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Origin of the gravitational constant and its behaviour at very high energies*

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Abstract. Recent progress in the unification of strong, weak and electromagnetic interactions suggests that the high energy behaviour of these interactions would be different from their observed low energy characteristics. For instance their coupling constants would become a function of the energy (momentum) of the interacting particles and would become comparable at a particular large energy scale. Extension of these ideas to gravitation would imply that the high energy behaviour of gravity (well described by Einstein's general relativity, which is a non-Abelian gauge theory at low energies) may also be different and in particular, the gravitational constant will become energy dependent. At very large energy scales involved in the earliest epochs of the universe and in gravitational collapse, *i.e.* near singularities, these modifications to the low energy theory could have interesting consequence for quantum gravity.

Key words : unified theories—coupling constants

1. Introduction

Recent work on the unification of the fundamental interactions of the physical world has led to the interesting conclusion that they manifest themselves as different interactions with diverse strengths and properties only because they are being observed at low energies. The so called grand unified theories (GUTs) suggest that the weak, electromagnetic and strong interactions merge into one unified Yang—Mills gauge theory at some large mass (energy) scale (Georgi *et al.* 1974). This would mean that the high energy behaviour of these interactions would be different from what is seen at lower energies and in particular that the coupling constants characterizing the interaction strength would be energy (momentum) dependent (Ellis *et al.* 1980). For instance, in this picture the strong interactions are "strong" only at comparitively lower energies; the coupling constant is expected to decrease with increasing energy of the interacting particles. Again the weak interactions with a small coupling constant at ordinary energies are expected to grow in strength and equal the

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electromagnetic coupling at energies of the order of a 100 GeV. The large energy ($\sim 10^{15}$ GeV) at which the GUTs unification occurs is inaccessible by a large margin to contemporary accelerators but would have been present in the very early stages of the big bang. At these energies baryon nonconserving interactions (a hallmark of GUTs) would have created the observed asymmetry between matter and antimatter in the universe. Most attempts at unification however leave out the gravitational interaction characterized by a single universal coupling constant G. As is well known, gravitation dominates all astrophysical phenomena and by virtue of the universality of its coupling, whereby all forms of matter and energy momentum couple to gravitation as well as give rise to it, is the most important interaction in determining the large scale structure of the universe. The gravitational interaction is well described within observational errors by Einstein's general theory of relativity (GTR) which assumes a constant, energy-independent G. Modifications of GTR, such as the Brans-Dicke theory do have a varying G, but this is a variation with time with reference to the cosmological epoch. Thus all these theories describe gravity at low energies. In analogy with other interactions which have different behaviour at high energies one may perhaps reasonably expect gravity also to differ at very high energies (Sivaram 1983a). Many astrophysical scenarios involve matter at extremely high energies. Examples are the earliest phase of the big bang and matter disappearing into the singularity beyond the event horizons of blackholes. The usual approach to problems arising from these extreme conditions is to merely state that the theory probably breaks down at these energies. Thus an understanding of how gravity might behave at such large energies could be of considerable importance in solving some seminal problems in astrophysics. Of course any modifications to the theory at these energies should vanish at low energies when one recovers the usual theory —GTR in this case.

2. Origin of the gravitational constant

Before comparing gravitation with other interactions with a view to guessing its high energy behaviour it might be worthwhile to summarize present ideas on the origin of the gravitational constant which is considered as something given in both the Newtonian theory and the GTR. We can distinguish between 'macroscopic' and 'microscopic' models for determining G.

MACROSCOPIC THEORIES : In these theories [examples are Brans-Dicke (1961) and Hoyle-Narlikar (1964)] the constant G is related to the distribution of the matter in the universe, the idea being to incorporate Mach's principle which seeks to relate local and global properties of matter. Since the distribution of matter changes with time in an expanding dynamic universe, G is also time varying in these theories. In theories such as Brans-Dicke the varying gravitational constant is related to massless scalar field as $G \sim \phi^{-1}$; ϕ obeys an equation like $\Box\phi = T$ and is connected with the overall distribution of matter in the universe through a relation like $\phi \sim M/RC^2$. Thus $G \sim RC^2/M$, imposing the constraint $GM/RC^2 \sim 1$ on the universe. More generally, G will be a function of the field ϕ, *i.e.* $G \sim f(\phi)$ and the gravitational Lagrangian will have the form :

$$L = \int d^4x \sqrt{-g}\, (F(\phi)\, R + L_{\text{matter}}),$$

R being the scalar curvature. This leads to the field equations

$$R_{\mu\nu} - \tfrac{1}{2} g_{\mu\nu}R = -\frac{1}{2F}[T_{\mu\nu} + 2F_{\mu;\nu} - 2g_{\mu\nu}F^{\alpha}_{;\alpha}],$$

with G again related to ϕ. Similar equations are obtained in the Hoyle-Narlikar theory, G now being related to a mass field,

$$F = \tfrac{1}{2} \sum_{a \neq b} \sum m^{(a)} m^{(b)}, \quad \text{and } G = 3/8\pi F.$$

In all these theories G therefore becomes a time dependent quantity, since it is related to a changing matter distribution. We can also understand these results in terms of broken scale invariance.

Consider a scale invariant Lagrangian like $\int d^4x \sqrt{-g}\,[g^{\mu\nu}\phi_{,\mu}\phi_{,\nu} + \frac{1}{6} R\phi^2]$, which under the scale transformation $g_{\mu\nu} \rightarrow \bar{g}_{\mu\nu}\, \phi^{-2}$ is formally equivalent to Einstein's Lagrangian with the resulting field equations $R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 6\phi^{-2}\,[T_{\mu\nu}(\phi) + T_{\mu\nu}$ (matter)]. The scale invariance is broken (*i.e.* T^{μ}_{μ} (matter) $\neq 0$), *e.g.* by adding a mass term for the scale field in the Lagrangian, *i.e.* $m^2 \int d^4x \sqrt{-g}\, \phi^2$, implying $m^2\phi^2 = -T^{\mu}_{\mu}$. If N be the total number of particles, then $\langle T^{\mu}_{\mu} \rangle_{\text{av.}} \sim NmR_{\text{H}}^{-3}$, R_{H} being the Hubble radius. Thus $G \sim R_{\text{H}}C^2/Nm$, One can see that G fixes the magnitude of the scale invariance braking as given by the range of the ϕ field. This same values of G realized from macroscopic considerations is used in determining energy and length scales at which quantum effects in gravity become important, the so called Planck energy $(\hbar C^5/G)^{1/2} \sim 10^{19}$ GeV, and Planck length $(\hbar G/C^3)^{1/2} \sim 10^{-33}$ cm, at which the theory (*e.g.* GTR) is customarily expected to 'break down' (Wheeler 1962). The Planck length is used as the ultimate, ultraviolet cutoff to wavenumber in expressions for the self energy of particles in gravity-modified quantum electrodynamics. At Planck energies, gravity is as strong as the other interactions. In these often quoted deductions about the behaviour of gravity in the microworld, one uses value of G as obtained from the macroscopic picture, *i.e.* by relating it to the averaged matter distribution in the universe. In other words, G is assumed to be energy independent right up to the highest energies, contrary to the behaviour of unified gauge theories where the coupling constant is a function of energy.

An alternative viewpoint advocated by Sakharov (1968) suggests that G be a derived constant from microphysics. Then the curvature of space-time alters the zero point energies, so that one has

$$L(R) = A\hbar \int k^3\, dk + B\hbar R \int k\, dk + \text{higher order terms},$$

The first term would be dropped from usual renormalization arguments whereas the second term is identical to the Hilbert action of GTR provided G is now defined as $G = C^3/16\pi B\hbar \int k\, dk$, where the cutoff k_{C} in the formally divergent integral is taken to be of order of magnitude of the reciprocal Planck length, *i.e.* $k_{\text{C}} \sim (C^3/\hbar G)^{1/2} \sim 10^{33}\ \text{cm}^{-1}$. Here it is assumed that the value of this cutoff arises purely out of the physics of all other fields and particles. In this connection it is of interest that the energies at which GUTs expect all interactions to 'merge' are 10^{15}–10^{16} GeV (Buras *et al.* 1978) not far from the Planck energy. Thus inclusion of gravity through a larger group structure such as in theories of supergravity could actually raise these energies to the Planck energy (Cremmer & Julia 1979), so that breakdown of the supergroup below this energy into gravity and other interactions would now generate the gravitational constant. Here unlike the earlier case, the cutoff wave number or energy is more fundamental, being related to the energy at which unification occurs. The gravitational constant here is derived from this cutoff energy. We shall elaborate these ideas in the next two sections.

3. Gravity in relation to other interactions

As is well known, gravity can be considered a non-Abelian gauge field. The gravitons couple to themselves like the massless coloured gluons of quantum chromodynamics. The group of general coordinate transformations is a conserved non-Abelian group like colour SU (3) which is assumed to be an exact symmetry of the strong interactions binding quarks. The dimensionless strong interaction coupling constant $\alpha_S (= g^2/4\pi)$ is expected to decrease at large energy (momentum) of the interacting particles approximately as

$$\alpha_S(E) \simeq 1\ [1 + \pi \ln (E^2/\Lambda^2)]^{-1}$$

Λ being of order of 1 GeV, so that as $E \to \infty$, $\alpha_S \to 0$ (asymptotic freedom).

In the unified theories, the electromagnetic coupling constant is also a weak function of the energy. It obeys a similar logarthmic relation, but only this time it slowly increases with the energy (being an Abelian gauge field), and at the large unification energy ($\sim 10^{15}$ GeV) both the coupling constants merge into a single constant characterizing the unification group. Einstein's gravitational theory and Fermi's theory of weak interactions (FWI) have one thing in common: unlike electrodynamics and QCD, they both contain dimensional coupling constants with dimension of mass^{-2}, the Fermi coupling constant being $G_F \sim (300\, m_p)^{-2}$, and Newtonian gravitational constant being $G_N \sim (10^{19}\, m_p)^{-2}$, both in units of GeV^{-2}, m_p with the proton mass ~ 1 GeV. Unification of eletromagnetic and week interactions has suggested that the smallness of G_F is due to the large mass of the intermediate boson M_W *i.e.* $G_F \sim e^2/M_W^2$. At energies $\sim M_W$ both interactions have the same coupling e^2, the electric charge, *i.e.* $G_F M_W^2 \sim e^2$. The mass is in turn generated by the non-zero vacuum expectation value (VEV) of a scalar field ϕ: $G_F = e^2/M_W^2 = 1/\phi^2$. The analogy suggests that the smallness of G_N, the Newtonian constant, can also be attributed to a very massive particle $\sim$ a Planck mass (M_{Pl}), which mediates the interaction at the very high GU energies $G_N = \hbar C/M_{Pl}^2$, M_{Pl} being generated by the large VEV of a Higgs scalar field at those energies. While considering scale invariance we already had a Lagrangian like $\frac{1}{2}\, \epsilon\phi^2 R + \frac{1}{2}\, \phi_{;\mu}\phi^{;\mu}$. Now we add a 'cosmological' term of the form $\Lambda\phi^4$, this being the tranformation of Λ under the scale field. This would give an action like $\frac{1}{2}\, \phi_{;\mu}\phi^{;\nu} + \frac{1}{6}\, \epsilon\phi^2 R + \Lambda\phi^4$, which looks just like the Higgs scalar field which supplies masses to the mediating particles in the unified field theories.

More generally, in place of the Hilbert action, one may write

$$I = \int [\tfrac{1}{2}\, \phi^2 R + \tfrac{1}{2}\, \phi_{;1}\phi^{;1} - V(\phi) + L_{matter}]\, \sqrt{-g}\, d^4x,$$

where the potential function $V(\phi)$ is of the form

$$V(\phi) = - m^2(E)\phi^2 + \lambda\phi^4;$$

m being the energy-dependent (or temperature-dependent) mass parameter of the field (Sivaram 1977; Zee 1979). Then as in the unified theories, the gravitational constant G now depends on the VEV of ϕ as $G = 1/8\pi\epsilon\phi_0^2$. Thus the parallel with the Fermi interaction has been made explicit.

Again it is to be noted that in the Einstein-Cartan-Weyl theory, there is a surprising similarity (Shivaram 1975, 1977) in the form of the effective spin-spin interaction $\kappa(\bar{\psi}\gamma_\mu\gamma_5\psi)^2$ to the Fermi weak interaction term. Thus the high energy behaviour of the Fermi theory should be a guide to what can happen to gravity. We know that cross-sections for processes like $e^+ + e^- \rightarrow$ neutrinos in the Fermi theory grow like $\sigma \sim G_F^2 E^2$ with centre-of-mass energy E. We also know that the unitarity bound is violated for $E \sim 1/\sqrt{G_F} \sim 300$ GeV. Apparently something new must happen at $E \leqslant 300$ GeV. Cross-sections for quantum gravitational processess will also grow like $\sigma \sim G^2E^2$ (like in the case, $e^+ + e^- \rightarrow$ gravitons), and unitarity will now be violated for $E \sim 1/\sqrt{G} \sim 10^{19}$ GeV. Again, one invokes four-Fermion interactions for baryon-number-violating interactions in GUTs. Here also the appropriate coupling $f \sim (M_{Gu})^{-1}$ is very weak. The apparent violation of the S-wave unitary bound $\sigma \sim f^2 \leqslant E^2$, at $E \sim f^{-1}$ can be circumvented only if at $E \geqslant M_{Gu}$ the four-Fermion coupling ceases to be a good approximation.

In fact Heisenberg pointed out long back that in those interactions where the couplings have dimensions of negative powers of mass, the mass scale which enters in the couplings would set a bound on the applicability of the theory. This for Einstein's theory would happen at $E \sim 10^{19}$ GeV. If the coupling constant of a field theory has dimensions of mass^d, then the integral for a Feynman diagram of order N will behave at large moment like $\int p^{-Nd}dp$. Thus for interactions with $d < 0$, like gravitational and Fermi weak, the integrals for any process will diverge at high order, that is, a dimensionless amplitude of order G^n diverges as G^nE^{2n}. If one now insists that at asymptotically high energies the cross-sections for S-waves for all interactions be Froissart bounded, $\sigma \sim 1/s \sim E^{-2}$, then one can see that both the renormalizable theories like QED and QCD satisfy this bound :

$$\sigma_S = \alpha_S^2 \,.\, 1/s; \quad \sigma_{ee} = \alpha^2.\ 1/s.$$

At energies around 10^{15} GeV, both σ_S and σ_{ee} meet. Now the Fermi interaction is current-current below ~ 100 GeV. At about this energy σ_{weak} would meet σ_{ee} and σ_{weak} should again be proportional to E^{-2}, which means G_F should become energy independent.

As seen above, the cross-section for gravitational processes would parallel that for weak interactions and by analogy at energies above 10^{19} GeV, the cross-section should now be proportional to E^{-2}, and the dimensionless coupling constant $\alpha_G = GE^2/\hbar c$ should be energy independent. This is possible only if $G \propto E^{-2}$ above this energy. We can therefore write for the energy dependent gravitational constant $G(E) = G_N/(1 + E/E_P)^2$, where E_P is the Planck energy. At the usual low energies,

$E \ll E_P$. One then recovers the usual Newtonian constant G_N of Einstein's GTR. At very high energies $E > E_P$, $G(E) = G_N(E_P/E)^2 \propto 1/E^2$.

We can justify this conclusion also from what was said earlier about G being related to the VEV of the scalar field ϕ : $G = 1/8\pi\epsilon\phi^2$. Since $\phi \sim E$, it generates the mass of the field quanta, so that $G \propto E^{-2}$. Again for large E, the S-matrix elements in quantum gravity tend to $\kappa^{e+l-1}E^{l+3}$ (l is the number of loops, e the number of external lines). This is also true for extended supergravity. It is easily shown that the S-matrix elements would behave in the Froissart manner (as for other interactions) if the 'running' gravitational constant $G(E)$ behaves like E^{-2} for large E. This would also ensure finiteness of all the cross-sections at very large E, that is $E > E_P$. It has been pointed out that in a renormalized model of quantum gravity the renormalized gravity constant must scale like $G(E) \propto 1/(E^2)^\alpha$, $\alpha > 1$ (Salam & Strathdee 1978). What will be the other consequences of this high energy behaviour of the gravitational constant?. For one thing, it would be of significance in the very early epochs of the universe during and before the quantum gravity era, $t \sim 10^{-43}$ s. It may resolve the horizon problem as can be briefly seen as follows.

We have the relation between r and t as $r = at^{1/2}$, $a =$ (const. $\times$ $G)^{1/4}$. If this relation remains true for all t, then r becomes larger than light velocity when $t < (a/2c)^2$ implying that in very early times of the universe not all particles were in causal contact. But if G is energy- and hence temperature-dependent at $E > E_P$, that is $G = G_N\,(T_P/T)^2$, then one can see that $r \propto t$ and causality is reestablished even for the smallest values of t. Since the rate of expansion depends on G and if G decreases monotonically above a critical energy, the expansion can be prolonged like in some recent inflationary universe models, establishing causality and delaying phase transitions.

The monotonic weakening of gravity above a certain critical energy, in analogy with the other interactions, would doubtless ameliorate the formation of singularities in gravitational collapse, which are inevitable in GTR. Usually one glossess over the behaviour of matter after it crosses the event horizon with the statement that it collapses with the comoving observer to a singular state of infinite density. In the present picture, once Planck energies are reached at the densest regions and exceeded, the interaction steadily weakens and the matter can rebound.

In fact in the spirit of asymptotic freedom characterizing other non-Abelian gauge theories, of which gravity is a prime example, the coupling vanishes as energies tend to very high values. A detailed investigation of this scenario is in progress (Sivaram 1983b).

Another consequence of the present picture is that, as explained earlier, one ends up with a finite theory of quantum gravitational processess. No doubt a detailed study of the possible modifications of gravity at very high energies would provide answers to some very fundamental questions in physics and astrophysics.

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