attempt to construct broken-symmetric scalar-tensor theories of gravity where G is related to the vacuum expectation value (VEV) of a scalar field as $G \approx 1/\phi^2 \approx \hbar c/M^2_{(\text{Planck})}$ analogous to the Fermi constant being generated by a Higgs field VEV as $G_F \approx e^2/M^2_W$ which gives masses to the intermediate bosons. However, there is an important difference between the behaviour of the coupling constants in the Fermi theory and in the Einstein-Hilbert theory. To see this we write the effective dimensionless actions in the two cases as ($\hbar = c = 1$)

$$I_{\text{fermi}} = \int d^4x \, \frac{G_F}{\sqrt{2}} \, \bar{\psi}_1 \psi_2 \bar{\psi}_3 \psi_4; \qquad \dots (1)$$

where ψ is fermion field.

We have a four-fermion interaction which characterizes the Fermi theory. ψ has dimensions of $(\text{mass})^{3/2}$, being a spinor field.

The integration measure d^4x has dimension $(\text{mass})^{-4}$ thus the action density $G_F(\psi)^4$ must have dimensions $(\text{mass})^4$, so that I is dimensionless. As $(\psi)^4$ has dimensions $(\text{mass})^6$, it follows that G_F has dimension $(\text{mass})^{-2}$. Now in the Einstein-Hilbert action for gravity,

$$I_{\text{grav}} = \int d^4x \, (-g)^{1/2} \, \frac{1}{16\pi G} R. \qquad \dots (2)$$

R has dimension $(\text{mass})^2$, $d^4x(-g)^{3/2}$ has dimension $(\text{mass})^{-4}$, so that $(16\pi G)^{-1}$ has dimension $(\text{mass})^2$. As G_F has dimensionality of mass to a *negative* power, the dominant contributions to the weak interaction will arise from the *lowest mass* intermediate states, i.e. from intermediate bosons of mass ~ 80 GeV. That means that even a hierarchy of intermediate bosons with larger and larger masses (if they exist) will not affect the value of G_F much and hence the weak interaction strength. Also one cannot have weak intermediate bosons of masses less than ~ 80 GeV, as they will push up the value of G_F to more than what is observed. On the contrary, the inverse Newton's constant $(16\pi G)^{-1}$ which appears in the Hilbert action for gravity has dimensionality of mass to a *positive* power and thus the dominant contribution to G^{-1}, will come from the *highest* mass scales, which would be of the order of the Planck mass $M_{\text{Pl}} \approx 10^{19}$ GeV, to account for the observed G.

All the hierarchy of *lower mass scales* including M_W's, the GUTs mass scales, *etc.* would not affect G and hence the gravitational action. An interesting application of this feature is to show that effects of quantum gravity at Planck mass scales would not alter much the proton decay rate predicted by GUTs models without including gravity. The reason is that in these theories the nucleon decays occur through effective four-fermion interactions of the Fermi type but mediated by exchanges of superheavy bosons of mass $M_X \approx 10^{15}$ GeV, rather than intermediate bosons of 80 GeV. So by analogy with the Fermi action [equation (1)] it is easily seen that the coupling constant G_U in this case would have again dimensions of $(\text{mass})^{-2}$, so that the dominant contributions to the nucleon decay would come from the *smallest* mass scales (*i.e.* $\sim M_X$) so that the much larger Planck mass scale ($\approx 10^4$ larger) would affect the coupling only by a factor of $\approx 10^{-8}$. Thus the effects of quantum gravity at M_{pl}, would not much alter the predicted nucleon rates of GUT models. The eifect of the spacetime metric $g_{\mu\nu}$ on the axial vector constant and on weak interaction rates through terms of the type $\bar{\psi}\gamma^\mu [C_V \pm \sqrt{-g}\, C_A\gamma^5]\psi$ has been pointed out by Sivaram & Sinha (1979).

3. Gravity and strong interactions

Analogous to the emergence of an effective low energy strong interaction theory of pions from the underlying high energy gauge theory of QCD, we may expect Einstein's theory to emerge as the low-energy effective theory of an underlying high energy gauge theory of gravity. We explain this below. The strong interactions have a global chiral SU(2) × SU(2) symmetry in the limit of vanishing U and d quark masses or, generalizing to the limit of massless quarks of n flavours, the QCD Lagrangian has the symmetry: $[\mathrm{SU(n)_L} \times \mathrm{SU(2)_R}]_{(\mathrm{global})} \times [\mathrm{SU(3)}]_{\mathrm{colour(local)}}$. At the energy scale below about $\lambda_{\mathrm{QCD}} \sim 0.5$ GeV, the colour coupling constant between quarks becomes strong, and massless scalar bound states form.

In other words the local $\mathrm{SU(3)_{colour}}$ singlet operator $\phi(x) = \lambda_{\mathrm{QCD}}^{-3}\, \psi_i(x)\, \bar{\psi}_j(x)$ $(i, j = 1, \ldots, N)$ develops a VEV, $\langle \phi_{ij}(x) \rangle \sim \delta_{ij}$, thus breaking the global chiral symmetry down to diagonal $\mathrm{SU}(n)_{L+R}$, producing $(N^2 - 1)$ massless Goldstone bosons; and the low energy (*i.e.* $\leqslant \lambda_{\mathrm{QCD}}$) effective action must *retain the full original chiral symmetry*. This constrains it to be of the type

$$I_{\mathrm{eff}} \sim \int d^4x \lambda_{\mathrm{QCD}}^2\ \mathrm{Tr}\,(\partial_\mu\phi\, \partial_\mu\phi^+) + \ldots, \text{ which is invariant under } \phi \to S_L \phi\, S_R^+,$$

$S_{L,R} \in \mathrm{SU(n)_{L,R}}$. The pion fields represent small fluctuation about the vacuum and are related to ϕ through the 'phase' $\phi(x) = \exp\,(i\lambda_{\mathrm{QCD}}^{-1})\,\pi(x)$. The π-fields transform linearly under the unbroken $\mathrm{SU(n)_{L+R}}$ but nonlinearly under the broken $\mathrm{SU(n)_L} \times \mathrm{SU(n)_R}/\mathrm{SU(n)_{L+R}} = \mathrm{SU(n)_{L-R}}$ (*e.g.* Peskin 1982). A similar situation can be envisaged for gravity, with the group of general coordinate transformations (GCT) playing the role of the global $[\mathrm{SU(n)_L} \times \mathrm{SU(n)_R}]$ chiral symmetry in QCD and the corresponding counterpart of $\mathrm{SU(3)_{colour}}$ would be S, the subgroup of the conformal group generated by Lorentz (M_{ab}), dilatation (D), and special conformal (k_a) transformations (*i.g.* all generators except translations). Then in four dimensions the unique action which has the invariance GCT × S_{local} and has no dimensional coupling is the Weyl action (de Wit 1981)

$$I_W = \alpha \int d^4x \sqrt{-g}\, C^{\alpha\beta\gamma\delta} C_{\alpha\beta\gamma\delta}$$

where $\alpha(q)^2$, the running coupling constant for S_{local} is dimensionless and asymptotically free. tIis also the unique *locally scale invariant* action. $C^{\alpha\beta\gamma\delta}$ is constructed out of the corresponding Riemann curvature tensor, the covariant derivatives involving the gauge fields associated with the generators [P, M, K, D] of the conformal group. Analogous to QCD, various local S invariant operators would acquire non-zero VEVs at energy scales below $E_{\rm Planck}$, where α becomes large and dimensional. A set of second rank S invariant tensor operators would be given by

$$S_{\mu\nu}(x) = f^{-1/2} f^{a}_{\alpha}(x) f^{b}_{\beta}(x)\, \eta_{ab}\,; \quad f = \det f^{a}_{\alpha}\,;$$

fs are the vierbeins gauge fields for P. $S_{\mu\nu}$ can now develop a non-zero VEV, a simple choice being $\langle S_{\mu\nu}(x) \rangle \sim \eta_{\mu\nu}$.

We note that here the metric is no longer a fundamental field; the vierbeins (*i.e.* spinor fields) are the basic entities, the expectation values of their product (*i.e.* analogous to bound states) generate the metric at energies $\sim M_{\rm Planck}$. The above VEV does not break $S_{\rm local}$. It breaks GCT invariance which has been broken to the Poincare subgroup; and associated with this symmetry breaking there are massless spin-2 Goldstone fields described by $S_{\mu\nu}$ (*i.e.* gravitons). But in analogy with QCD the low energy effective action must retain the full original invariance, (*i.e.* $SU(n)_L \times SU(n)_R$ in the case of strong interactions and general coordinate transformations in case of gravity). Thus the effective low energy action which in this case must retain the GCT invariance is constrained to be of the form

$$I_{\rm eff} = \int d^4x \sqrt{-S} \times (\alpha M^4_{\rm Pl} + \beta M^2_{\rm Pl} R + \gamma R^2 + \delta R^2_{\mu\nu} + M \epsilon^{-2}_{\rm Pl} R^3 + \ldots + \omega M^{4-2n}_{\rm Pl} R^n).$$

$\alpha M^4_{\rm Pl}$ would be identified with the cosmological constant term, and $\beta M^2_{\rm Pl} R$ with the Hilbert action characterizing the low energy Einstein gravity with $\beta M^2_{\rm Pl} = (16\pi G)^{-1}$. It turns out that the terms involving higher powers of the curvature are suppressed at ordinary energies E by powers of $(E/M_{\rm Planck})$, the term with R^n by factor of $(E/M_{\rm Planck})^{2n-4} \ll 1$. Thus for all practical purposes the low energy effective action for gravity would just have the usual Hilbert term $R/16\pi G$ of general relativity and a cosmological constant. However, observational limits on the cosmological constant would require the constant to be ridiculously small, *i.e.* $\alpha_0 \approx 10^{-120}$. We shall discuss this in the next section.

We have thus made precise the philosophy that the usual theory of general relativity only emerges at large distances and that at distances of the Planck length, conformal gauge theory of gravity dominates. If we write out above action with the couplings explicitly containing G, it would have the form (omitting the cosmological constant for present)

$$I_{\rm eff} = \int d^4x \sqrt{-g} \left[\frac{R}{16\pi G} + \hbar R^2 + \frac{\hbar^2 G}{C^3} R^3 + \ldots + \hbar \left(\frac{\hbar G}{C^3} \right)^{n-2} R^n \right].$$

We note the remarkable fact that apart from the usual Hilbert-Einstein term involving R, the higher powers of the curvature have G appearing in the numerator. This is similar to the case of the weak interaction action where G_F (which like G has dimensions of (mass)$^{-2}$) also appeared in the numerator. In the weak interaction case we had argued that this implied that the dominant contribution will arise from the lowest mass intermediate states. Here the lowest mass states are of the Planck mass, which means that even at energies much larger than Planck energies, the gravitational interaction would chiefly be determined by Planck mass states as the effects of larger mass states fall off as M^{4-2n} for the R^n term.

The interaction above Planck energies does not have the R term, so it will be dominated by the quadratic R^2 terms with dimensionless coupling constant. At energies below the Planck scale, the Hilbert term is induced and will be determined by the highest mass scale now present as G now apears in the denominator, this being the Planck mass as already discussed. We also note the Hilbert term is the only one not having ħ; thus the other terms are to be pictured as quantum gravity corrections appearing at Planck energy and above. We therefore see why gravity at low energies is determined only by the Hilbert term (the usual GR). At very high energies the quadratic terms will dominate. The field equations at very high energies will be of the form:

$$\sum_n M_{\rm pl}^{(4-2n)} \left\{ R^{n-1}\left(R_{ab} - \frac{1}{2n} g_{ab} R\right) - g^{cd} g_{ab} (R^{n-1})_{;cd} - (R^{n-1})_{;ab} \right\} = 0.$$

Of course for simlpicity we have ignored terms of type $R_{\mu\nu}R^{\mu\nu}$ and higher products.

4. Concluding remarks

The scale invariant Weyl action which is quadratic in the curvature is the gravity analogue of the QCD action quadratic in the Yang-Mills field. At the appropriate high energy scale Weyl and QCD actions describe gravity and strong interactions respectivly. They have some remarkable properties in common. For instance in QCD, colour strong interaction between quarks is linear, $V \propto r$, *i.e.* only systems with zero total colour have finite energy. For the scale invariant Weyl gravity, the potential also grows linearly with distance as the corresponding Poisson equation is $\nabla^{-4} m\delta^3(\gamma) \sim m\gamma$, the field equations being of fourth order. Thus for scale invariant gravity only systems with zero total energy have finite energy, *i.e.* energy is confined, analogous to colour in QCD. The Einstein-Hilbert term breaks the scale invariance of the Weyl action.

Another way of looking at it is as follows. We can insist by analogy with Yang-Mills fields describing other interactions that the 'true' gravity action be quadratic in the fields and scale invariant. This would make it of the form $I \sim aC^{\alpha\beta\gamma\delta}C_{\alpha\beta\gamma\delta} + bR^2$.

However, scale and conformal invariance is always broken by quantum fluctuations. The only term which is not scale invariant but invariant under general coordinate transformations alone is the Einstein term κR. Thus when quantum fluctuations are turned on, the Einstein term is brought into the original action as the scale invariance is now broken. With regard to Einstein's well known attitude to quantum mechanics it is indeed ironic that quantum fluctuations should lead to general relativity.

Of course a large cosmological constant should also be generated in the breaking of scale invariance. To understand then why it is so small, we draw the analogy again with QCD, where one expects strong violations of CP symmetry induced by actions of the type $\theta F_{\mu\nu} \tilde{F}_{\mu\nu}$, whereas in reality it is very weak, *i.e.* $\theta < 10^{-8}$. The solution proposed in this case (Peccei & Quinn 1977) involves absorbing into the phases of a Higgs field $\phi(x)$ by performing a chiral transformation removing such a term, *i.e.* $\phi(x) \rightarrow \phi(x) \exp[i(\alpha(x) + \theta)]$, $\alpha(x)$ being the phase. A similar attempt can be made to remove the cosmological constant (Sivaram 1985, in preparation).

We had pointed out in the last section that the quadratic terms will dominate at well above Planck energies. It may be pertinent to ask what would be the consequences of the modified field equations with say the Robertson-Walker metric for the very early universe. It turns out that the usual singularity still persists, and one cannot get a nonsingular solution. The only effect of the quadratic and higher terms seems to be a modulation of the collapse by small oscillations around the standard Friedmann solution. Briefly the usual radiation dominated solution $R \approx (2at)^{1/2}$ is modified to

$$R \approx (2at)^{1/2} [1 + (\sqrt{\epsilon}/t)^{3/4} \sin (t/\sqrt{\epsilon} + \phi)], \; \epsilon = 6\gamma + 2\delta,$$

γ, δ being the coefficients of the R^2 and $R_{\mu\nu} R^{\mu\nu}$ term.

The following two points should be noted.

(i) A combination of the Hilbert and Weyl terms, *i.e.* $xR + aR^2 +$ etc., gives field equations whose solution has a 'Newtonian' limit *i.e.* given by modified Poisson equation $\nabla^4 \phi + \nabla^2 \phi = 0$ with the potential given by $\phi \approx \frac{A}{r} - \frac{B}{r} \exp(-M_{Pl}/r)$ which simply means that we have the usual GR and Newton's law at distances $\gg$ Planck length. This once again explains why gravity behaves the way it does.

(ii) As stated in the last section, all classical solutions of the field equations following from the action $I \sim aC^2 + bR^2$, have zero total energy for $ab > 0$. This would have interesting consequences for the very earliest phases of the universe when gravity would have been described by such equations. The initial state would have been a fluctuation with zero total energy, thus explaining the equality between kinetic and potential energies to about 10^{-60} at the Plank era. Breaking of the scale invariance near about the Planck epoch would have brought in the Einstein term which would then have dominated the large-distance behaviour subsequently as discussed earlier.

References

De Subbata, V. & Gasperni, M. (1979) *Gen. Rel. Grav.* **10,** 731.
De Wit, B. (1981) in Supergravity (ed.: J. G. Taylor), Camb. Univ. Press.
Peccei, R. & Quinn, H. (1977) *Phys. Rev. Lett.* **38,** 1440.
Peskin, M. E. (1982) in *Gauge Theories* (ed. : J. Ellis) North Holland.
Sivaram, C. (1975, 1977) *Nuovo Cim. Lett.* **13,** 351; Ph.D. Thesis, I.I.Sc., Bangalore.
Sivaram, C. & Sinha, K. P. (1979) *Phys. Rep.* **51C,** 111.
Zee, A. (1979) *Phys. Rev. Lett.* **42,** 417.