have about five years before it actually fell into the Sun. If it was only as large as Uranus, it would only become visible to the naked eye as Uranus is now visible, and would reach the Sun in three years from that time.

We are therefore bound, you see, to have some warning of a collision of a dark body with the Sun causing such an outburst as would destroy the world. The larger the dark body, the greater the collision, and the more extensive the final catastrophy, the longer our warning would be, extending to perhaps twenty or thirty years. On the other hand if the dark body were of the smallest size that by its direct collision with the sun could cause the evolution of enough heat to destroy the earth we might only have a couple of years of warning.

Dynamical Theory and Tidal Friction.

Questions regarding the tides have been recognised as being of the very first importance by great sea-faring nations like the English from the remotest antiquity. The ancients must have early discovered that there was a connection between the ebb and flow of the tides and the diurnal motion of the Moon. Cæsar shows us in his De bello Gallico that he possessed a rough and ready idea of that connection. He must have noticed that the intervals between the times of high water were equal to half those between the Moon's meridian passage. Of course he did not know that the Moon could cause high tides when on the meridian below the horizon. For high water is not produced merely under the Moon, but equally (or to be more accurate, almost equally) on the side of the Earth furthest removed from the Moon. These great tidal waves are separated from each other by ½ circumference of the Earth. As the Earth rotates, every part of its surface that is roughly in the same plane with the Moon, passes successively under these tidal waves. And then it is high tide at these particular places. But if the Moon's absolute attraction caused the tides, there would be only one high tide, whereas there are two tides daily. Again if the Moon's tidal force were equal on all the component parts of the Earth, there would be no tides at all. Why then are these two lunar tides (we are, for simplicity's sake at present neglecting the solar tides) daily? If the solid part of the Earth were fixed in space, and if the Moon were also fixed then there would be but one high-tide heaped up always under the Moon. But Earth and Moon are all the time tracing out ellipses round each other, or to be more accurate, around their common centre of gravity, which is about 3,000 miles distant from the Earth's centre, and so the conditions are quite different and more complicated. The great mathematician Newton was the first to give us anything like a rigorous theory of the tides. But he necessarily left this very difficult problem imperfectly solved. He showed that every particle of water on the Earth is attracted towards the Moon with a force proportional to her mass, and to the inverse square of her distance from the particle. Hence she attracts a particle nearest to her, more than a particle at the Earth's centre, and still more than a particle on the furthest side of the Earth away from her. Therefore she diminishes the Earth's attraction on any particle of water nearest her, and on any particle on the furthest side from her. Now, if you have two bodies not far apart, and attach to them two long strings, and if then you pull the strings towards you, you will thereby also pull the two bodies towards each other. And this accelerating force driving the balls together, varies in the ratio of the distance of the balls from each other, and the inverse ratio of the length of the strings. Thus the Moon's tidal force on the waters which lie at 90° from the common plane of the Earth and Moon reinforces the Earth's attraction there. But the Moon's contractive tidal force at these points is only $\frac{1}{2}\times$ her tidal differential or separative force above mentioned. Of course the Moon exerts a tidal force upon the Earth's matter as well as on the ocean, but the firmness of the Earth's solid matter destroys any apparent reaction to this force. But the fluid ocean is free to obey this tidal force, and thus to exhibit the change in its form. Now let us go more accurately and mathematically into the Moon's and Sun's tidal force on the ocean. We will, for the present confine ourselves to the tide-raising force of the Moon only. The attraction of the Moon on the Earth as a whole is the product of their masses divided by the square of their distance. Let M and m represent the masses of the Earth and Moon and c be the Earth's centre, and n be the point nearest the Moon, and f be the point furthest from her. Now the attraction by the Moon, on the Earth as a whole will be $K\frac{M\,m}{(m\,c)^2}$ the Earth moving with acceleration $K\frac{m}{(m\,c)^2}$ towards the common centre of gravity of Earth and Moon. Now at point n, the Moon's attraction will be $K\frac{m}{(m\,n)^2}$ and is greater than at c, because the denominator m n is less than m c. Hence the Moon tends to accelerate the point n more than the point c, and consequently tends to draw a particle at point n away from the Earth's centre. And this relative acceleration can be expressed by the formula $2k \frac{m}{d^3}$. Again, at *f* the Moon's acceleration is $K\frac{m}{(m\ r)^2}$, and this is less than $K\frac{m}{(m\ c)^2}$ because the denominator m f is greater than m c. Therefore the Moon tends to draw away the Earth from the point f, or in other words, the Earth's attraction on f is diminished by the Moon's tidal force. Thus we have proved that water both at n and f tends to be separated from the centre of the Earth. We have said above that water at the points 90° from n and f tends to be driven towards the Earth's centre by the Moon's tidal force. Let p be the one point, and q the other. Then the Moon's accelerating force on p is $K\frac{m}{(p\ m)^2}$. This force can be conveniently resolved into its 2 components, which are parallel to c m and p c, respectively. But the component that is parallel to c m simply makes the particle at p move along with the rest of the Earth, and therefore produces no relative acceleration at all. Therefore it is only the 2nd component, *viz.* :—that which is parallel to p c, that has any effect on the relative acceleration of p. And this constrictive force can be represented mathematically by the formula $K\ m\ \frac{pc}{(pm)^3} = K\,m = \frac{p\ c}{(d)^3}\ K\ m\ \frac{r}{(d)^3}$. Similarly at *q* the Moon tidally attracts *q* with relative acceleration $K\ m\ \frac{q\ c}{(q\ m)^3} = K\ m\ \frac{q\ c}{(q)^3} = K\ m\ \frac{r}{(d)^3}$. Hence at either points p or q, a particle of the ocean tends to approach the Earth by a force $= \frac{Kmr}{(d)^3}$. Of course similar results arise from the attraction of the Sun, the solar wave tending to follow the apparent motion of the Sun, but to a much less degree, because this tide-raising force is in the ratio of the masses, but in the inverse triplicate ratio of the distances. And this distance in the case of the Moon is only 30 × Earth's diameter, but in the case of the Sun 12,000 × Earth's diameter.

The mathematical ratio of the two tide-raising forces of the Sun and Moon, respectively, can be shown thus—

$$\text{Moon} : \text{Sun} = g \times \frac{2 \times \frac{1}{80}}{(60)^3} : g \times \frac{2 \times 322{,}000}{(23{,}000)^3} = 5 : 2$$

(nearly). Thus although the Sun is so much heavier than the Moon, the proportion of the distances cubed is far greater than the simple proportion of their masses. The Moon's tidal wave is about 58 inches, that of the Sun only about 23 inches. Whilst gravity = 15 million × Sun's tide-raising force, it only = 6 million × Moon's tide-raising force, or about 4 million × tidal force of Sun and Moon together. As a matter of fact however, as the Earth is a spheriod of revolution and not a sphere, the Earth's gravity is more than 4 million ×

tidal force of Sun and Moon together, being nearer 5 million times greater. It can be calculated mathematically, that the Moon's tide-raising force $= \frac{1}{30}$ of her whole attraction, because it is as twice the Earth's radius divided by the Moon's distance, or $\frac{2}{60} = \frac{1}{30}$. And as the Moon's mass is $\frac{1}{80} \times$ Earth's mass and her distance $= 60 \times$ Earth's radius, therefore the fraction of the Earth's gravity represented by the Moon's differential tide-raising force $= g \times \frac{\frac{1}{30} \times \frac{1}{80}}{(60)^2} = g \times \frac{1}{8,640,000}$. And since the Moon's constrictive tide-raising force $= \frac{1}{2}$ her differential, her constrictive tide-raising force $= g \times \frac{1}{17,280,000}$. Thus if the constrictive force $= 1$, the differential will be 2, and there is a force altogether of 3 tending to make the water under the Moon higher than the water 90° distant. We can easily compare the tide-raising force of the Moon at points *n*, *c* and *f*. For the attraction of the Moon—

$$\text{at n} = g \times \frac{\frac{1}{80}}{(59)^2} = g \times 0{\cdot}00000359.$$

$$\text{at c} = g \times \frac{\frac{1}{80}}{(60)^2} = g \times 0{\cdot}00000347.$$

$$\text{at f} = g \times \frac{\frac{1}{80}}{(61)^2} = g \times 0{\cdot}00000335.$$

It is impossible however actually to observe by experiment the variations in the force of gravity due to the Moon's tidal force, owing to the fact that they are such very small quantities. Darwin thus failed to measure them experimentally, because other causes of far larger amounts interfered. It should be remembered that the tide-raising force due to the attraction of the Moon or Sun varies inversely not as the square but as the cube of the distance, as we have shown above by the formulas $2k \frac{m}{(d)^3} r$, (representing the differential force) and $k \frac{m}{(d)^3} r$ (the constrictive). This can easily be shown thus.

$$\text{Since attraction at n} = \frac{m}{(d-r)^2}$$

$$\text{and attraction at c} = \frac{m}{d^2}$$

$$\therefore \text{tide raising-force at n} = m\left(\frac{1}{(d-r)^2} - \frac{1}{d^2}\right)$$

$$= m\left(\frac{2d\,r - r^2}{d^4 - 2d^3\,r + d^2\,r^2}\right)$$

=(Since r is very small compared with d) $m\frac{2r}{d^3}$. It may be interesting to compare the difference of the tide-raising forces of Moon and Sun at perigee and apogee. Since the eccentricity of the Moon's orbit is $\frac{1}{16}$, her distances at apogee and perigee are in the ratio of $(1+\frac{1}{16}):(1-\frac{1}{16})=17:15$, therefore the tide-raising forces are as $(17)^3:(15)^3$ = (about) 7 : 5. Since the eccentricity of the Sun's orbit is $\frac{1}{20}$, his distances at apogee and perigee are in the ratio of $(1+\frac{1}{20}):(1-\frac{1}{20})=21:19$. We have now investigated the amount and the ratios of Moon's and Sun's tidal-forces at the four cardinal points, *viz.*:—those in the plane of the attracting bodies, and those at 90° from these points. It now remains to enquire as to what will be the effect of these tidal forces at other points than the four cardinal points. The tidal forces will then be tangential forces tending to draw the water towards the plane of the attracting bodies. This tangential force is the resultant of two component forces, one of which is parallel to the direction of the attracting bodies and the other perpendicular to this direction. This force will then have the effect, not of raising or lowering the water, but of imparting to it a motion. Its amount and direction can be represented by the length and direction of the resultant diagonal, divided by the cube of the distance, thus :—$Km\ \frac{\text{resultant diagonal}}{(d)^3}$. We have hitherto been chiefly considering the effect of the Moon's tidal-attraction, we will now consider the tides as due to both Moon and Sun. We have already shown that the Sun has its 2 tidal waves as well as the Moon (although in actuality they coalesce to form one ellipse) and that their ratios are as 7 : 3. At half Moons the Sun is at 90° or 6 hours from the Moon. The Sun is therefore pulling crosswise to or athwart the Moon, trying to make it high water when the Moon is trying to make it low water. But the Moon's tidal-force is $\frac{7}{3}$ × Sun's tidal-force, and as the actual tide is then due to the difference of the 2 forces, the effect of the Sun is to lower the tide, so that it then rises and falls least, the height of the tide being then the difference of the lunar and solar tides. This is called "Neap tide." When the Moon is in syzygies (either new or full Moon) the Sun is then reinforcing her tidal-attraction and the result is the "Spring Tide", when the tide rises and falls most. The height is then equal to the sum of the solar and lunar tides. The height of the spring tide $=(1+\frac{3}{7})=\frac{10}{7}$ × that of the lunar tide alone. The height of the neap tide $=(1-\frac{3}{7})=\frac{4}{7}$ × the lunar tide. Hence the ratio between the spring and the neap tide $=\frac{10}{7}:\frac{4}{7}=10:4$. When however the Moon is neither at Syzygies nor quadratures, the Sun's tidal-interference with the Moon's tide is of a different nature at every period of the month, the actual high water is either accelerated or retarded by the Sun. Thus in the 1st and 3rd quarters of the Moon, the Sun will tend to pull the tide westwards of the lunar direction, when the time of high-tide will then be accelerated for us on the Earth's surface, and the opposite effect will take place in the second and last quarters, when the time of high-tide will be retarded. Because, when the two tidal waves do not coincide, their combined culmination must be at a point intermediate between them. Consequently the tide is not 49 minutes later every day, as it would be if it exactly followed the Moon, but some times it is as much as 11 minutes earlier than this 49 minutes period (*i.e.*, 24 hours 38 minutes) and sometimes later by 11 minutes (*i.e.*, 25 hours). This is called the priming and lagging of the tides. The approximate time of high water at any particular port in the afternoon of the day when the Moon is in syzygies is called the "Establishment of the Port." It represents the mean interval between the moment when the Moon crosses the Meridian, and the moment when it is high-water at that port. This interval depends upon local circusmtances, as to how the tide has to reach the place. Except for the interfering effect of wind and barometric pressure, this "Establishment of Port" is constant. This comparative lateness of high-tide is due to the interference with the free motion of the tide due to large continents, the narrowness of channels, their length and depth etc. At London the establishment of port is 1 hour 58 minutes, so that mean high tide occurs 1 hour 58 minutes later than the transit of the Moon, *i.e.*, when the Moon's hour angle is 1 hour 58 minutes or 29° 30′ angular distance from the meridian. By correcting for the priming and lagging of the tides the lunar time of high-water can be calculated for any phase of the Moon. It is very important for maritime purposes to make accurate observations in reference to the establishment of ports, and these observations unfortunately are sometimes made incorrectly, owing to observers often confusing the time of high-tide with the time of "Slack-water" which is quite a different thing, and means merely the time when the tide ceases to flow one way or the other. The heights of the spring and neap tides are proportional to the distances of Sun and Moon from the Earth. When both are in perigee, spring tides will be at their highest. When the Moon is in apogee, and the Sun in perigee, the neap tides will then be at their lowest. The Spring tides are greatest, we have said, when Sun and Moon are in perigee, but they are in reality still greater at the Equinoxes, when the Sun is on the equator and the Moon therefore necessarily only 5° from it, because then their pull is direct (that is not oblique). They are greatest of all, when (added to all these favourable circumstances) the Moon's nodes are at the Equinoxes, which happens every $9\frac{3}{4}$ years. The variation in the height of tides due to the declination of the Sun and Moon, is in consequence of the fact that the vertex of the tide-wave tends to take its position immediately under or normal to the Sun and Moon, and thus when the Sun or Moon shifts North or South the tide must shift too For example, if the Moon is in North declination, and the place on the Earth is in North latitude ; of two consecutive tides, that which happens when the Moon is near the Zenith will be greater than that which happens when the Moon is near the Nadir, because the Moon's least Zenith distance is then less than her least Nadir distance The difference will be greatest when Sun and Moon are in opposition and in opposite declinations. This is called the " Diurnal Inequality." These inequalities in height of the tides can be regarded as half periods, of a day or fortnight. But, after a fortnight the Moon's declination will have the same value but opposite sign. Hence a fortnightly tide. In the same way these inequalities in the heights of the tides can be referred to the Sun, being then semi-diurnal and six-monthly. When both the Sun's and Moon's declination is Zero, of course there is no such difference in the diurnal, fortnightly or half-yearly tides. It is interesting to note that the first attempts at finding the relative masses of Sun and Moon were made by means of measurements of the relative heights of the Solar and lunar tides. Thus, if the spring tide anywhere is say 41 feet, and neap-tide is 15 feet, then the ratio between lunar and solar tide $=(41+15) : (41-15) = 28$ 13. But tidal forces are proportional to the masses and inversely proportional to the cubes of the distances Now the Sun's distance = 385 × Moon's distance. Therefore the Moon's Mass $= \frac{28}{13} \times \frac{1}{(385)^3} \times$ Sun's mass $= \frac{1}{26,500,000} \times$ Sun's mass, and as the Sun's mass is known by other means to be 324,000 × Earth's mass, therefore the Moon's mass $= \frac{324000}{26,500,000} \times$ Earth's mass $= \frac{1}{81 \cdot 79} \times$ Earth's mass. Again if the ratio of the Sun and Moon's mass is known, then by a similar method the ratio of their distances can be calculated We should mention here that in speaking of " diurnal " tides, reference is made not to solar, but lunar days, because the time of high-tides depends upon the Moon's motion relative to the meridian, the sun merely modifying these effects. The lunar day is the interval occuring between two upper transits of the Moon across the meridian. And since $28\frac{1}{2}$ lunar days $= 29\frac{1}{2}$ mean solar days, therefore one lunar day $= (1 + \frac{1}{29\frac{1}{2}})$ mean solar days $= 1\frac{2}{57}$ mean solar days $=$ 24 hours $50\frac{1}{2}$ minutes. Besides irregularities in the tides due to the local configuration of the sea, base, and shore, etc., the tides are also affected by the state of the atmosphere and the wind. The rise of the tide is roughly in inverse proportion to the (height of the barometer). Thus a fall of the barometer of $\frac{2}{3}$ inch will correspond to a rise of the tide of about $\frac{2}{3}$ feet $=$ 8 inches. The ratio being about one foot for every inch of the mercury. Again the wind, if it blows into port, will make the tide rise often several feet above the normal. But then the tide will be delayed (curious as it may seem), because although the depth of the high water at the bar is reached before the proper time, yet the greatest depth is not attained till some time afterwards. And opposite effects take place in the opposite circumstances. If a tide has to run into a narrowing channel, it will attain a height much greater than the normal. If on the contrary the tide has to widen out into a large sea through a small channel (such as through the straits of Gibraltar into the Mediterranean), the tides will be almost imperceptible. It is very difficult to detect the tides in lakes or land-locked seas. Their height ought theoretically to be in the same ratio to the height of a mid-ocean-tide, as the length of the particular sea is to the length of the Earth's diameter. But practically this theoretical valuation is so much masked by the effects of wind and barometric pressure (which are far greater in their results) that it necessitates a long series of careful observations to separate real tides from the effects of these other causes. Tides travel up rivers at a rate proportional to the depth of the river, the amount of friction encountered by the rivers's bed, and the river's velocity. It usually ascends the river as far as the point where the velocity of the river $=$ the velocity of the tide. At this point it will be "slack water," as we before mentioned. It is the point where anything on the river's surface would cease to float upwards. The tidal velocity in rivers is something between 10 and 20 miles an hour, and it will attain a height far greater than the apex of the tide at the river's mouth. We have hitherto been treating chiefly of the great ocean in its statical aspect, *i.e.*, as forming the figure of a prolate spheriod around the Earth (this prolate spheriod being generated by the revolution of the ocean's ellipse about its major axis), which points in the direction of the tidal attraction. This no doubt would be a perfectly accurate description of the form which the ocean would assume, if the Earth did not rotate. But when we take into account at the same time the Earth's rotation, we find that the form of the ocean's surface is greatly modified thereby, and instead of the major axis of the prolate spheroid pointing towards the Moon, we learn on the contrary the surprising fact that it would be low tide in direction of the tide-raising force and high tide at 90° angular distance from it. However, whether the Earth rotated or not, the tide-raising bodies would always tend to draw the water away from the poles towards the equator. Moreover from the equilibrium theory of the tides, certain useful deductions can be made. For example, the height of the tide is proportional to the ratio of the tide raising force to gravity, and hence inversely proportional to the intensity of gravity, which is itself proportional to the density of the Earth. Another mathematical consequence is, that the height of the tide is proportional to the Earth's radius, because both the intensity of gravity and the intensity of the tide-raising forces are themselves proportional to the Earth's radius. We are thus enabled to calculate tidal quantities on different celestial bodies. For if M represent the mass of the attracting body and R its distance, and r represent the radius of the body whose tides we are considering and d its density, then we have the formula $\frac{Mr}{dR^3}$ to denote the height of the tide on that particular body. But as we have said this statical theory of the tides is wholly incomplete, because it fails to take into account the fact that the Moon and Sun besides merely raising the tides, have to keep up their motion round the Earth, in the opposite direction from its rotation (*viz* :—from East to West). The statical theory also fails to take into account the fact, that a tidal wave has impetus of its own. This impetus varies according to the depth of the water and the force of gravity, and the amount of friction it has to encounter. The dynamical theory then which takes into account all these things is a much more complicated and difficult problem. But what has been said above in connection with the statical theory is absolutely necessary to be thoroughly comprehended before we can understand the true dynamical theory. Now if the Earth had no continents, but was entirely covered with water, the tidal waves would travel round the globe at a constant velocity. And this velocity would be the apparent velocity of the Moon, the vertex of the tidal wave keeping always under her, if the depth of the water were as much as or more than 14 to 12½ miles. If the depth were less than 12½ miles, the tidal wave would travel 90° behind the Moon, whilst the water near the poles, since it has a far less distance to travel in each rotation than the water at the Equator would still be able to keep up with the Moon. There would thus be a neutral latitude of tideless water between these two extremes. We must however distinguish between a wave whose motion is one of "forced" or on the other hand "free oscillation." If an earth-quake should suddenly disturb the ocean and then the wave generated were to be left to take its own course, this would be an instance of "free oscillation." In this case it can be mathematically proved that its $(\text{velocity})^2$ will vary in the ratio of the depth of the ocean, and will be given by the formula $v^2 = gh$ or $v = \sqrt{gh}$, which is the velocity which a body would acquire in its fall through the depth of the ocean. Thus in a depth of water of say 100 ft. the wave's velocity would be = 40 miles an hour, and in 10,000 ft. would = nearly 400 miles an hour. Now as the Moon passes daily from over the continent of America into the Pacific ocean, she generates after the manner of a forced oscillation an initial tide in the Pacific. The wave then continues its course much after the manner of a free wave, but not quite, since the Moon still accelerates it, because she travels slower than does the surface of the Earth at the Equator, and therefore relatively Westwards. Another tide wave is generated by her in the same place 12 hours later when she is on the meridian below the horizon. It also commences as a forced and continues merely as a free oscillatory wave. If we enquire more accurately into the manner of the tidal wave's motion round the globe, we notice that in the first and third quadrants of the Earth's equatorial circumference, the Moon's tidal attraction will pull the wave in a direction contrary to that of the Earth's rotation, and in the 2nd and 4th quadrants the Moon will pull in a similar direction to the Earth's rotation. Therefore after the first and third terrestrial quadrants the wave will be going comparatively slowly (having been retarded by the Moon), but after the 2nd and 4th quadrants, the wave will be going at its fastest, having been accelerated by the Moon. Thus at quadratures the wave will be travelling slowly, and at Syzygies it will be travelling fast. And if the water be travelling fast (say for simplicity's sake) in a uniform canal it must be shallow when it is travelling fast, and deep when it is travelling slowly. But as we have shown, the place where it is travelling fast is in the direction of the Moon, and when it is travelling slowly is at 90° to that direction. Therefore from the dynamical theory of the tidal motion, we arrive at a result which is the exact opposite from the result obtained on the statical theory, *viz* :—that under the Moon it will be lowtide, and at 90° from the Moon it will be hightide. This dynamical theory however, though being nearer the truth than the statical theory, does not give a result which is absolutely and mathematically correct in fact, owing to the extensive complications that arise from the varying depths of the ocean, etc., etc. Supposing for simplicity's sake, the tidal wave to be running in a canal round the world of constant depth and width, let us see the results. You will easily perceive that the deeper the canal, the greater the amount of water lifted in any vertical column by the Moon, because more water is displaced in the case of a deep vertical column than a shallow one : and hence the more it will be elevated relatively to the water at its side. Now the only way the raised water is able to fall again to the normal level of the ocean, is by pushing forward and upward its adjacent column. And hence the motion of the tidal wave depends upon the ratio of the height of the tidal wave to the normal height of the surface of the ocean. And this depends upon the depth, as I stated above. To keep up with the Moon the tidal wave would have to travel over 1,000 miles an hour, which would necessitate a depth of from 12 to 14 miles. But the sea is nowhere anything approaching this depth, being nowhere probably deeper than 5 miles. And this would give a velocity of only 600 miles an hour, even if it were as deep as 5 miles all over. In other words gravity is quite unequal to the task of making the tidal-wave keep pace with the Moon, except perhaps very near the poles.

Let us now consider what is the motion of any individual particle composing the tidal-wave. When a particle is above the mean level of the ocean, it will be advancing, since the wave is produced by the pushing of a whole vertical column of particles. But when the particles are below the mean level, they are then receding. Hence, an individual particle describes a long ellipse. If, however, the water has an independent current of its own, the ellipse will travel along in company with the current either backwards or forwards. The velocity of any individual particle can be mathematically calculated. It bears the same ratio to the velocity of the tidal-wave, as the wave's height above the mean level of the ocean bears to the whole ocean's depth. We will now enquire more fully into the components of the Moon's tidal-attraction. At the four cardinal points, *i.e.*, the two points in the Moon's plane (or at 0° and 180°), and the other two points at 90° and 270°, the water is pulled neither East nor West by the Moon's action. But between these points and most of all at the half-quarters, the force is tangential towards the plane of the Moon. Therefore at these half-quarter points, the tidal-wave cannot rest in equilibrium, for it is being pulled backwards or forwards by the tangential force. And this tangential force is counteracted by no other force, since, gravity is indifferent to motion forward or backward along the Earth's circumference. And herein lies the most glaring defects of the statical theory when taken alone, in that it absolutely neglects this tangential force which is really the most important force of them all, because it is quite independent of gravity. Thus whilst the tidal wave cannot keep under the Moon, it cannot remain (owing to tangential force) at any point from there up to 90°. Hence, not only, as we have said above, the true dynamical theory of the tides proves that it is low water under the Moon but it proves the still more startling fact, that the particles of the water under the Moon are flowing backwards, because water below the level of the advancing wave flows backwards in an elliptical curve, though of course much more slowly than the tidal-wave advances. The only matter that remains to be noticed, is the effect of friction, which is always acting as a drag upon the motions of the water. We have showed above that the head of the tidal prolate spheroid cannot remain in a line with the Moon, nor owing to tangential force can it lie between that point and 90° Eastwards. But even at 90° from the plane of the Moon's direction, the wave cannot remain, because being a cardinal point, there is no tangential force there and gravity exerts no influence backwards or forwards, but the water is advancing with its greatest velocity there, and therefore meets with the greatest amount of friction in opposition. Consequently friction unopposed by any other force would carry the major axis of the tidal prolate spheroid still further back than 90°. But between 90° and 180°, both friction and tangential force would continue to urge the wavehead still further backwards, and so it cannot stay there. And as we have seen above, it could not stay at 180°. But it can at last find a resting place, so to speak, between 180° and 270°, because at a point intermediate (say 225°), friction is acting backwards or Eastwards with the Earth's rotation, and the tangential force is acting Westwards, and so from these two forces in opposition, equilibrium is at last attained. Thus according to the true dynamical theory, the wave-head of the tidal spheroid has fallen back, 225° from the Zero point facing the Moon, which was the position which according to the statical theory it should have assumed. And this looks for all practical purposes (because for practical purposes, it is of no consequence what particular end of the major axis points in either direction) as though the wave-head had only fallen back 45° from the direction of the Moon. We will next consider the actual course of a tidal wave. It is as follows. It starts on the West coast of South America, and travels West through the deepest water of the Pacific Ocean at a rate of 850 miles an hour, so that the tidal-wave reaches New Zealand in about 12 hours from the start. It then passes on by Australia through the Indian Ocean and arrives at the Cape of Good Hope in 29 hours. It then enters the Atlantic and verges towards the North at about 700 miles an hour. It arrives at Florida in about 40 hours from the start. But in order to get to London it has to go round the North of Scotland and then southwards through the North Sea, and arrives there in 58 hours from the start. Hence in the great oceans there are four great tides following each other at the same time, and nearly in the same paths. Their paths are however modified continually by the changing declination and distances of Sun and Moon. In the middle of the great oceans the height of the tidal-vertex is not more than 2 or 3 feet, but on the continental shores it is a great deal more pronounced. And in bays which narrow very rapidly (for example the Bay of Fundy) the tide is nearly 100 ft. At Bristol it is 50 ft. The shallower the bottom, the less will be the velocity and the greater will be the height of the tidal-wave. The height of the tidal-wave varies theoretically inversely as the fourth root of the depth. But this theoretical calculation is greatly modified in practice, by tides interfering with one another. For example, when a tidal wave beats up against a coast, specially if the water be deep there, it will be reflected for a very considerable distance back again into the ocean, and will meet the next tide coming in. And these so-called "interferences" greatly vary the height of the tides even at places comparatively near to each other. Again there are places, where the tide comes in by two different routes. For example on the East coast of England the tide not only comes round by the North of Scotland, but in a minor degree also up the English Channel. This has the effect of producing tides at some places of twice the normal height, and at other places the tide is reduced to almost nil. We must not forget to mention a most important scientific fact, which is proved by the action of the tides, namely that the Earth's interior core is exceedingly rigid ; a fact which goes clean against the theories of the geologists, who held that the interior of the Earth was a viscous molten mass. The Earth's rigidity is proved in this way. If the Earth's core were viscous, it would yield somewhat to the tidal-attracting forces. And if it yielded the tides would be diminished. But the most careful observation shows that the tides are not diminished by the minutest fraction from what they ought to be if the Earth were as rigid as steel. Now we must consider the very interesting results which follow from tidal-friction. As the Moon revolves 27½ times slower than the Earth from West to East, she of course moves relatively to the Earth's rotation in a direction East to West, tending to drag the water of the ocean (or rather a portion of it) in a direction contrary to that of the Earth's rotation. Thus friction is set up between the Earth's surface and the ocean. Hence whatever water stays behind with the Moon must retard the Earth's rotational velocity. This is due to the fact that liquids are viscous, and resist change of form, and in doing so convert part of their kinetic energy into heat. Owing to this friction, the Earth's rotating surface drags the water along with it, whilst the Moon drags the water back. Hence a couple is formed, which tends to diminish the angular velocity of the Earth's rotation. If the tidal motion consisted merely in the small variations of height twice a day in mid-ocean, tidal friction would be a quite negligible factor. The case is quite different however, when we consider the action of the tides against the continents. Here immense waves of water flow with great friction against the shores involving a great amount of energy which has to be made up for by the expenditure of an equal amount of energy from the Earth's rotating motion. This means a lessening of rotational velocity. As the amount however is really comparatively very small, the length of the day is affected to only a very small extent. The day cannot be more than $\frac{1}{10,000}$ second of time longer than it was a hundred years ago. This excessive minuteness of the retardation of the Earth's angular velocity is no doubt due to the fact that there is another cause which acts in an opposite direction tending to *accelerate* the Earth's velocity. For the fact that the Earth is always giving out heat into space, means that the Earth is always contracting, and hence the Earth's radius is diminishing and hence (as far as this cause is concerned) its angular velocity is increasing. However the frictional cause on the whole preponderates, resulting as we have said in the Earth's rotation being diminished by $\frac{1}{10,000}$ of a second in a century. But if the Moon exerts a couple on the Earth, it is obvious that the Earth will exert a counter couple on the Moon. Now the Earth's rotating surface tends to carry the head of the major axis of the tidal spheroid Eastward of the Moon, and since the particles there are nearer to the Moon than the particles at the other end of the major axis, the tidal head tends to pull the Moon forward or Eastward, and hence increases her velocity, and her distance (because her areal velocity remains the same) and her period. Thus the month is lengthened by tidal friction, as well as the day. Now, since the Moon's distance from the centre of gravity of Earth and Moon is far greater than the distance of the ocean from its axis of rotation which passes through the centre of the Earth, the effect of tidal friction upon the Earth's rotation is far greater than it is upon the Moon's orbital motion. Hence the final effect of tidal friction is to make the day equal in length of time to the month. Hence after many millions of years to come, the day and the month will be equal, and it can be calculated mathematically that they will both be 1,400 hours long. The Earth will then always present the same hemisphere to the Moon, and lunar tides and lunar tidal friction will then have ceased. But solar tidal friction will still continue. And its effect will be to continue to diminish the Earth's rotatary velocity. And it will have the opposite effect upon the Moon's orbital motion from what the lunar tide had, namely to retard instead of accelerate her velocity and to diminish instead of increase her distance. The Moon will thus ultimately rejoin her parent Earth, and owing to the immense gravitational pressure which will then be set up between Earth and Moon, will to a certain extent coalesce with the Earth, and the two bodies will together form one somewhat misshapen ellipsoid. The last step will be when the rotation of this Earth-Moon body or in other words its day, will coincide with its year, and will follow the example of the two inner planets Mercury and Venus in turning always the same face to the Sun, one hemisphere remaining in eternal day and the other in eternal night.

Observation of the Transit of Mercury, 7th November 1914,

BY R. J. POCOCK, B.A., B.SC., F.R.A.S.,
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HYDERABAD.

At Hyderabad the first and second contacts alone were visible, the third and fourth taking place after sunset.