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M. J. Johnson, Esq. President, in the Chair.

Lieut. Tennant, Bengal Engineers, India;
Rev. S. Newth, New College, London; and
Capt. W. Noble, R.A., Woolwich,

were balloted for and duly elected Fellows of the Society.

On the Computation of Double Star Orbits. By Capt. W. S. Jacob, East India Company's Astronomer.

"In bringing this subject to the notice of the Society, it is proper to premise that I have no decidedly new method to propose; but it has occurred to me, having computed many orbits in various ways, that my experience may be of some service to others, who may feel disposed to take up the subject, by suggesting certain facilities in minor matters of detail, by which time may be saved or greater accuracy attained.

"The plan I follow is, in the main, Herschel's first method given in vol. v. of the Society's *Memoirs;* his second, or improved method, in vol. xviii. I have also tried; but, though more elegant and scientific in form, it does not appear to me to insure greater accuracy, while it has the disadvantage of working *in the dark*, instead of allowing scope for the judgment to be exercised at every step, which, considering the loose character of much of the materials to be dealt with, appears to me of great importance. In order that this second method may be advantageously applied, the observations would have to be all of about the same order of goodness, and pretty equally distributed round a considerable part of the periphery of the ellipse,—conditions which are scarcely as yet attainable in a single instance; and it would also be essential that all the computations should be made in duplicate and compared at every step, since a small numerical error would vitiate the whole results, and would be by no means easy of detection.

"I have, therefore, after a few trials, abandoned this method, and returned to the old or graphical mode with very slight alterations. In the preliminary operations up to the first approximation to the apparent orbit, Herschel's rules are exactly followed; occasionally, when the distance-measures have been sufficiently good and numerous, I have projected them, as well as the angles, into a curve, and have brought the two curves into unison by slight alterations and successive trials; but the occasion for this has rarely offered, and it is in general easy to introduce the effect of the distances in the final corrections. Having then laid down the corrected places from the position curve, I cut out in paper an ellipse of about the proper size, and adapt this so as to agree as nearly as possible with the points laid down; it is then easy to see, not only the proper direction of the major axis, but also whether any variation is required in the dimensions of the ellipse: having thus got approximately the apparent orbit, the real elements are obtained by Herschel's rules, with the following exceptions: having found a' and α graphically, I find it lead to greater accuracy to compute b' and β by the following formulæ rather than to get *them* also by the graphical mode:—

$$\tan \mathrm{B} = \frac{t^2}{c^2} \tan \mathrm{A}$$

$$a' = \frac{t \,.\, \sec \mathrm{A}}{\sqrt{1 + \frac{t^2}{c^2} \tan^2 \mathrm{A}}}$$

$$\frac{a'}{b'} = \frac{t \,.\, \sec \mathrm{A}}{c \,.\, \sec \mathrm{B}}$$

where t and c are the transverse and conjugate axes of the apparent ellipse, a' and b' the projected major and minor axes of the true orbit, A the inclination of a' to t, and B that of b' to c.

"To obtain the mean motion and epoch, Herschel proposes two methods: first, to cut out of card and weigh in a nice balance the whole ellipse and the portions included between the extreme dates of observation and the projected major axis; or second, analytical, by the resolution of certain equations: the first appears to have been the plan generally preferred; the second having been probably rejected by common consent as needlessly laborious. I have not followed either, not having, in general, had at hand a balance accurate enough for the purpose, and being also doubtful of the advantage of the statical mode over that of simple mensuration:—whether, even supposing the weights could be obtained with perfect accuracy, it would be possible to cut out the ellipse more exactly, or even so exactly as it could be measured. My plan is to divide the portion of the ellipse included between the observations into triangles and segments, and measure them in the ordinary way: the area of the elliptic segment being approximately $= 0{\cdot}67\, ch$, where c is the chord and h the height or versed sine of the segment; this is exact for the parabola, and practically so for the ellipse when the segment is small enough not to differ sensibly from that of a parabola,—say, in general, when c exceeds $10h$. Supposing the observations spread over a large portion of the ellipse, so as to include within them (or nearly so) the area of one or more quadrants, it will evidently be sufficient to measure the *difference* of the total area described from that of the included quadrants.

"For drawing ellipses I have found the best mode, both as regards convenience and accuracy, to be that of co-ordinates, depending on the fact that all ellipses with a common axis (major or minor) have the ordinates on the same abscisses proportional to the *other* axis; consequently, since the circle is one of them, if the absciss* $x = a \sin X$, then $y = b \cos X$.

"There are several of these co-ordinates which can be easily retained in the memory, so that no time need be lost in referring to tables; thus,—

If $x = a \times$ ·28	then $y = b \times$	·96
·50	... ...	·866
·60	... ...	·80
·707	... ...	·707
·80	... ...	·60
·866	... ...	·50
·96	... ...	·28

"It will be observed that there are only 4 ratios to be remembered, and they furnish 7 points in each quadrant, besides the extremities of the axes, or 32 points in the whole periphery, through which the curve may be readily drawn with a curve-ruler, such as is used by architects for tracing their mouldings.

"For approximately solving the equation $u - e \,.\, \sin u = m$, Sir J. Herschel mentions having constructed a machine of wheel-work: this is, doubtless, a very ingenious contrivance, and probably useful for other purposes; but it is not at all essential here, since a solution may be obtained by the common slide-rule. The line of sines on a Bate's 10-in. rule will give the value by simple inspection to about $0^s{\cdot}1$, or even less; whereas the machine was said to give it only to the nearest degree.

"There is nothing further to be noticed till we come to the final corrections. After comparing the computed with the observed places, I formerly used to correct the elements by the method of least squares. This proved unsatisfactory; the labour was great, and the result by no means commensurate, since the changes required in the elements were often so large as to render sensible the effect of variations of the second and higher orders, and it was not easy to introduce these into the equations of condition; and I now adopt a partly graphical mode, by laying down carefully on a large scale the apparent orbit from the computed places. This will not agree exactly with the one first assumed,

* x having its origin at the centre.

because slight and obvious corrections will probably have been introduced into the elements (especially ☊) in the course of computation: the eye will then readily detect the alterations required in the course or dimensions of the curve to make it agree more closely with observation, and it will sometimes be an assistance to cut out in paper an ellipse of about the right size, and see whether by turning it this way or that the proper amount of deviation can be given. In this way, after a few trials, a result will be obtained as good as the materials are capable of giving.

"It will generally be advisable to compute the coefficients of variation for the different elements, to serve as guides in making these final alterations. They are very easily computed as follows:—

$$\frac{\Delta\theta}{\Delta\tau} = \text{annual motion (apparent)}$$

$$\frac{\Delta\theta}{\Delta n} = \text{annual motion} \times \frac{t-\tau}{n}$$

$$\frac{\Delta\theta}{\Delta e} = \frac{\text{annual motion}}{n} \times \sin u\left(1 + \frac{\sin u}{2\sin v\,.\,\sqrt{1-e^2}}\right)$$

$$\frac{\Delta\theta}{\Delta(\log\cos\gamma)} = \frac{1}{\text{tab. diff. log tan}\,(\theta - \Omega)}$$

$$\frac{\Delta\theta}{\Delta\lambda} = \frac{\text{tab. diff. log tan}\,(v+\lambda)}{\text{tab. diff. log tan}\,(\theta - \Omega)}$$

"The annual motion is easily deduced from the computed change of angle for any given interval, and the mean distance for that interval, remembering that the rate of motion is always in the inverse ratio of the squares of the distances.

"The coefficients thus obtained may be employed, if it be thought fit, in forming equations of condition for solution by the method of least squares, but it will in general be found preferable to use them, in conjunction with the graphical mode above described, to correct the elements by trial and error; for having found graphically the general direction of the alterations required, the coefficients will enable us to judge on which of the elements the changes can most advantageously be thrown, so as to produce the greatest proportional effect on the resulting angles.

"The following elements for the orbit of γ *Coronæ Australis* were obtained in the manner above described, being the result of the second trial:—

τ =	1863$^{\text{yr}}$·08	
P =	100·80	
n =	3°·5715	
π =	256° 12′	
☊ =	352 13	
λ =	266 25	
γ =	53 35	l. cos = 9·7736
e =	0·602	
a =	2″·549	

Comparison.

Date.	θo.	θc − θo. ∠	θc − θo. arc.	ρo.	ρc − ρo.
	° ′			″	
1834·47	37 6	+46	·034		
1835·55	36 48	−48	·035		
1836·43	34 30	− 3	·002		
1837·43	32 42	+ 1	·001	2·66	−·14
1847·32	14 6	+ 5	·003	2·30	−·01
1850·46	5 52	+77	·047	2·29	−·20
1851·54	4 28	− 3	·002	2·26	−·20
1852·27	3 27	−59	·034	1·89	+·12
1852·72	0 58	+13	·007	1·91	+·04
1853·25	359 35	+ 3	·002	1·83	+·08
1853·78	358 30	−30	·016	1·82	+·05
1854·26	356 10	+16	·008	1·71	+·12

"The signs of the distance-errors would indicate some further slight correction of the elements, but the early distances are not worthy of much confidence.

"*London, 7th June,* 1855."

M. Drach has received a letter from Dr. Donati, of the Florence Observatory, informing him of his having discovered, on the 3d instant, a new comet in the constellation *Telescopium Herschelii.* With the ring micrometer, and a low magnifying power, and through mist and clouds, he obtained the following positions:—

	Florence M.T.	Comet — Star. In R.A.	Comet — Star. In Decl.
1855.	h m s	m s	′ ″
June 3	10 4 10	−2 17·18	+22 0·0
4	9 55 12	+2 37·78	− 9 22·9
5	9 18 36	−4 2·41	−20 12·5

Comet's App. R.A.	App. Decl.	No. of Comparisons.
h m s	° ′ ″	
......	......	2 with (*a*)
6 56 56·27	+36 22 5·5	1 with (*b*)
7 10 32·73	+36 15 15·1	1 with (*c*)

Apparent Positions of the Comparison Stars.

	R.A.	Decl.	Catalogue of Stars.
	h m s	° ′ ″	
(*b*)	6 54 18·49	+36 31 28·4	Lalande 13569
(*c*)	7 14 35·14	+36 35 27·6	— 14298