to but a few hundredths of a second, as a minimum. In consequence of the variability of the coefficient of the annual term above-mentioned, the apparent average period between 1840 and 1855 approximated to 380 or 390 days ; widely fluctuated from 1855 to 1865; from 1865 to 1885 was very nearly 427 days, with minor fluctuations; afterwards increased to nearly 440 days, and recently fell to somewhat below 400 days.

An Enquiry into the Equilibrium and Stability of the Solar System.

By Rev. A. C. Ridsdale, M.A., F.R.A.S., F.R.Met. Soc., F.Ph.S., M. Lond. Math. Soc., A.L.C.M., Foreign Member Societe Astronomique de France.

The mechanism of the solar system is independent of forces from without and is self-sufficing. And this, because it is surrounded on all sides by an immense void, which effectually shuts off any gravitational interference of sensible amount on the part of the hosts of heaven. The solar system is under the almost absolute sway of the Sun, except for the small mutual perturbations of his attendant satellites. It is well known that the prevailing view of sciene at present is in the main in accord with the theories of Laplace, that the mass of the solar system was in remote ages an enormous globe of nebulous matter, revolving very slowly from west to east. By cooling it contracted, and because its moment of inertia became thereby diminished, its rotational velocity increased, and when its rotational velocity exceeded a certain critical point, centrifugal force overpowered centripetal, and rings were thrown off to form planets. The present distances therefore of the planets represent the lengths of the radii of this primitive nebula at the moment when each particular planet was abandoned. Their periods also represent the period of rotation of the nebula at the moment of their birth. It has generally been held, judging from the Sun's present period of rotation, that when it was as wide as the various planets' orbits, it would have rotated with like periods. Thus when it extended out to Neptune's distance it rotated once in 164 years, when as wide as Saturn's orbit or Jupiter's orbit once in 30 years and 12 years respectively. There would have been the same equilibrium between centrifugal and centripetal forces as exist now, for a globe attracts as if it were condensed

into its centre. But of the density of a globe increase towards the centre, the inner particles will rotate quicker than the outer, dragging the outer particles along with them, increasing their rotational motion, and hence their centrifugal force beyond the proportion of their distance from the centre, and thus causing them to fly off at a tangent, when they would henceforward maintain their distance from the centre, in spite of further contraction on the part of the main mass. What existed before this primitive enormously expanded and diffused nebula whose density must have been nearly two thousand million times less than our atmosphere, who can say? All that can be safely asserted is that its heat, and motion and gravity, etc., could not have cause itself, but must have depended from the beginning upon the Prime Author of all things, amongst which things is included the process we name Evolution. And further, the mechanism of the solar system is not all so obedient to the mathematical laws of Laplace as was once supposed. Since his time observational instruments of much greater power have been made, and consequently many hitherto unknown anomalies in the mechanism of the system have been detected, which are still a sore puzzle to the mathematician. For example, the satellites of the outermost planets revolve in such a way as to indicate that the axis of their primaries were greatly displaced at the time when they separated off. But how their axis of rotation could ever have been displaced whilst they were revolving is indeed an enigma, for revolving bodies resist any displacement of their plane of rotation. Again it has lately been discovered that the exterior satellite of Saturn actually revolves in the exact opposite direction from that of all the other satellities. It seems impossible to account for this astounding phenomenon, otherwise than by supposing the direction of Saturn's rotation to have once been the opposite to what it is at present. And the force of tidal friction is deemed by some competent to have produced this striking alteration, in spite of the fact that at the distance of Saturn its power is not the twenty-thousandth part of what it is at the Earth's distance from the Sun. Thus when Phoebe was thrown off from Saturn, he then had a radius of eight million miles, and revolved in a period of a year and a half from east to west. When the next satellite Iapetus became detached from Saturn, he was then revolving in eighty days which is the period of that satellite, but no longer from east to west, but from west to east! Again since Laplace's time the asteroids have been discovered, which to a great extent set at defiance the rules regulating the planetary motions, both in respect of the eccentricity and inclination of their orbits. Again there is nothing more utterly perplexing to the mathe-

matician than that the angular velocity of one of Mars' two Moons should actually exceed that of Mars' himself. For it is contrary to all known laws to suppose that Mars should have rotated with greater speed than at present. Again, two small satellites of Jupiter have lately been discovered, far beyond the bounds of the Galilacan satellites, which outrage all the proprieties as hitherto understood of satellite motion. For, owing to the great difference of their eccentricities, they pass in and out of one another's orbits. Again quite recently it has been discovered that the two largest planets are surrounded by systems of asteroids, which resemble in many respects only on a smaller scale the Sun's asteroidal family which circulates between Mars and Jupiter. In both cases they probably represent the abortive attempt at the formation of a planet, and travel in orbits of almost every inclination to the ecliptic. In fact it is remarkable that only bodies of small mass are permitted to move in orbits that are greatly inclined to the "invariable plane" of the solar system. Otherwise, the equilibrium of the system would be jeopardized. Nevertheless it is a moot point, whether the equilibrium really *is* stable or not. M. Poincaré has recently investigated this question from a vigourous mathematical point of view, and his results have greatly shaken confidence in the assertions of Laplace (who be it remembered was utterly unacquainted in those days with many important considerations which have since come to light) to the effect that in the long run all perturbations mutually compensate each other. Laplace maintained that the mean distances and periods of the members of the solar system are free from secular disturbance. They are only affected by small periodic inequalities. In other words they are disturbed only by their relative positions in their orbits, and not by the position of the orbits themselves. It is now known, however, that perturbations are also of the nature of secular inequalities due to variations in the relative positions of lines of nodes and apsides, although the periods are very great, ranging from tens to hundreds of thousands of years. The short period planetary perturbations are almost negligible except in the case of the disturbance of Saturn by Jupiter, which amounts in favourable circumstances to the one hundred and thirtieth of the Sun's attraction. Other perturbations never exceed a thousandth part of the Sun's pull. Whilst the perturbations of Saturn are as much as 48 minutes of arc, those of Mercury and Venus are only 15 and 30 seconds respectively, and those of our Earth can range up to one minute. The periodic perturbations of the asteroids, however, far exceed those of the planets, being several degrees. Laplace has rightly shown that all such perturbations are nearly self-compensating after a few revolutions, and wholly

so within definite cycles of such periods. The nodes of all planets are continuously regressing, and, with the solitary exception of Venus, the apsides are advancing. the periods of the former ranging from 37 to 170 thousand years, and of the latter from 67 to over half a million of years. The inclinations and eccentricities of planets' orbits are also continuously changing, but only within very narrow limits, and in a very irregular manner. The mean distances and periods, however, are constant. And since the revolution of perihelia and the nodes cannot affect the stability of the solar system, and the changes of inclinations and eccentricities are of insufficient amount to do so, we may say in general that from this point of view the system is one of stable equilibrium. The fundamental plane of the system (to which the ecliptic is now inclined two degrees) is invariable. And it can be shown that if the planets be projected upon it, the sum of the products of the planets' masses into the areas which their radii vectores describe upon this plane in a unit of time will be a maximum. Also that the sum of the product of the planets' masses into the square root of their orbital semi-axes major multiplied by either the square of their orbital eccentricities or by the square of the tangent of the inclination of the planet's orbit to the fundamental plane of the system will be constant. Hence no eccentricities can become very great, and when some are increasing, others must be correspondingly decreasing. Nevertheless, as the eminent French mathematician Poincaré has lately demonstrated, recent discoveries have rendered the confident conclusions of Laplace in regard to the indefinite stability of the solar system no longer tenable. For example, the solar system is being subjected to a continuous diminution of its *vis viva* as the effect of tidal friction. Thus the force, which is effective in accelerating the Moon's orbital velocity, is actually less than 4 per cent. of that which diminishes the Earth's axial velocity. Over 96 per cent., therefore, of this force must be lost dynamically to the system to become converted into some other form of energy. Another secular cause of the final destruction of the stability of the solar system may lie in the property of a "resisting medium" which interplanetary space seems to possess, as a result of the immense quantity of refuse cosmic matter therein derived from cometary and meteoric disintegration, and also perhaps from solar and even planetary ejections.