A MAGNETO-HYDRODYNAMIC STUDY OF PRE-FLARE LOOPS

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Introduction

The aim of this paper is to discuss the equilibrium and stability of a configuration which may apply to pre-flare loops on the Sun. The analysis is based on the following working scenario:

a) The loops exist much prior to the flare and in equilibrium with their surroundings.

b) Gradually, starting probably a few hours before the flare, the configuration acquires energy in the form of currents.

c) During the period of energy build-up, the loops are MHD stable.

d) The flare occurs only after there is adequate energy in the currents.

Equilibrium

Assuming cylindrical geometry, we choose the following equilibrium distributions for the magnetic field and pressure within the loop:

$$B_z = B_o/(1 + \varepsilon^2 r^2) \qquad (2.1)$$

$$B_\theta = B_o \mu r/(1 + \varepsilon^2 r^2) \qquad (2.2)$$

$$P = P_o + (B_o^2/2\mu_o)\, \frac{(\varepsilon^2-\mu^2)}{\varepsilon^2}\, [1 - \frac{1}{(1+\varepsilon^2 r^2)^2}] \qquad (2.3)$$

where the subscript o denotes the quantities on the axis of the loop and μ and ε are constants. The pitch of the magnetic field is $2\pi/\mu$, which we have assumed to be constant. For $\mu=\varepsilon$, we get the well-known force-free field of Gold and Hoyle (1958).

We shall now attempt to find typical values for μ and ε for the loops under study, using observations of sub-flares.

The magnetic energy associated with the azimuthal field is given by:

$$W_\theta = \frac{F^2 L}{2\pi a^2 \mu_o} \frac{\mu^2 a^2}{\ln(1+\varepsilon^2 a^2)} \left[1 - \frac{\varepsilon^2 a^2}{1+\varepsilon^2 a^2} \frac{1}{\ln(1+\varepsilon^2 a^2)} \right] \qquad (2.4)$$

where a is the internal radius of the loop, L is its length, F is the longitudinal magnetic flux and μ_o is the permeability constant.

Let us take a ~ 2000 km, L ~ 50000 km, $W_\theta \sim 10^{21}$ Joules, which is a typical energy associated with with a sub-flare and F ~ 3 x 10^{10} Webers corresponding to an average longitudinal field of about 25 Gauss. If the field is approximately force-free, we have

$$\mu a \sim \varepsilon a \sim 2$$

Stability

The MHD stability of the system will be examined using the following equation for the vector displacement $\vec{\xi}$ of a plasma element (Bernstein et al., 1958):

$$\rho \frac{\partial^2 \vec{\xi}}{\partial t^2} = \vec{\nabla}\ (\gamma p \vec{\nabla} \cdot \vec{\xi} + \vec{\xi} \cdot \vec{\nabla} p) - \mu_o^{-1} (\vec{\nabla} \wedge (\vec{\xi} \wedge \vec{B})) \wedge (\vec{\nabla} \wedge \vec{B}) - \mu_o^{-1} \vec{B} \wedge (\vec{\nabla} \wedge (\vec{\nabla} \wedge (\vec{\xi} \wedge \vec{B}))) \qquad (3.1)$$

Results and Discussion

Equation (3.1) along with suitable boundary conditions was solved as an eigen-value problem for ω, the frequency of the normal modes. The results of the calculations are shown in the table, in which $K_{\parallel} = \vec{k}.\vec{B}$,(measured in units of B_o/a), the wave-number corresponding to marginal stability, is shown for a few typical values of μa and εa. The last column gives the ratio P_e/P_o of the pressures at the surface and axis of the loop respectively. Instability occurs for $K_{\parallel,1} < K_{\parallel} < K_{\parallel,2}$. It may be observed that as the pressure-gradient increases, the wave-number region where instability occurs becomes smaller and finally disappears altogether.

We, thus, see that the force-free configuration is unstable for all cases considered, with the wave-number region for instability increasing inversely with pitch (i.e. $2\pi/\mu$). The minimum pressure-gradient that is necessary for stability also increases with μa and for $\mu a \gg 1$, the stable configuration is far from being force-free. For $\mu a \sim 1$, however, a very modest pressure-gradient provides stability and the configuration can still be regarded as approximately force-free. Incidentally, it is worth noting that even if we assume that the ends of the loop are anchored firmly, the force-free configuration corresponding to $\mu a \gtrsim 1$ cannot be stabilized as it would require a length which is roughly the same or even less than its radius, contrary to observations.

In conclusion, the analysis in this paper has shown that stable configurations for loops, possessing adequate energy for a flare, are possible provided that positive pressure gradients of the magnitude indicated exist.

Table 1. The wave-numbers $K_{\parallel,1}$ and $K_{\parallel,2}$ corresponding to marginal stability are shown for different values of μa and ϵa. The pressure ratio is calculated from equation (2.3), assuming a value of $\beta = 2\mu_o P_o/B_o = 0.01$.

μa	ϵa	$K_{\parallel,1}$	$K_{\parallel,2}$	P_e/P_o
	1.0	0.0	0.21	1
1.0	1.02	0.04	0.18	3.95
	1.04	-	-	6.8
	2.0	0.0	1.0	1
2.0	2.2	0.4	0.8	17.85
	2.21	0.5	0.7	18.58
	2.23	-	-	20.01
	3.5	0.0	2.5	1
3.5	4.5	2.0	2.2	40.42
	4.6	-	-	43.02
	5.0	0.0	4.0	1
5.0	6.0	1.8	3.8	31.53
	6.7	-	-	45.29

References:

Bernstein, I.B., Frieman, E.A., Kruskal, M.D. and Kulsrud, R.M.: 1958, Proc.Roy.Soc. London, A224, 1.

Gold, T. and Hoyle, F.: 1958, Mon.Not.Roy.Astron.Soc. 120, 89.