LARGE NUMBER COINCIDENCES AND UNIFICATION OF THE PARAMETERS UNDERLYING ELEMENTARY PARTICLES ASTROPHYSICS AND COSMOLOGY

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ABSTRACT

The ubiquitous occurrence of the Dirac-Eddington large dimensionless numbers when relating the physical parameters such as mass, radius, angular momentum etc. of typical astrophysical objects like stars and galaxies to the fundamental constants of atomic physics is currently interpreted in terms of constraints imposed on these parameters as a result of physical processes underlying the existence of these objects rather than as chance coincidences, i.e. these relations can be understood in terms of the underlying physics governing these objects. Again various cosmological parameters such as the total number of nucleons, the photon-to-baryon ratio etc. can be expressed in terms of these numbers, which again can be understood in terms of the physics involved. In fact it would appear that Eddington's *cloud bound observer* can also get a good idea about the overall mass, size and background temperature of the universe, apart from his classic deductions on the masses and luminosities of stars sans observations. Further the weak and strong interaction coupling constants can be included in the large number hypothesis (LNH) and dimensionless relations connecting these constants to cosmological parameters can be constructed. The gross parameters characterizing the universe such as overall size and mass can be arrived at from microphysical considerations involving the fundamental interactions of elementary particle physics with interesting relations for the

Hubble radius and closure density obtained entirely in terms of the coupling constants underlying these interactions. SEveral other interesting coincidences and relationships connecting the parameters of cosmology and elementary particle physics are pointed out. The significance of these inter-relations is explored especially in connection with the time variation of the fundamental constants and the unification of cosmology and quantum physics. The above topics being close to Eddington's interests seem particularly appropriate for discussion at this meeting.

1. INTRODUCTION

The Eddington-Dirac dimensionless large numbers arose in discussions involving physics and cosmology as follows: If one considers the ratio of the electrostatic force between a proton and an electron to the gravitational force between them one obtains a large number, i.e. $e^2/Gm_pm_e \approx 10^{40}$, the electrostatic force being proportional to e^2, e being the electric charge (same for both proton and electron) and the gravitational force being proportional to the product of the proton and electron's masses (m_p and m_e respectively) multiplied by the universal gravitational constant G. The inverse squared distance dependence being same for both these long range forces of course cancels out. Now one can form another large dimensionless number by expressing the so called Hubble age of the universe (i.e. the time elapsed since the universal expansion began) given by the inverse of the Hubble's constant H (i.e. 1/H) in units of the so called atomic time, given for instance by the time it takes light to cross a typical elementary particle dimension (say the classical electron radius $e^2/m_ec^2 \sim 3 \times 10^{-13}$ cms) which is $\approx 10^{-23}$ secs. The ratio of the two times is again a large number, $\sim 10^{40}$ which is remarkably the same as the first large number. Another large number is the total number of nucleons in the universe which is estimated as $\sim 10^{80}$. This number is the <u>square</u> of the previous two large numbers, again quite remarkable as there was no apriori reason to expect this or for that matter the equality of the previous two large numbers. A dimensionless large number (LN) involving the Planck's constant $\hbar$, m_p G and C is ($\hbar c/Gm_p^2$) $\approx 10^{38}$. It being the inverse of the gravitational fine structure constant ($Gm_p^2/\hbar c \approx 10^{-38}$) and hence expressing the strength of the gravitational binding between protons we may expect this number (or its appropriate powers) to crop up in situation involving the gravitational assembly of a large number of nucleons, i.e. in celestial bodies. Indeed the masses of most stars turn out to be within a numerical factor of the mass $M_s = (\hbar c/Gm_p^2)^{3/2}\, m_p \approx M_\theta$, M_θ being the solar mass. The Chandrasekhar limiting mass for white dwarf stars (where the gravitational 'charge' Gm^2 is balanced by the quantum 'charge' $\hbar c$ of the degenerate Fermi gas) is again precisely M_s. If we denote the LN i.e ($\hbar c/Gm_p^2$) by N_1 it turns out that several physical parameters characterising stars can be expressed as simple powers of N_1 multiplied by the appropriate quantum physical fundamental constants.

To give some examples:

$$\text{Mass:}\quad M_s = (\hbar c/Gm_p^2)^{3/2} \times m_p = N_1^{3/2}.\, m_p$$

Radius:

(Typical Main-Sequence Star) $R_s = N_1^{1/2} \times$ Bohr radius $= N_1^{1/2} \times \hbar^2/m_e e^2$

$R_{\text{white dwarf}} = N_1^{1/2} \times \hbar/m_e c$ $\quad R_{\text{neutron star}} \approx N_1^{1/2} \times \hbar/m_\pi c$

Angular momentum : $\quad J_{\text{Star}} = N_1^2 \hbar$ ($\hbar$ = quantum unit of angular momentum)

Life time of hottest stars $\quad t_s = N_1 \times \hbar/m_p c^2$

and several such relations as we shall see later, including typical relations for galaxies (mass $N_1^{7/4}\, m_p$, etc.,) and the universe (mass $N_1^2 m_p$, etc.,).

II. LARGE NUMBER COINCIDENCES IN ASTROPHYSICS

Now we can understand the above typical relations for stellar objects in terms of the physics involved in their structure and evolution. For instance the virial theorem tells us that for the star to be in equilibrium the radiation pressure (given by $\sim (KT)^4 (hc)^{-3}$ T being the temperature) should not exceed the kinetic gas pressure $N_s R_s^{-3}$ KT, N_s and R_s being the total number of nucleons and the radius respectively. Using the virial theorem relation, $KT = GN_s m_p^2 R_s^{-1}$ and equating the above two pressures then gives the equilibrium number of nucleons for which the star is stable as:

$$N_s = (Gm_p^2/\hbar c)^{-3/2} \approx N_1^{3/2} \approx 10^{57},$$

which is the coincidence we observed earlier. Again the Chandrasekhar mass for a white dwarf is obtained as $N_1^{3/2} m_p$ by balancing the pressure of the relativistic degenerate fermion gas (proportional to number density $n^{4/3}$), with the gravitational force proportional to n^2. The radius of the equilibrium configuration can also be obtained in the above cases and gives $N_1^{1/2}$ x Bohr radius and $N_1^{1/2} \times \hbar/m_e c$ for mainsequence stars and white dwarfs respectively. The relation involving the stellar life time can be accounted for as follows. If L be the luminosity of the star and η be the fraction of rest mass energy converted into radiation in nuclear fusion reactions (η= 0.008) then the lifetime t_s of the star is $t_s = (\eta\ N_s m_p c^2/L)$. Now the maximal luminosity of a star of given mass is given by the so called 'Eddington luminosity' for which the radiation pressure on the stellar material just balances the inward gravitational pull (at higher luminosities than the Eddington limit the stellar material will be blown away by radiation pressure). The Eddington limit on the luminosity is given by $L = N_s\ G\ m_p^2\ c/k$ and if the opacity k is the one corresponding to Thompson (electron-photon) scattering as is indeed the case for the hottest stars, then $k = \sigma_t/m_p$, $\sigma_t = \frac{8\pi}{3} (e^2/m_e c^2)^2$ is the Thompson scattering cross-section. Writing σ_T as $\sigma_T = \alpha^2 \beta^2 (\hbar/m_p c)^2$; $\alpha = e^2/\hbar c$, $\beta = m_p/m_e$ and substituting for K and L in the formula for t_s :

Now $\eta\alpha^2 \beta^2 \simeq$ unity (!) so that

$$t_s \approx (\hbar/m_p c^2) \times (\hbar c/Gm_p^2)$$

$\approx N_1 \times (\hbar/m_p c^2)$, which was the relation given earlier. So it is no coincidence that $t_s \approx N_1 \times$ atomic time ! For the angular momentum (J_s) of a typical star we have $J_s = M_s V_s R_s$, where M_s, V_s and R_s are typical mass, rotational velocity and radius. As seen above; $M_s \approx (\hbar c/Gm_p^2)^{3/2} m_p$; $R_s \approx (\hbar c/Gm_p^2)^{1/2} . \left(\frac{\hbar^2}{m_e e^2}\right)$ and $V_s \approx (GM_s/R_s)^{1/2} \approx G^{1/2} (\hbar c/Gm_p^2)^{3/4} . m_p^{1/2} (\hbar c/Gm_p^2)^{-1/4} \times \left(\frac{\hbar^2}{m_e e^2}\right)^{-1/2}$ Substituting the above expressions for M_s, R_s and V_s we have :

$$J_s \text{ (main sequence) } \approx (\hbar c/Gm_p^2)^{5/2} \left(\frac{m_p^3}{m_e}\right)^{1/2} \frac{G^{1/2}}{e}$$

$$\approx 10^{78} \hbar \approx N_1^2 \hbar.$$

Similarly for a neutron star (NS)

$$M_{Ns} \approx (\hbar c/Gm_p^2)^{3/2} m_p; \quad R_{Ns} \approx \left(\frac{\hbar c}{Gm_p^2}\right)^{1/2} . \frac{\hbar}{m_\pi c}$$

m_π is the pion mass (the neutrons are separated by a distance $\hbar/m_\pi c$, the range of nuclear interactions on the average, thus the value of R_{NS}) and V_{NS} $(GM_{Ns}/R_{Na})^{1/2}$. As before putting together the formulae for M_{Ns}, R_{Ns} and V_{Ns} give :

$$J_{Ns} \approx \left(\frac{\hbar c}{Gm_p^2}\right)^{5/2} \left(\frac{m_p^3}{m_\pi}\right)^{1/2} (G/\hbar c)^{1/2} \approx 10^{76} \hbar$$

Similarly for a white dwarf: (WD)

$$J_{WD} \approx (\hbar c/Gm_p^2)^{5/2} \left(\frac{m_p^3}{m_e}\right)^{1/2} \left(\frac{G}{\hbar c}\right)^{1/2}$$

$\approx 10^{77}$ $\hbar$, so that angular momentum like the mass is more or less the same for all stars being N_1^2 $\hbar$, white dwarfs and neutron stars having lower angular momentum the reason being that the star during its evolution loses mass and for a neutron star it is distinctly lower as seen because a substantial portion is transferred to the expanding envelope during the supernova explosion. We shall see later how other physical parameters like magnetic moment of stars can also be expressed in terms of powers of N_1 times the corresponding microscopic quantity, i.e., Bohr magneton $e\hbar/2mc$ for magnetic moment etc. Again a typical interstellar magnetic field (10^{-6} gauss) when compared with the upper limit for magnetic fields in neutron stars the so called critical Schwinger field (i.e. when the Larmor radius mc^2/eB becomes equal to the elecron's Compton wavelength $\hbar/mc$, giving $B \sim m^2c^3/e\hbar \sim 10^{13}$ gauss) once more gives a ratio $N_1^{1/2} \approx 10^{19}$.

The above analysis can also be extended to galaxies, imposing constraints on their physical parameters which also turn out to be related to the ubiquitous N_1. An estimate of the typical mass and size of a galaxy can be arrived at by considering a collapsing gas cloud of mass M contracting to radius R (so that the virial temperature $T \approx \frac{GMm_p}{K_B R}$, K_B = Boltzmann constant) and $K_B T \sim 1$ Rydberg $\sim \alpha^2 m_e c^2 \approx 10\text{eV}$, the heated up cloud cools chiefly by bremsstrahlung emission, the cooling time being $t_c \approx (m_e c^2/K_B T)^{-1/2} \cdot \hbar/n\sigma_T e^2$ (n = number density) Now for the cloud to be supported by pressure, t_c should be less than the free-fall time, $t_{ff} \approx (R^3/GM)^{1/2}$, i.e $t_c < t_{ff}$ giving:

$$R \simeq \alpha^4.(\hbar c/Gm_p^2)\ (m_p/m_e)^{1/2}\ \frac{\hbar^2}{m_e e^2} \lesssim 10^2 \text{ kiloparsec (Kpc)}.$$

With corresponding :

$$M \gtrsim \alpha^5\ (\hbar c/Gm_p^2)^2 (M_p/m_e)^{1/2}\ m_p \gtrsim 10^{10}\ M_\odot \approx N_1^{7/4}.\ m_p$$

which agrees well with the obsered scales for galaxies.

As is well known, the so called Jeans mass characterises the initial fluctuations or inhomogenities which ultimately evolved into galaxies, masses smaller than the Jeans mass are dissipated by pressure forces. The Jeans length is given by $l_J \approx C_s/\sqrt{G\rho}$, C_s being the velocity of sound, $C_s = (K_B T/m_p)^{1/2}$, T being the average temperature of the ambient gas. The Jeans mass is $M_J \sim \rho l_J^3 \sim C_s^3/G^{3/2}\rho^{1/2}$, the rotational velocity of the mass $\sim C_s$. So we can write for the angular momentum of a typical galaxy objects, $J_G \sim M_J l_J\ C_s \sim C_s^5/G^2\rho$. Another well known coincidence involving N_1 to which we shall return later is that the ratio of the nucleon number density n to the photon number density $(KT/\hbar c)^3$ is $N_1^{-1/4}$ i.e $(Gm_p^2/\hbar c)^{1/4}$; so that $\rho = (KT/\hbar c)^3\ (Gm_p^2/\hbar c)^{1/4}.\ m_p$. Hence substituting for and C_s in J_G, we get the following formula for the angular momentum of a typical galaxy, expressed in terms of $\hbar$ as:

$$J_G = T^{1/2}\left(\frac{\hbar^9\ c^{13}}{K_B^2\ G^9\ m_p^{16}}\right)^{1/4}\hbar \approx 10^{100}.\ \hbar \approx N_1^{5/2}.\ \hbar.$$

This would explain the empirical observation that the mass and angular momentum of galaxies are respectively $N_1^{7/4}m_p$ and $N_1^{5/2}\hbar$, whereas the mass and angular momentum of stars are $N_1^{3/2}m_p$ and $N_1^2\hbar$. Thus the mass of a typical galaxy is $N_1^{1/4}$ times the mass of a star whereas its angular momentum is $N_1^{1/2}$ times that of a star. This is consistent with another empirical relation which is now well known, i.e. that the angular momentum of a wide range of celestial objects ranging from planets to galaxies, goes as the square of their mass, i.e $J \sim M^2$. This is also valid for black holes where one can write: $J = G/c\ M^2$; (i.e for extreme kerr black holes). For other bodies ($J = (G/V)\ M^2$) In the next section we shall see that a similar results holds for elementary particle resonances, their angular momenta rising as mass squared for the higher spin states, the slope of the so called Regge trajectories being $(G\sqrt{N_1}/\hbar c) \sim (1\text{Gev})^{-2}$. Now according to Blackett, the angular momentum and magnetic moment (H) of astrophysical objects are related according to:

$$H = \frac{G^{1/2}}{c} J, \qquad \text{one.}$$

We can write this as:

$$H = \frac{e\hbar}{2m_p c}\left(\frac{J}{\hbar}\right)\left(\frac{Gm_p^2}{\hbar c}\right)^{1/2}\left(\frac{1}{\alpha}\right)^{1/2}$$

i.e Bohr magneton x angular mom. in units of $\hbar$ x $1/\alpha$ x $N_1^{1/2}$!

As magnetic moment = Magnetic Field (B) x volume, we can estimate the magnetic fields of various celestial bodies. For the Earth, this gives about 0.5 gauss, for the sun about 5 gauss for neutron stars $\sim 10^{12}$ gauss and for the galaxy about 10^{-6} gauss in agreement with what is observed. We shall now try to understand how Eddington's cloud bound observer could proceed further to deduce that the mass and radius of the universe would be given by $N_1 \cdot e^2/m_e c^2$ and $N_1^2\ m_p$. As a bonus he would also arrive at $N_1^{1/4}$ for the ratio of the number densities of photons to nucleons. Assume that the microwave background could have been produced by pregalactic supermassive stars which may have formed in the period between decoupling and galaxy formation. Clusters of these objects could form culminating their evolution eventually as black holes. Being massive these objects would be radiation dominated and the total luminosity of a large number of these objects in a cluster would be given by the Eddington value, $L_E = 4\pi GMc/k = \frac{4\pi Gc}{k}\Sigma M$ where $\Sigma M = M_T$ = total cluster mass. If M_T is the total cluster mass, then general relativity imposes as is well known a lower limit on its spatial localisation or size given by $R_m \sim GM_T/c^2$ and the shortest time-scale that can be associated with the cluster is then $t_m \sim GM_T/c^3$. If during t_m a substantial portion η (~ 1) of the mass is converted into energy, then the maximum possible luminosity is given by $L_M \sim \eta M_T c^2/GM_T/c^3 \sim c^5/G$; with $\eta = 1$ this becomes the so called 'Gunn luminosity', which gives the upper limit to the power that can be radiated by the cluster, and as all the individual objects are radiating at their maximal Eddington luminosity we can equate above to L_E above to L_M. Thus with $L_E = L_M$ and substituting $k = \sigma_T/m_p$ $\sigma_T = \frac{8\pi}{3}(e^2/m_e c^2)^2$ the upper limit to the cluster mass then turns out to be:

$$M = M_T \approx \frac{e^4}{Gm_p m_e^2} \sim \left(\frac{e^2}{Gm_e^2}\right)\left(\frac{e^2}{Gm_p^2}\right) m_p \approx 4 \times 10^{78}\ m_p \approx N_1^2 m_p \approx 10^{21} M_\odot$$

The effective cluster size corresponding to the maximal luminosity would be: (GM_T/c^2):

$$R_m \approx (e^2/m_e c^2)\ (e^2/Gm_p m_e) \approx N_1 \cdot \frac{e^2}{m_e c^2}$$

Very interesting to note that these relations for the critical mass and radius of the cluster compare well with the mass and radius of the universe quite naturally accounting for the Eddington-Dirac Large Number relations. The effective temperature of the total radiation produced by the cluster (assuming thermalisation has taken place via grains, etc.) would be given by:

$$T_{eff} = \left(L_T / 4\pi\sigma_{SB} R_m^2 \right)^{1/4} \quad \text{, where}$$

$\sigma_{SB} = \pi^2 K_B^4 / 45\hbar^3 C^2$ is the stefan Boltzmann constant. Using the given expressions for L_T and R_m gives for T_{eff},

$$T_{eff} = \left(\frac{Gm_p^2 \hbar^3}{4\pi^3} \right)^{1/4} \left(\frac{m_e}{K_B e^2} \right) C^{11/4} \approx 10^{\circ} \text{ K and}$$

as baryon number is conserved we can estimate the maximal entropy per baryon:

$$S_{max} = 4a\ T^3 / 3n\ K_B = \left(\frac{e^2}{m_e c^2} \right) \left(\frac{1}{L_p \gamma_p} \right)^{1/2} \quad \text{, where}$$

$L_p = (\hbar\ G/C^3)^{1/2}$ (the Planck length) and $\gamma_p = \hbar / m_p c$

Substituting values give $S_{max} \approx 10^9$ photons/baryon $\approx N_1^{1/4}$, comparing well with what is observed. Rees has obtained a somewhat similar relation for the entropy per baryon i.e.

$$S \sim \left(e^2 / Gm_p^2 \right)^{1/4} \left(m_p / m_e \right) \left(\frac{e^2}{\hbar c} \right)$$

by considering the characteristic nuclear burning time-scale (so called Salpeter time $(t \sim C\sigma_T / 4\pi G m_p)$ for radiation dominated objects.

The angular momentum for such a super cluster following the earlier approach for stars and galaxies, is shown to be $J \approx N_1^3 \hbar \approx 10^{120} \hbar$, corresponding to that for the whole universe and the Blackett relation gives the magnetic field (intergalactic) as 10^{-7} gauss.

III. A UNIFICATION OF THE PARAMETERS OF ELEMENTARY PARTICLES AND COSMOLOGY

In his well known book Gravitation and Cosmology, Weinberg has drawn attention to a curious empirical relation connecting the mass of a typical elementary particle to cosmological parameters (Eq.16.4.2 of Weinberg):

$m_\pi \approx \left(\frac{\hbar^2 H_o}{GC}\right)^{1/3}$; m_π is the pion mass and H_o = Hubble's constant. This relation can be understood in the sense of an operational requirement that the gravitational self energy ($Gm^3c/\hbar$) of a particle of spread h/mc (So: $\frac{Gm^2}{r} \approx \frac{Gm^3c}{\hbar}$) be at least measurable over a Hubble time ($1/H_o$). The time-energy uncertainty principle then gives the above relation. This relation also arises naturally as a cosmological constraint on the upper limiting temperature of evaporating black holes giving rise to a characteristic or fundamental length, given by:

$$l_o = \left(\frac{3G\hbar}{32\pi C^2 H_o}\right)^{1/3} = \frac{e^2}{2m_e C^2} = \frac{\hbar}{m_\pi C} , \tag{1}$$

($1/H_o \approx 10^{18}$ secs), the limiting temperature being

$T_{max} = \frac{m_\pi c^2}{K_B} \sim 2\times10^{12}$ °K, interestingly the same as the Hagedorn temperature which arises in several bootstrap models of elementary particles. Stellar mass black holes would have a Hawking temperature of $\sim 10^{-7}$ °K and so the ratio of this temperature to the Hagedorn temperature is again $N_1^{1/2} \sim 10^{19}$. The question arises as to whether from microphysical considerations involving the fundamental interactions of elementary particles we can arrive at the gross parameters characterising the universe. We mention below two ways in which this might be done. As gravity is a very long range force, the mediating quanta (the gravitons) must have a vanishingly small rest mass ($m_g \to o$) (corresponding to the smallest possible rest mass). Now a mass m in general relativity cannot be localised in space to a distance smaller than Gm/c^2 and thus with the smallest possible m, i.e. m_g we would obtain the smallest possible distance or length scale in Nature, i.e. Gm_g/c^2. On the contrary, in quantum mechanics a particle or system of mass M cannot be localised over a distance smaller than $\hbar/Mc$ in contrast to classical physics where a point particle can be identified with a vanishingly small mass (localisation proportional to m in Gm/c^2) and localisation is inversely proportional to mass in quantum physics. The smallest possible length in the quantum picture corresponding to the largest possible mass whcih we assume as the mass of the universe M_U would then be $\hbar/M_U C$. Now if we insist for consistency that these two smallest length scales (defined in different ways) be the same we would have:

$$Gm_g/c^2 = \hbar/M_U C, \text{ or } Gm_g M_U = \hbar c \quad \text{or}$$

$$m_g M_U = m_{pl}^2, \text{ where } m_{pl}^2 = (\hbar c/G) \tag{2}$$

To get an estimate for m_g, we note that any two nucleons (in the universe) of mass m_p while they interact gravitationally by exchanging quanta, their mass would fluctuate by an amount: $\Delta m = m_p/\sqrt{N}$, where N is the total number of nucleons in the universe and the fluctuation Δm could be identified with m_g, the mass of the mediating particles exchanged. Thus $m_g = m_p/\sqrt{N}$ and equation(2) then gives (noting that $M_U = Nm_p$):

$$\sqrt{N}\, m_p^2 = \hbar c/G \text{ or } N = (\hbar c/Gm_p^2)^2 \simeq N_1^2 \tag{3}$$

thereby explaining apriori the coincidence noted earlier; a deduction from microphysics. Considering that the proton and electron are the only stable conserved particles one can construct three and only three possible types of gravitational charges which would in the dimensionless form be:

$$Gm_e^2/\hbar c, \; Gm_p^2/\hbar c \text{ and } Gm_p m_e/\hbar c$$

and these would dominate the long range interactions between all these particles in the universe. If N be the total number of particles one can write the local fluctuations in these couplings as:

$$\sqrt{N}\text{-}Gm_e^2/\hbar c, \; \sqrt{N}.Gm_p^2/\hbar c \text{ and } \sqrt{N}.Gm_p m_e/\hbar c.$$

If $N = 10^{79}$, we get the following intriguing relations for the three possible dimensionless constants:

$$\sqrt{N}.Gm_e^2/\hbar c \approx 10^{-5} \quad \text{identified with} \quad \left(\frac{G_F}{\hbar c}\right)\left(\frac{m_p c}{\hbar}\right)^2 = \frac{g_W^2}{\hbar c}$$

$$= \frac{g_w^2}{\hbar c} = \text{dimensionless weak interaction decay coupling constant}$$

(G_F = universal Fermi constant = 1.5×10^{-49} ergs cm^3)

$\sqrt{N}.Gm_p^2/\hbar c \approx 15$ identify with $\frac{g_s^2}{\hbar c}$ = strong interaction pion-necleon coupling constant

$\sqrt{N}.Gm_p m_e/\hbar c \approx 10^{-2}$ identify with $e^2/\hbar c$ (only protons interact strongly) electromagnetic coupling constant (4)

It is to be noted that strengths of strong and weak interactions are in the ratio $(m_p/m_e)^2 \sim 10^6$ and strength of strong and electromagnetic interactions in the ratio $(m_p/m_e) \sim 10^3$. So apart from gravitation, there are three possible types of gravitational charges and eqs (4) give their values in remarkable agreement with those observed. In Sivaram 1982(a), a formula was obtained for the gravitation mass m_g from the gravitational charge $g_g^2 = Gm_p m_e$ as $m_g = g_g^2/l_o c^2$; where l_o is the fundamental length in equation (1) (also using $g_s^2/m_p c^2 = e^2/m_e c^2$; g_s is the strong interaction charge given in eq. (4)) thereby giving

$$m_g = \frac{Gm_p m_e m_\pi}{\hbar c} \quad (5)$$

which gives from the uncertainty principle the maximal range associated with such fluctuations of energy due to the gravitational interaction as (this is to be identified with the Hubble radius of the universe):

$$R_H = \frac{\hbar^2}{Gm_p m_e m_\pi} \quad (6)$$

One can use the following relations (eq.7) seen to arise from the unification of weak, electromagnetic and strong interactions

$$e^2/2m_p c^2 = (G_F/\hbar c)^{1/2}$$

$$g_s^2/2m_p c^2 = e^2/2m_e c^2 = \hbar/m_\pi C \quad (7)$$

to eliminate the masses m_p, m_e and m_π from eq.(6) to give

$$R_H = \frac{g^4}{Ge^8}\left(\frac{c^7 G_F{}^3}{\hbar}\right)^{1/2} \simeq 10^{28}\text{ cm} \tag{8}$$

and the closure mass $M = c^2 R_H/G$ (from general relativity) gives for the closure density:

$$\rho_H = \frac{3Ge^{16}\hbar}{4\pi c^5 g^8 G_F{}^3} \sim 3 \times 10^{-29}\text{ g/cc.} \tag{9}$$

Eqs.(8) and (9) express cosmological parameters, solely in terms of the coupling constants of the four fundamental interactions. (truly in the Eddington spirit). Also substituting for m_g in eq.(2) from the relation given in eq.(5), we get the elegant relation:

$$G\, m_p m_e\, m_\pi\, m_U = m_{pl}{}^2\, \hbar c$$

or $$m_p m_e\, m_\pi\, m_U = m_{pl}{}^4 = (\hbar c/G)^2 \tag{10}$$

Writing m_U as Nm_p we have

$$N = \frac{m_{pl}{}^4}{m_p{}^2\, m_\pi\, m_e} \tag{11}$$

and further from the equality: $e^2/2\ m_e c^2 = \hbar/m_\pi C$ (cf.eq.(1) and eq.(7)):

$$N = \frac{\alpha}{2}\left[\frac{m_{pl}{}^2}{m_p m_e}\right]^2 \tag{12}$$

relating the electromagnetic fine structure constant to the total number of particles in the universe through the proton, electron and Planck masses. Again the cosmological Robertson-Walker models, the position and momentum are not quite independent of each other but connected by Hubble's law $r = (R/C)V$ and if V fluctuates by ΔV, the distance also fluctuates by $\Delta r = (R/C)\,\Delta V$ and the kinetic energy of a particle fluctuates by $1/2\ m\langle\Delta V\rangle^2 \sim mc^2/\sqrt{N}$

$$m\Delta V \Delta r = m\,(\Delta V)^2\, R/C \sim 2\, mCR/\sqrt{N} = nh$$

(for consistency with the uncertainty principle) h is then determined

then determined by R and N and is $\simeq 10^{-27}$ erg sec.
What can all these relations tell us about the variation of the fundamental constants? In the original Dirac cosmology, in order to preserve the equality of the ratios of e^2/Gm_pm_e to Hubble age/atomic time, it was suggested that G vary as t^{-1} (t=epoch) and the total number of particles vary as t^2. In all the above relations, we see tha the coupling constants of the other fundamental interactions always occur in the form of the product $\sqrt{N}$. G, which means that they vary as t°, i.e. are constant with respect to epoch. Even if G does not vary with time (as some very recent experiments based on radar time delay from the viking probes on Mars seem to suggest: (see for eg. Hellings, et al. PRL 51, 1609 1983) and N is strictly conserved (which is reasonable) the product $\sqrt{N}$ G is constant with epoch. This would imply that the coupling constants of the strong, weak and electromagnetic interactions are constant in time (cf. eqs.(12), (4)). (In eqs. (8) and (9) R_H is to be interpreted as the maximal value of the closed universe radius and therefore a constant). The tightest limits claimed for the constancy of the weak(w), strong(s) and electromagnetic (E) couplings are based upon the abundance ratio of the Samarium isotopes Sm^{149} and Sm^{148} from the Oklo Uranium Mine, the ratio of these isotopes is $\sim$ 0.02 as compared to the natural ratio $\sim$ 0.9 the depletion being due to bombardment from thermal neutrons over the several millions of years of the running of the natural 'reactor'. The capture cross-section for thermal neutrons on Sm^{149} is dominated by a strong capture resonance and the Oklo samples imply that it has not shifted by more than 0.02 eV over the past $2\mathrm{x}10^9$ years. As the position of this resonance sensitively determines relative binding energies of the different Sm isotopes with respect to W, S and E interactions, this would imply time variations constrained by $E/E \lesssim 10^{-17}$ yr^{-1}, $W/W \lesssim 10^{-12} yr^{-1}$ and $S/S \lesssim 10^{-19} yr^{-1}$.

ADDITIONAL NOTES

I. Some other large number coincidences are:

(i) Entropy of a solar mass black hole $\simeq 10^{77}K_B \simeq N_1^2 K_B \simeq 10^{19}$ (i.e. $N_1^{1/2}$) times that of $1M_\odot$ star

(ii) Entropy of a black hole of Universe Mass $\simeq 10^{120}\ K_B \simeq N_1^3 K_B$

(iii) Entropy of a galactic mass black hole $\simeq 10^{100}\ K_B \simeq N_1^{5/2}\ K_B$

(iv) PROTON decay time predicted by GUTS (10^{38} secs) to Planck time (10^{-42}s) RATIO is $\simeq 10^{80} \sim (N_1^{\ 2}.)$

(v) Maximal decay time of proton (by quantum gravity tunneling) to Planck time RATIO is $\simeq 10^{100}$

(vi) Observational limit on cosmological constant $\Lambda \simeq 10^{-56}\text{cm}^{-2}$ $\therefore$ Λ/Planck curvature $\simeq 10^{120} \simeq N_1^{\ 3}$.

(vii) Nuclear density/Mean density of interstellar space $\simeq 10^{38} \simeq N_1$

II The Googol (i.e. $10^{100} \sim N_1^{\ 5/2}$) in Astrophysics.

It was noticed that several of the large numbers (especially those concerning galaxies) involved the googol 10^{100}, i.e. angular momentum (typical galaxy) $\simeq 10^{100}\hbar$, entropy of galactic blackhole $\simeq$ googol K_B etc. An interesting combination giving the googol is:

$$\frac{1}{16}\left(\frac{3}{4\pi}\right)^{1/3}\left(\frac{g^4e^2}{m_e^{\ 2}m_p}\right)\left(\frac{G_F^{\ 3}c^{4}}{H_0^{\ 2}G^{13}\hbar^{16}}\right)^{1/6} \simeq \text{googol} \quad (10^{100})$$

III Eq.(1) implies for α (the electromagnetic fine structure):

$$\alpha = \left(\frac{3m_e^{\ 3}Gc}{4\pi\hbar^2 H_0}\right)^{1/3}$$

IV In the true Eddington spirit, the mass given by: (Sivaram, Physics Today 34, 108 (1981)

$$M = \frac{\hbar^2}{m_e}\ \frac{\alpha\alpha_s}{\sqrt{GG_F}} \simeq \text{One gram.} \quad \text{(a unified support}$$

$(\alpha_s = g_s^{\ 2}/\hbar c \simeq 14)$ for the metric mass!)