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Eigenfrequencies of radial pulsations of strange quark stars

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We calculate the range of eigenfrequencies of radial pulsations of stable strange quark stars, using the general relativistic pulsation equation and adopting a realistic equation of state for degenerate strange quark matter.

Nuclear matter at high densities is expected to undergo a phase transition to its constituent quark matter, of which "strange matter", consisting of an approximately equal number of u, d and s quarks (together with electrons to provide electrical charge neutrality), would be the lowest and true ground state of matter [1]. Further studies of strange matter [2–4] support this idea and suggest the possibility that strange matter may exist in various forms ranging from "strangelets" of size 5–200 fm to huge "strange quark stars" of mass $\sim M_\odot$ ($M_\odot$ = solar mass) and radius $\simeq 10$ km. There so-called strange stars have a rather different mass–radius relationship than neutron stars, but for stars of mass = 1.4 $M_\odot$, the structure parameters of quark stars are very similar to those of neutron stars. Since pulsars are believed to be (rotating) neutron stars, and since available binary pulsar data suggest their masses to be close to 1.4 $M_\odot$, it has been conjectured [4] that at least some pulsars could be quark stars. Among the criteria suggested for distinguishing a quark star from a neutron star are the neutrino cooling rate [5–7], transport properties such as bulk viscocity [8,9], and sub-millisecond period rotation rates [10].

Recently, Haensel et al. [11] have emphasized that pulsation properties of a neutron star can yield information about the interior composition, namely, whether the interior has undergone a phase transition to quarks. The main idea is to know the damping times, which will be modified if there is a quark matter core. In their study, Haensel et al. [11] used a polytropic model for the equation of state (EOS) for nuclear matter as well as quark matter, and the newtonian pulsation equation to calculate the eigenfrequencies. The strange quark mass and the quark interactions are important for the structure of quark stars [4]. This suggests that the EOS of strange quark matter will have a role to play in determining the pulsation features of quark stars. Clearly, for a more exact understanding of the vibrational properties of quark stars, use of a realistic EOS for quark matter, and the general relativistic pulsation equation, are desirable. Cutler et al. [12] have calculated the frequencies and damping times of radial pulsations of some quark star configurations, using the general relativistic pulsation equation, but for quark matter, adopted the MIT bag model in its simplest form, namely, non-interacting and massless quarks. The purpose of this paper is to calculate the range of eigenfrequencies of radial pulsations of stable quark stars (using the general relativistic pulsation equation) and to investigate the sensitivity of the eigenfrequencies on the EOS.

The EOS used by us incorporates short-range quark–gluon interactions perturbatively to second order in the coupling constant α_c, and the long-range interactions are taken into account phenomenologically by the bag pressure term (B). We incorporate the density dependence of α_c by solving the Gell-Mann–Low equation for the screened charge. The parameters involved, the strange quark mass (m_s), B and the renormalization point (μ_0), are obtained by demanding that bulk strange matter be stable at zero temperature and pressure, with energy per baryon less than the lowest energy per baryon found in nuclear matter. For completeness, we also do the calculations for the MIT bag model.

The spacetime for a spherically symmetric gravitating system is described by the Schwarzschild metric

$$ds^2 = e^{\nu} c^2 dt^2 - r^2 (d\theta^2 + \sin^2\theta \, d\phi^2) - e^{\lambda} dr^2 , \quad (1)$$

where ν, λ are functions of r only [13]. The equations for the hydrostatic equilibrium of degenerate stars in general relativity are [13]

$$\frac{dp}{dr} = -\frac{G(\rho + p/c^2)(m + 4\pi r^3 p/c^2)}{r^2 (1 - 2Gm/rc^2)} , \quad (2)$$

$$\frac{dm}{dr} = 4\pi r^2 \rho . \quad (3)$$

Here p and ρ are the pressure and total mass energy density. Given an EOS $p(\rho)$, eqs. (2) and (3) can be numerically integrated, for a given central density, to obtain the radius (R) and gravitational mass $M = m(R)$ of the star.

The equation governing infinitesimal radial pulsations of a nonrotating star in general relativity was given by Chandrasekhar [14], and it has the following form:

$$F \frac{d^2\xi}{dr^2} + G \frac{d\xi}{dr} + H\xi = \sigma^2 \xi , \quad (4)$$

where $\xi(r)$ is the lagrangian fluid displacement and $c\sigma$ is the characteristic eigenfrequency. The quantities F, G, H depend on the equilibrium profiles of the pressure and density of the star, and are given by

$$F = -e^{-\lambda} e^{\nu} \Gamma p / (p + \rho c^2) , \quad (5)$$

$$G = -e^{-\lambda} e^{\nu} \left[\Gamma p \left(\frac{1}{2} \frac{d\nu}{dr} + \frac{1}{2} \frac{d\lambda}{dr} + \frac{2}{r} \right) + p \frac{d\Gamma}{dr} + \Gamma \frac{dp}{dr} \right] \times (p + \rho c^2)^{-1} , \quad (6)$$

$$H = \frac{e^{-\lambda} e^{\nu}}{p + \rho c^2} \left(\frac{4}{r} \frac{dp}{dr} - \frac{(dp/dr)^2}{p + \rho c^2} - A \right) + \frac{8\pi G}{c^4} e^{\nu} p , \quad (7)$$

where Γ is the adiabatic index, defined in the general relativistic case as

$$\Gamma = (1 + \rho c^2 / p) \frac{dp}{d(\rho c^2)} \quad (8)$$

and

$$A = \frac{d\lambda}{dr} \frac{\Gamma p}{r} + \frac{2p}{r} \frac{d\Gamma}{dr} + \frac{2\Gamma}{r} \frac{dp}{dr} - \frac{2\Gamma p}{r^2} - \frac{1}{4} \frac{d\nu}{dr} \left(\frac{d\lambda}{dr} \Gamma p + 2p \frac{d\Gamma}{dr} + 2\Gamma \frac{dp}{dr} - \frac{8\Gamma p}{r} \right) - \tfrac{1}{2} \Gamma p \left(\frac{d\nu}{dr} \right)^2 - \tfrac{1}{2} \Gamma p \frac{d^2\nu}{dr^2} . \quad (9)$$

The boundary conditions to solve the pulsation equation (4) are

$$\xi(r=0) = 0 , \quad (10)$$

$$\delta p(r=R) = -\xi \frac{dp}{dr} - \Gamma p \frac{e^{\nu/2}}{r^2} \frac{\partial}{\partial r} (r^2 e^{-\nu/2} \xi) \Big|_{r=R} = 0 . \quad (11)$$

Eq. (4) is of the Sturm–Liouville type and has real eigenvalues $\sigma_0^2 < \sigma_1^2 < \ldots < \sigma_n^2 < \ldots$, with the corresponding eigenfunctions $\xi_0(r)$, $\xi_1(r)$, ... $\xi_n(r)$, ..., where $\xi_n(r)$ has n nodes.

At high baryonic densities, bulk strange matter is in an overall colour singlet state, and can be treated as a relativistic Fermi gas interacting perturbatively, the quark confinement property being simulated by the phenomenological bag model constant (B). Chemical equilibrium between the three quark flavours and electrical charge neutrality allows us to calculate the EOS from the thermodynamic potential of the system as a function of the quark masses, the bag pressure term (B) and the renormalization point μ_0. To second order in α_c, and assuming u and d quarks to be massless, the thermodynamic potential is given by [15]

$$\Omega = \Omega_u + \Omega_d + \Omega_s + \Omega_{int} + \Omega_e , \quad (12)$$

where Ω_i (i = u, d, s, e) represents the contributions of u, d, s quarks and electrons and Ω_{int} is the contribution due to interference between u and d quarks and is of order α_c^2:

$$\Omega_u = -\frac{1}{4\pi^2}\mu_u^4\left[1-\frac{8\alpha_c}{\pi} - 16\left(\frac{\alpha_c}{\pi}\right)^2\ln\left(\frac{\alpha_c}{\pi}\right) - 31.3\left(\frac{\alpha_c}{\pi}\right)^2\right], \quad (13)$$

$$\Omega_d = \Omega_u(\mu_u \leftrightarrow \mu_d), \quad (14)$$

$$\Omega_s = -\frac{\mu_s^4}{4\pi^2}\{(1-\lambda^2)^{1/2}(1-\tfrac{5}{2}\lambda^2) + \tfrac{3}{2}\lambda^4\ln\{[1+(1-\lambda^2)^{1/2}]/\lambda\} - \frac{8\alpha_c}{\pi}[3((1-\lambda^2)^{1/2} - \lambda^2\ln\{[1+(1-\lambda^2)^{1/2}]/\lambda\})^2 - 2(1-\lambda^2)^2]\}, \quad (15)$$

$$\Omega_{int} = \frac{1}{\pi^2}\left(\frac{\alpha_c}{\pi}\right)^2\left[8\mu_u^2\mu_d^2\ln\left(\frac{\alpha_c}{\pi}\right) - 1.9\mu_u^2\mu_d^2 - 19.3\{\mu_u^4\ln[\mu_u^2/(\mu_u^2+\mu_d^2)] + \mu_d^4\ln[\mu_d^2/(\mu_u^2+\mu_d^2)]\} - 4(\mu_u^2+\mu_d^2)\{\mu_u^2\ln[\mu_u^2/(\mu_u^2+\mu_d^2)] + \mu_d^2\ln[\mu_d^2/(\mu_u^2+\mu_d^2)]\} + \tfrac{4}{3}(\mu_u-\mu_d)^4\ln(|\mu_u^2-\mu_d^2|/\mu_u\mu_d) + \tfrac{16}{3}\mu_u\mu_d(\mu_u^2+\mu_d^2)\ln[(\mu_u+\mu_d)^2/\mu_u\mu_d] - \tfrac{4}{3}(\mu_u^4-\mu_d^4)\ln\left(\frac{\mu_u}{\mu_d}\right)\right], \quad (16)$$

$$\Omega_e = -\frac{\mu_e^4}{12\pi^2}. \quad (17)$$

Here μ_i is the chemical potential of the ith particle species and $\lambda = m_s/\mu_s$. We neglect the strange-quark contribution to order α_c^2 and higher in the thermodynamic potential Ω_s. The screened charge α_c is obtained by solving the Gell-Mann–Low equation [15]:

$$\mu\frac{d\alpha_c(\mu)}{d\mu} = \left(-\frac{58}{3\pi} - 8\pi\mu\frac{d}{d\mu}\pi_s(\mu)\right)\alpha_c^2 - \frac{460}{3\pi^2}\alpha_c^3(\mu), \quad (18)$$

which includes the effects of the strange-quark mass in the lowest order. The higher order contribution to the Gell-Mann–Low equation due to strange quarks may be ignored because these are important only at low densities where the coupling is strong but the pair production of massive strange quarks is unimportant (see ref. [15] for further discussions).

The vacuum polarization tensor $\pi_s(\mu)$ for the strange quarks is given by

$$\pi_s(\mu) = \frac{1}{4\pi^2}\left(\frac{5}{9} - \frac{4m_s^2}{3\mu^2} - \tfrac{2}{3}[(1-2m_s^2)/\mu^2](1+4m_s^2/\mu^2)^{1/2} \times \operatorname{arctanh}(1+4m_s^2/\mu^2)\right)^{-1/2}. \quad (19)$$

In eq. (18), $\alpha_c(\mu_0)$ is the value of α_c at the renormalization point μ_0, where it is taken to be equal to 1.

The total energy density and the external pressure of the system are given by

$$\varepsilon = \Omega + B + \sum_i \mu_i n_i, \quad (20)$$

$$p = -\Omega - B, \quad (21)$$

where n_i is the number density of the ith particle species. For specific choices of the parameters of the theory (namely, m_s, B and μ_0), the EOS is now obtained by calculating ε and p for a given value of μ,

$$\mu \equiv \mu_d = \mu_s = \mu_u + \mu_e, \quad (22)$$

by solving for μ_e from the condition that the total electric charge of the system is zero.

There is an unphysical dependence of the EOS on the renormalization point μ_0, which, in principle, should not affect the calculations of physical observables if the calculations are performed to all orders in α_c [4,16]. In practice, the calculations are done perturbatively and, therefore, in order to minimize the dependence on μ_0 the renormalization point should be chosen close to the natural energy scale, which could be either $\mu_0 \simeq B^{1/4}$ or the average kinetic energy of quarks in the bag, in which case $\mu_0 \simeq 313$ MeV. In the present study, our choice of μ_0 is dictated by the requirement that stable strange matter obtains at zero temperature and pressure with a positive baryon electric charge [17]. This leads to the following representative choice of the parameter values:

Table 1
Equations of state for degenerate strange quark matter.

ρ (10^{14} g cm^{-3})	P(10^{36} dyn cm^{-2})		
	model 1	model 2	MIT bag (B=56 MeV fm^{-3})
6.0	4.44	2.23	6.01
8.0	10.13	7.83	12.00
10.0	15.88	13.49	17.99
12.0	21.63	19.17	23.99
14.0	27.41	24.88	29.98
16.0	33.20	30.16	35.97
18.0	39.00	36.36	41.96
20.0	44.82	42.12	47.95
22.0	50.64	47.89	53.95
24.0	56.47	53.67	59.94
26.0	62.30	59.46	65.93
28.0	68.14	65.26	71.92
30.0	73.98	71.06	77.91
32.0	79.83	76.87	83.90
36.0	91.53	88.51	95.89
40.0	100.32	100.16	107.87
50.0	132.58	129.36	137.83

Table 2
Equilibrium strange quark star models.

Equation of state	ρ_c (10^{14} g cm^{-3})	$M/M_\odot$	R (km)	Surface redshift (z)	P_0 (ms)
model 1	24.0	1.958	10.55	0.487	0.488
	20.0	1.967	10.78	0.472	0.503
	16.0	1.951	11.02	0.448	0.522
	12.0	1.864	11.22	0.401	0.548
	8.0	1.521	11.02	0.299	0.591
	6.0	0.997	9.93	0.192	0.624
	5.0	0.485	7.99	0.104	0.646
model 2	24.0	1.863	10.09	0.483	0.468
	20.0	1.862	10.29	0.465	0.482
	16.0	1.829	10.49	0.435	0.500
	12.0	1.710	10.62	0.381	0.527
	8.0	1.281	10.14	0.263	0.568
	6.0	0.645	8.37	0.138	0.600
	5.0	0.092	4.48	0.032	0.622
MIT bag model	24.0	2.021	10.81	0.493	0.500
(B=56 MeV fm^{-3})	20.0	2.033	10.04	0.480	0.514
	16.0	2.023	11.29	0.450	0.533
	12.0	1.947	11.52	0.410	0.558
	8.0	1.635	11.41	0.310	0.604
	6.0	1.150	10.52	0.210	0.636
	5.0	0.666	8.98	0.130	0.659

EOS model 1: $B=56\ \mathrm{MeV\ fm}^{-3}$;

$m_s=150\ \mathrm{MeV}$, $\mu_0=150\ \mathrm{MeV}$.

EOS model 2: $B=67\ \mathrm{MeV\ fm}^{-3}$;

$m_s=150\ \mathrm{MeV}$, $\alpha_c=0$.

Model 2 corresponds to no quark interactions, but a non-zero mass for the strange quark.

In the limit, $m_s\to 0$ and $\alpha_c\to 0$, the EOS has the analytical form

$$p=\tfrac{1}{3}(\varepsilon-4B)\,, \qquad (23)$$

where ε is the total energy density. Eq. (23) is the MIT bag model. It is independent of the number of quark flavours.

Numerical values of pressure (p) and total mass-energy density ($\rho=\varepsilon/c^2$) for the quark matter EOS models used here are listed in table 1. For the sake of comparison, we have included in this table the EOS corresponding to non-interacting, massless quarks as given by the simple MIT bag model with $B=56$ MeV fm^{-3}. Among these EOS, the bag model is "stiffest" followed by models 1 and 2. Equilibrium configurations of strange quark stars, corresponding to the above EOS, are presented in table 2, which lists the gravitational mass (M), radius (R), the surface redshift (z) given by

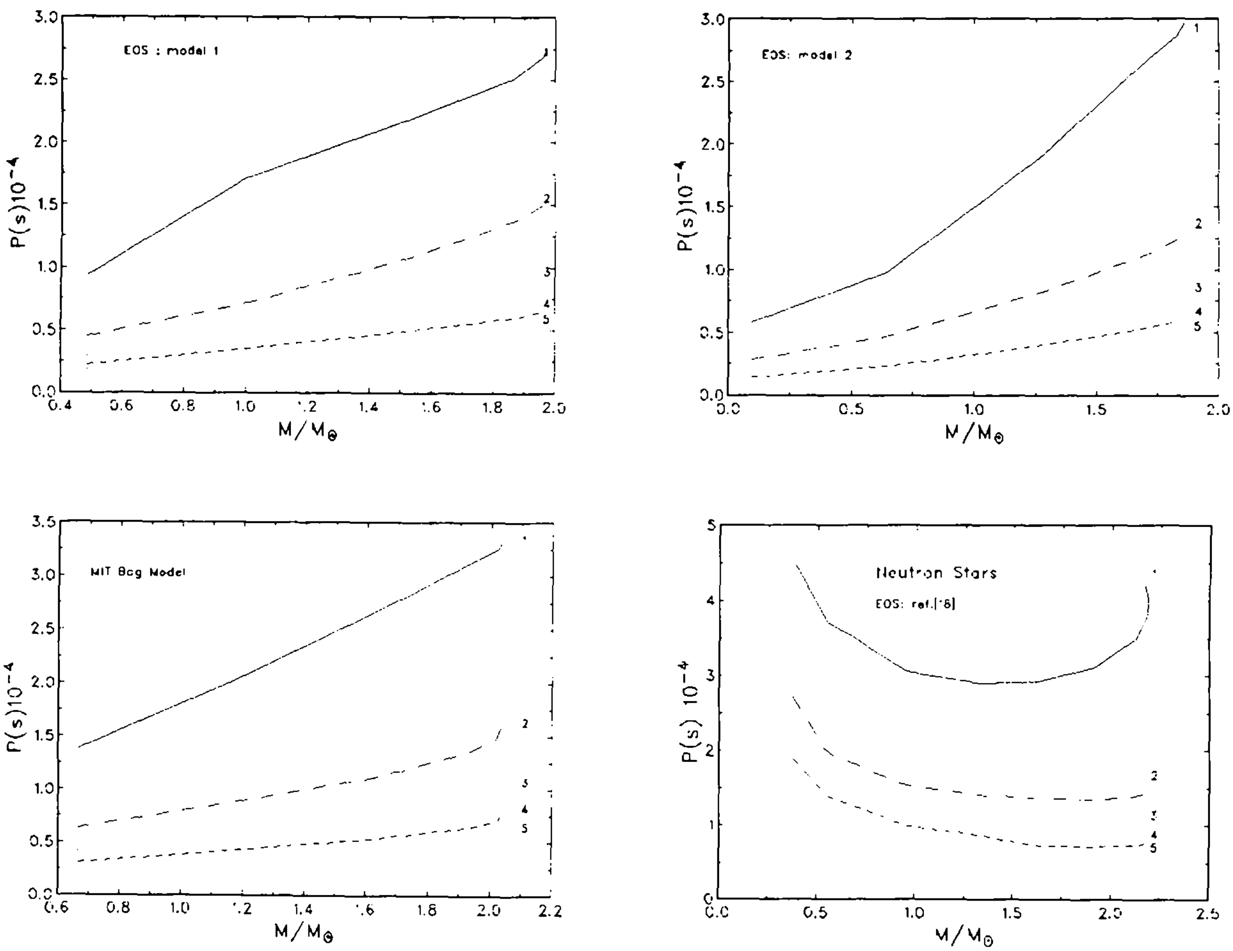

Fig. 1. Periods of radial pulsations as functions of the gravitational mass. The top two and bottom left boxes correspond to strange quark stars. The bottom right box is for stable neutron stars corresponding to beta-stable neutron matter, model UV14+UVII, ref. [18]. The labels 1, 2, 3, 4, 5 correspond respectively to the fundamental and the first four harmonics.

$$z=(1-2GM/c^2R)^{-1/2}-1 \tag{24}$$

and the period (P_0) corresponding to the fundamental frequency Ω_0 defined as [12]

$$\Omega_0=(3GM/4R^3)^{1/2} \tag{25}$$

as functions of the central density (ρ_c) of the star.

We solved eq. (4) for the eigenvalue σ by writing the differential equation as a set of difference equations. The equations were cast in tridiagonal form and the eigenvalue found by using the EISPACK routine. This routine finds the eigenvalues of a symmetric tridiagonal matrix by the implicit QL method.

Results for the oscillations of quark stars corresponding to EOS models 1 and 2 are illustrated in fig. 1. For purpose of comparison , we have included in fig. 1 the results for quark stars corresponding to (a) the simple MIT bag EOS (non-interacting, massless quarks and $B=56$ MeV fm^{-3}) and also (b) neutron stars corresponding to a recently given neutron matter EOS [18]. The plots in fig. 1 are for the oscillation time period ($=2\pi/c\sigma$) versus the gravitational mass (M). The fundamental mode and the first four harmonics are considered. The period is an increasing function of M, the rate of increase being progressively less for higher oscillation modes. The fundamental mode oscillation periods for quark stars are found to have the following range of values:

MIT bag model: 0.14–0.32 ms ,

EOS model 1: 0.10–0.27 ms ,

EOS model 2: 0.06–0.30 ms .

For neutron stars, we find that the range of periods for the $l=0$ mode is ~ 0.3 ms. For the higher modes, the periods are $\leqslant 0.2$ ms.

Inclusion of strange quark mass and the quark interactions make the EOS a little "softer" as compared to the simple MIT bag EOS (see table 1). This is reflected on the value of the maximum mass of the strange quark star (see table 2). For the pulsation of quark stars this gives, for $l=0$ mode eigenfrequencies, values as low as 0.06 ms. The main conclusion that emerges from our study, therefore, is that the use of realistic EOS can be important in deciding the range of eigenfrequencies, at least for the fundamental mode of radial pulsation. The results presented here thus form an improved first step of calculations on the lines presented by Haensel et al. [11], whose numerical conclusions are expected to get altered. Such a calculation for hybrid neutron stars (i.e., a neutron star with quark matter core) along with the damping times due to dissipative forces and gravitational radiation reaction is under preparation, and will be reported in a future paper.

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