Measurements of Heights of Lunar Mountains.

By S. C. Ghosh.

To Galileo is due the discovery that the surface of the Moon is covered with mountains and valleys. According to this astronomer the luminous points which are seen isolated near the border of light and darkness, on the lunar surface, are the summits of mountains which from their heights catch the sunlight first, and appear as stars like the bright spot P in the diagram I. As time passes the bright spot becomes larger as the light extends lower down the mountain side until the terminator reaches and passes it.

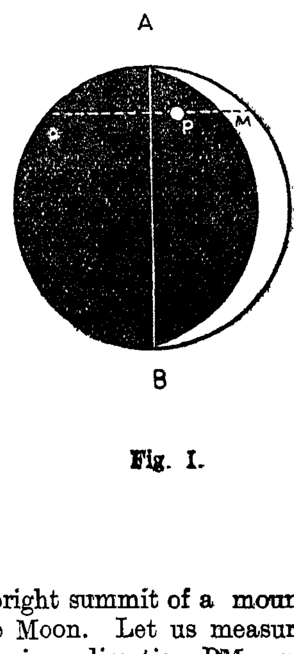

Fig. I.

Now P is the projection of the bright summit of a mountain peak on the visible disc of the Moon. Let us measure the distance of P with a micrometer in a direction PM perpendicular to the line of cusps, as well as the distance AB between the cusps. Then PM is the projection of a tangent to the sphere parallel to the Sun's rays, and therefore making with PM an angle equal to the complement of the Moon's angle of elongation. This is illustrated in Figure II in which

O represents the centre of Moon and the lines S′O, SP and S″E are parallel rays from the Sun.

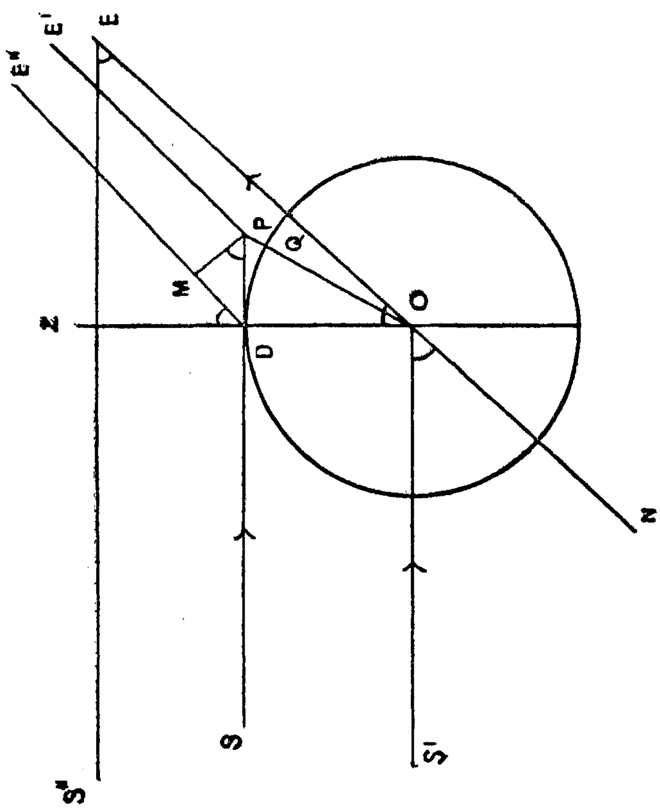

Fig. II.

The Moon's distance being only about $\frac{1}{400}$ of the Sun's, the line S″E is, for practical purposes, parallel to S′O. Therefore the angle S″EO = the complement of $\angle$ ZOE = the complement of $\angle$ ZDM = the complement of $\angle$ DPM.

Now putting down the height of the Moon (PQ) $= x$,

the radius of the Moon $\left(\frac{AB}{2}\right) = OD = r$

S″EO $= \theta$, and,

the distance P M $= d$

We have

$$PD = \frac{PM}{\text{Cos MPD}} = \frac{d}{\text{Sin } \theta} = d \text{ Cosec}$$

and $PO^2 = DO^2 + DP^2$

$$\text{or } (r + x)^2 = r^2 + d^2 \text{ Cosec}^2 \theta$$

$$\text{or } r^2 + x^2 + 2\,r\,x = r^2 + d^2 \text{ Cosec}^2 \theta$$

$$\text{or } \quad \frac{x^2}{2r} + x = \frac{d^2 \text{ Cosec}^2 \theta}{2r}$$

$$\text{or } \quad x = \frac{d^2 \text{ Cosec}^2 \theta}{2r} \text{(very nearly).}$$

$$\therefore \quad x = \left(\frac{d}{2r}\right)^2 2r \text{ Cosec}^2 \theta$$

$$= m^2\ 2r \text{ Cosec}^2 \theta$$

Where m is the ratio of the distance PM to the line of cusps AB, as observed by the micrometer, and θ the Moon's angle of elongation.

Galileo observed the distance PD to be equal to $\frac{1}{20}$th part of the diameter.

Therefore according to the above formula we find the height of the mountain

$$= \frac{PD^2}{2r} = \left(\frac{PD}{2r}\right)^2 2r = \left(\tfrac{1}{20}\right)^2 2r$$

$$= \frac{2153}{400} \text{ miles} = 5{\cdot}37 \text{ miles, or } 28{,}000 \text{ feet,}$$

which is a height equal to that of the highest summits of the Himalayan range. As the Moon's diameter is only $\frac{3}{11}$ of that of the Earth, we see that her mountains are comparatively very much loftier.

This remarkable result has been fully established by the accurate measurements of Mädler, who adopted the method of measuring the length of the shadows and the distance within the illuminated portion of the surface in which was the peak. This method is more accurate and is explained below.

Placing the cross-wire of the micrometer so as to be parallel to the line of cusps, the length of the shadow of the mountain is measured; next, the distance of the summit of the mountain from the terminator or border of illuminated surface is measured; and then, moving the micrometer into a position at right angles to the former, the distance of the mountain from the cusp of the Moon is determined. Converting these from micrometer revolution into arc and correcting for refraction, when necessary, the resulting distances may be put down as follows :—

σ = the length of the shadow.

T = the distance from the terminator.

α_0 = the distance from the cusp.

Now let us obtain the following date for the time of observation :—

θ_0 = the geocentric longitude of the Sun.

θ = the geocentric longitude of the Moon.

β = the geocentric latitude of the Moon.

p_0 = the horizontal parallax of the Sun.

p = horizontal parallax of the Moon.

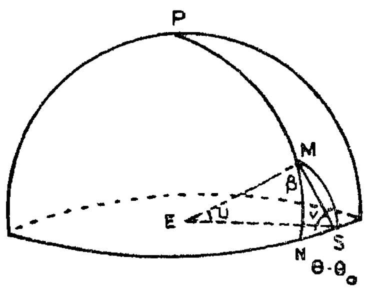

Fig. III.

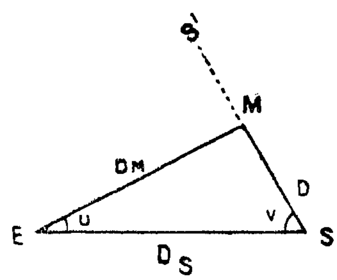

Fig. IV

Let E be the Earth,

,, S ,, Sun is the ecliptic and

,, M ,, Moon in the celestial sphere.

Then we have in the spherical triangle MNS,

$MN = \beta, \qquad NS = \theta - \theta_0, \qquad \angle MNS = 90°.$

Let us also put down

u = angular distance at the centre of the Earth between the Moon and the Sun.

v = angular distance at the Sun between the Earth and the Moon.

dm = the distance of the Moon from the Earth.

ds = the distance of the Sun from the Earth.

d = the distance between the Moon and the Sun.

Then we have from the spherical triangle MNS.

$$\text{Cos } u = \text{Cos } \beta \text{ Cos } (\theta - \theta_0) \qquad (1)$$

and from the triangle EMS

$$\frac{\sin u}{d} = \frac{\sin v}{dm}$$

$$dm^2 = ds^2 + d^2 - 2\, d.\, ds.\, \cos v.$$

$$= ds^2 + dm^2 \frac{\sin^2 u}{\sin^2 v} - 2.\ dm\ ds \sin u \cot v$$

$$\frac{dm^2}{ds^2} = 1 + \frac{dm^2}{ds^2} \cdot \frac{\sin^2 u}{\sin^2 v} - 2.\frac{dm}{ds} \sin u \cot v$$

$$\frac{dm^2}{ds^2}\left(1 - \frac{\sin^2 u}{\sin^2 v}\right) = 1 - 2.\frac{dm}{ds}.\ \sin u.\ \text{Cot}\ v$$

$$\frac{dm^2}{ds^2}[1 - \sin^2 u\,(1 + \cot^2 v)] = 1 - 2.\frac{dm}{ds} \sin u \cot v$$

$$\frac{dm^2}{ds^2} \cos^2 u = 1 - 2.\frac{dm}{ds} \sin u \cot v + \frac{dm^2}{ds^2} \sin^2 u \cot^2 v$$

$$\therefore\ 1 - \frac{dm}{ds} \sin u \cot v = \frac{dm}{ds} \cos u$$

$$\text{or}\quad 1 = \frac{dm}{ds} \sin u \cot v \text{ (nearly)}$$

$$\therefore\ \tan u = \sin u \frac{dm}{ds} = \sin u \frac{dm}{er} \cdot \frac{er}{ds} \text{(where}$$

er is the radius of the Earth).

$$= \sin u \tan p_0 \cot p. \text{——————} (2)$$

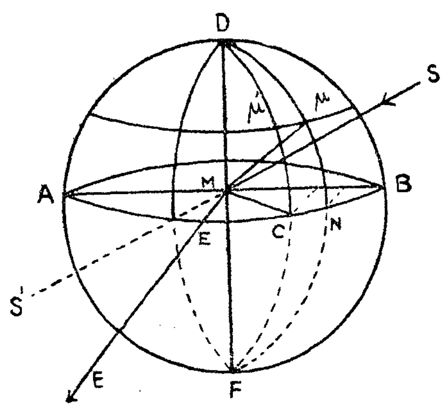

Fig. V.

Let A D B F be the Moon's disc visible to the observer and perpendicular to M E the line to the Earth.

Let S M be the direction of the Sun. Then D C F is the limit of the illuminated surface and is perpendicular to M S.

Therefore, $\angle$ B M C $= 90° - \angle$ BMS $= \angle$ EM S′ where MS′ is the elongation of the line S M,

$= u + v$ (see Figure IV).

$\therefore$ If we call the angle E C $= \omega$, then

$$\omega = 90^\circ - (u + v) \text{------------} \quad (3)$$

[$u + v$ being obtained for equations (1) and (2)]

Let us suppose that D μ NF is the great circle passing through the mountain at μ and let us call the arc μ N $= \nu$. This is obtained from the following equation :—

$$\sin \nu = \frac{s' - a_0}{s'} \text{ where } s' \text{ is the Moon's semi-diameter.—(4)}$$

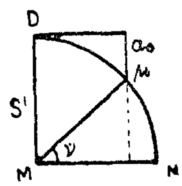

Fig. VI.

Let us represent the angular distance between the great circles passing through the mean terminater and the mountain by i. Then CN $= i$ and EN $=$ EC $+$ CN $= \omega + i$

Now projecting the arc ECN on the disc ADBF we have the projection of EN $=$ the projection of EC plus projection of CN or s$'$ $(\omega + i)$ $=$ s$'$ sin $\omega +$ projection of $\frac{\mu\ \mu'}{\cos\ \nu}$

$$\left[\because \quad \frac{\mu\ \mu'}{\sin D\mu'} = \frac{CN}{\sin Dc} \text{ and } \sin D\ \mu' = \cos \nu \text{ and } \sin D\ c = 1\right]$$

$$= s' \sin \omega + \frac{T}{\cos \nu} \text{------------} \quad (5)$$

[Where T is the observed distance of the mountain from the terminator and is therefore the projection of $\mu\,\mu'$ on ADBF].

Now the height of the Sun above the horizon at μ is computed as follows :—

Let ϕ be the altitude of the Sun at μ

Then $90^\circ - \phi =$ zenith distance of the Sun at μ

Let us take a point S$'$ in Fig. VII, so that CS$' = 90^\circ$. Then SMS$'$ is a diameter passing through the Sun and the Moon's centre at M. Let S μ s' be a great circle passing the mountain at μ. Then $S\mu s' = 180°$ and $\mu s = 90° - \phi$

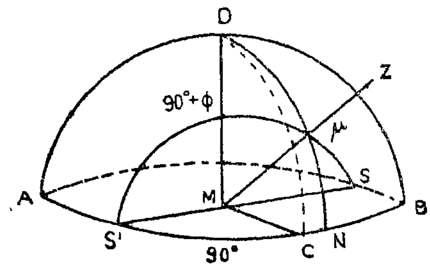

Fig. VII.

$\therefore$ $S'\mu = 90° + \phi$

$\therefore$ in the spherical triangle $S'\,\mu\,N$

$\cos \mu\, s' = \cos S'N \,\cos N\,\mu$ ($\cos S'N\mu$ being $= o$)

or $\cos(90° + \phi) = \cos(90° + i)\cos \nu$.

or $\sin \phi = \text{Sin } i \ \cos \nu$ ... ———————— (6)

When ϕ is known, since i and v are known from equations (4) and (5).

Now the length of the shadow as measured (σ) is not its actual length since what is seen is the projection of the actual shadow on the disc ADBF

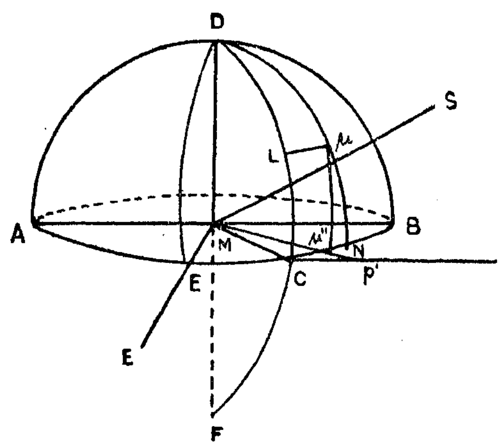

Fig. VIII.

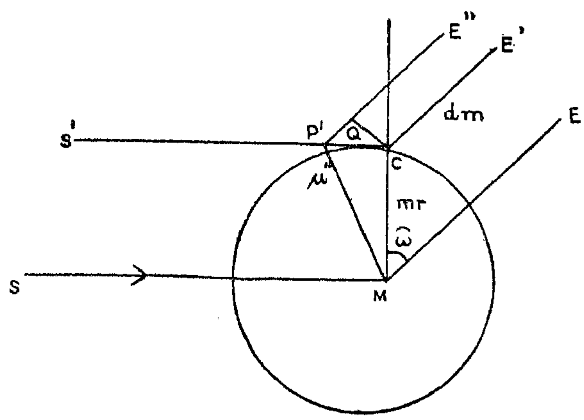

Fig. IX.

The plane of the shadow (L μ) is the central plane perpendicular to the plane D C F. (Fig. VIII). If $\mu\,\mu''$ be a plane perpendicular to the plane ACB, we have actual length of shadow

$$=L\mu$$
$$=c\mu''$$
$$=cP' \text{ ultimately.}$$

Now, $CQ = P'C \cos\omega$

$$\therefore \frac{CQ}{dm} = \frac{P'C}{dm}\cos\omega$$

$$= \frac{c\,\mu''}{Mc}\frac{Mc}{dm}\cos\omega = \frac{L\mu}{mr}\frac{mr}{dm}\cos\omega$$

where mr = radius of the Moon and dm = the distance of the Moon from the Earth.

$$\sigma = \sigma'\,s'\cos\omega$$

$$\text{or } \sigma' = \frac{\sigma}{s'\cos\omega} \qquad (7)$$

The height of the Sun above the horizon at μ (ϕ) and the true length of the shadow (σ') being now known, the height H of the mountain can be computed as follows :—

Let M μ z be the zenith at μ, P the top of the mountain and S the Sun.

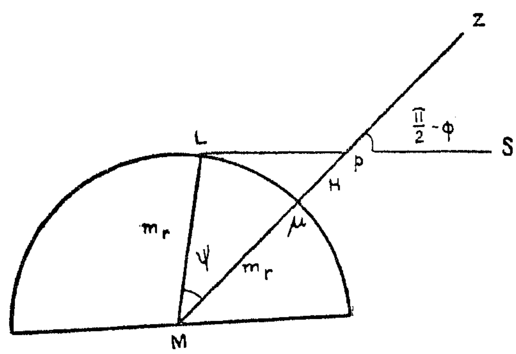

Fig. X.

Then $\angle$ SPZ $= 90° - \phi$

If we call the angle at the Moon's centre between the summit of the mountain and the end of the shadow $= \psi$

Then in the triangle L M p we have

$$\frac{\sin \psi}{\text{L}\,p} = \frac{\cos \phi}{\text{L M}}$$

$$\text{or } mr \sin \psi = \text{L}\,p \cos \phi$$

$$= \sigma' \, mr \cos \phi \text{ (ultimately)}$$

$$\therefore \sin \psi = \sigma' \cos \phi \qquad (8)$$

$$\text{Also, } \frac{\text{H} + mr}{\sin (90° - \phi + \psi)} = \frac{mr}{\sin (90° - \phi)}$$

$$\frac{\text{H}}{mr} + 1 = \frac{\cos \phi - \psi)}{\cos \phi}$$

$$\text{or H} = \left\{ \frac{\cos (\phi - \psi)}{\cos \phi} - 1 \right\} mr \qquad (8)$$

Mädler took the heights of upwards of 1,000 lunar mountains: of these six exceeded 3·62 miles (or 19,000 feet) in height and twenty-two exceeded 3·01 miles (or 16,000 feet), which latter is the height of Mont Blanc above the sea level.

The lunar mountain, called Dörfel, is situated near the south pole in the midst of a large plain and has a height of 4·75 miles (or 25,000 feet).

The lunar mountain, called Newton, is an annular crater and rises to a height of 4·54 miles (24,000 feet): the cavity of the crater lying below the general surface of the Moon.