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Time-Dependent Interaction of Granules with Magnetic Flux Tubes

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Abstract. The time-dependent interaction of the granulation velocity field with a magnetic flux tube is investigated here. It is seen that when a magnetic field line is displaced normal to itself so as to simulate the *buffeting* action of granules, a flow of gas is initiated along the field. By choosing a lateral velocity field which is consistent with observations of granules, it is found that the resulting gas motion is a downward flow with a velocity compatible with the observed downflow in isolated photospheric flux tubes. It is therefore proposed that the observed photospheric downflow is a manifestation of the interaction of granules with flux tubes.

Key words: Sun—small scale magnetic fields—granules—downdrafts—fluid dynamics—unsteady one-dimensional flow

1. Introduction

The dynamical state of a highly conducting fluid embedded in a magnetic field can be greatly influenced by changes in the field geometry. Such changes can often result in a significant flow being set up along the field lines (Hasan and Venkatakrishnan 1980, 1981). On the Sun, there are several processes which can change the magnetic field geometry, for example, hydromagnetic or thermal instabilities, or the *buffeting* of the field lines by an externally imposed flow. This paper is a study of the interaction of granular velocity fields with magnetic flux tubes. For purposes of mathematical simplicity and also to focus our attention on the vertical flow, we solve the MHD equations in a specified stream geometry as discussed by Kopp and Pneuman (1976). By assuming a form for the velocity field normal to the magnetic field based on the observations of granular velocities, we obtain estimates for the resulting fluid velocity along the magnetic field. These velocities compare favourably in magnitude and direction with the observed values of the fluid velocity within isolated flux tubes.

2. The model

At photospheric levels, granulation forms a velocity pattern which has a typical length scale of a few seconds of arc and a time scale of a few minutes. There are also concentrated magnetic flux elements in the photosphere which will undergo *buffeting* by the granules. A magnetic element situated in the intergranular lanes will be acted upon by these granules on its boundary. In this paper, we represent the element as a magnetic flux sheath in a plane normal to the intergranular lane. We attempt to model the result of nonlinear interaction between the velocity field of the granulation and the magnetic field of this flux sheath. As a model input, we assume a form for the motion of the magnetic field lines normal to themselves which is related to the observed form of the granular velocity field. The resulting model output is the velocity of gas parallel to the magnetic field.

When field lines are displaced normal to themselves, with a non-uniform velocity a centrifugal acceleration results along the direction of the field. This induces flow of gas along the field. The magnitude and direction of such induced flow depends on the amplitude of velocity fluctuations normal to the field lines and on the direction of positive gradient in this amplitude respectively (Hasan and Venkatakrishnan 1980, 1981). Since the granular velocity field decreases with height, when field lines are displaced with such a velocity field, the resulting flow will be a downdraft. The details of this downflow depend upon the details of the lateral velocity field like its amplitude at the base of the sheath and the scale height of its variation.

3. The basic equations

Let us assume that the flux sheath is in the y-z plane, with gravity acting in the negative z-direction. It is convenient to transform to a system of coordinates (s, n), where s is the distance measured along the sheath axis and n denotes the distance measured along a normal curve (in the same plane). Following Kopp and Pneuman (1976), we see that the unit vectors $\hat{\mathbf{s}}$ and $\hat{\mathbf{n}}$ satisfy the following geometric relations

$$\begin{pmatrix} \dfrac{\partial}{\partial t} \\ \dfrac{\partial}{\partial s} \\ \dfrac{\partial}{\partial n} \end{pmatrix} (\hat{\mathbf{s}}, \hat{\mathbf{n}}) = \begin{pmatrix} \dfrac{\partial \theta}{\partial t} \\ \dfrac{\partial \theta}{\partial s} \\ \dfrac{\partial \theta}{\partial n} \end{pmatrix} (\hat{\mathbf{n}}, -\hat{\mathbf{s}}) \tag{3.1}$$

where θ is the angle the field makes with the z-axis. The equation of motion along the field can now be expressed as

$$\frac{\partial V_s}{\partial t} + V_s \frac{\partial V_s}{\partial s} + V_n \frac{\partial V_s}{\partial n} = -\frac{1}{\rho}\frac{\partial p}{\partial s} - g \cos\theta + V_n \frac{\partial \theta}{\partial t} + V_n V_s \frac{\partial \theta}{\partial s} + V_n^2 \frac{\partial \theta}{\partial n} \tag{3.2}$$

where V_s is the gas velocity parallel to the field, V_n the velocity normal to the field, p the gas pressure, ρ the density and g the acceleration due to gravity. Let us now consider a frame of reference fixed to the sheath. The space and time derivatives in such a frame will be denoted as $\frac{D}{Ds}$ and $\frac{D}{Dt}$ respectively. These satisfy the following operator relationships

$$\frac{D}{Dt} \equiv \frac{\partial}{\partial t} + V_n \frac{\partial}{\partial n} \tag{3.3a}$$

and

$$\frac{D}{Ds} \equiv \frac{\partial}{\partial s}. \tag{3.3b}$$

From equations (3.2) and (3.3) we have

$$\frac{DV_s}{Dt} + V_s \frac{DV_s}{Ds} = -\frac{1}{\rho}\frac{Dp}{Ds} - g \cos\theta + V_n \frac{D\theta}{Dt} + V_n V_s \frac{D\theta}{Ds}. \tag{3.4}$$

In a similar manner the equation of continuity takes thè form (Kopp and Pneuman 1976)

$$\frac{D}{Dt}(\rho A) + \frac{D}{Ds}(\rho V_s A) - \rho V_n A \frac{\partial\theta}{\partial s} = 0, \tag{3.5}$$

where A is the cross-sectional area of the flux sheath. In the above equation we have assumed the constancy of all flow variables in a direction normal to the sheath axis. The evolution of the magnetic field, assuming infinite conductivity, is given by the induction equation

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{V} \times \mathbf{B}), \tag{3.6}$$

which can be resolved into the components

$$\frac{\partial B}{\partial t} = -\frac{\partial}{\partial n}(V_n B) \tag{3.7a}$$

along the field and

$$B\frac{\partial\theta}{\partial t} = \frac{\partial}{\partial s}(V_n B) \tag{3.7b}$$

normal to the field respectively. Using the condition for flux conservation in a sheath (BA = constant) and the geometric relation for the rate of change of the angle

$$\frac{D\theta}{Dt} = \frac{\partial V_n}{\partial s}, \tag{3.7c}$$

we can rewrite equations (3.7a) and (3.7b) as

$$\frac{D}{Dt}(\ln A) = \frac{\partial V_n}{\partial n} \tag{3.7d}$$

and

$$\frac{D}{Ds}(\ln A) = \frac{\partial \theta}{\partial n} \tag{3.7e}$$

respectively. Eliminating A between equations (3.7d), (3.7e) and (3.5) we have

$$\frac{DV_s}{Ds} + \frac{D}{Dt}(\ln \rho) + V_s \frac{D}{Ds}(\ln \rho) + V_s \frac{\partial \theta}{\partial n} - V_n \frac{D\theta}{Ds} + \frac{\partial V_n}{\partial n} = 0 \tag{3.8}$$

We relate density and pressure by a polytropic law

$$p/\rho^{\Gamma} = \text{constant}, \tag{3.9}$$

where Γ is the polytropic index. We thus have three equations (3.4, 3.8 and 3.9) in the four unknowns p, ρ, V_s and V_n respectively. The equation of motion normal to the field (which involves the Lorentz force and all other forces acting on the sheath) supplies the means of determining the four quantities uniquely. However, we replace this equation by a prescribed form for V_n (a similar procedure was adopted by Kopp and Pneuman 1976 to model post flare reconnection of coronal loops, as well as by Hasan and Venkatakrishnan 1981 to model spicule flow). Equations (3.4), (3.8) and (3.9) form a set of hyperbolic partial differential equations. These were solved numerically using the method of characteristics. The characteristic equations are

$$\frac{D\xi_+}{Dt} = A + B/a \quad \text{along} \quad \frac{Ds}{Dt} = u + a \tag{3.10a}$$

and

$$\frac{D\xi_-}{Dt} = A - B/a \quad \text{along} \quad \frac{Ds}{Dt} = u - a\,, \tag{3.10b}$$

where

$$\xi_\pm = \ln \rho \pm V_s/a, \tag{3.10c}$$

$$A = -V_s \frac{\partial \theta}{\partial n} + V_n \frac{\partial \theta}{\partial s} - \frac{\partial V_n}{\partial n} \tag{3.10d}$$

and

$$B = -g \cos \theta + V_n \frac{D\theta}{Dt} + V_n V_s \frac{\partial \theta}{\partial s}. \tag{3.10e}$$

The same boundary conditions, as in Hasan and Venkatakrishnan (1981), were used.

4. The stream geometry

At $t = 0$, the magnetic field is assumed to be potential with components

$$B_y = B_0 \exp(-kz) \sin \theta, \tag{4.1a}$$

$$B_z = B_0 \exp(-kz) \cos \theta \tag{4.1b}$$

and

$$\theta = ky. \tag{4.1c}$$

Here we have assigned a zero value for k so that the flux sheath is vertical initially. At later instants of time, the sheath geometry is completely determined by V_n. The coordinates of a given point on the sheath axis at different instants of time can be determined from the following equations

$$\frac{Dy}{Dt} = V_n \cos \theta, \tag{4.2a}$$

$$\frac{Dz}{Dt} = -V_n \sin \theta, \tag{4.2b}$$

$$\frac{D\theta}{Dt} = \frac{\partial V_n}{\partial s}. \tag{4.2c}$$

We integrated equation (4.2) numerically from a time t_0 to a time $t_0 + \Delta t$ where Δt is the step size for the grid used to integrate equations (3.10) (Hasan and Venkatakrishnan 1980).

Since the magnetic field is initially potential, it exerts no force on the gas. The initial stratification of the gas was assumed to be polytropic with a polytropic index $\Gamma = 1{\cdot}064$. This polytropic index was chosen to fit the *Harvard-Smithsonian Reference Atmosphere* (Gingerich *et al.* 1971) for a height of 500 km above the photosphere.

5. The choice of the lateral velocity field

Observations of the granular velocity variation with height in the solar atmosphere (Durrant *et al.* 1979) indicate a decrease of the rms velocity with height. The scale height of the variation is of the order of 500 km. The lifetime of the granule varies between 2 to 15 minutes. The time variation of a single granulation flow can therefore be approximated by a sine function with a half period equal to the granule lifetime. More specifically, one can represent the granular velocity V_g as

$$V_g = V_0 \exp(-z/H) \sin(\pi t/T), \tag{5.1}$$

where H is the scale height of variation and T is the lifetime of the external velocity field respectively. V_0 is the amplitude of the velocity at the base. In this paper we assume the form given by equation (5.1) for the velocity V_n of the field lines normal to the sheath axis.

6. Results and discussion

Fig. 1 shows the variation of the fluid velocity component in the direction of the magnetic field with distance along the field, at different instants of time. For this

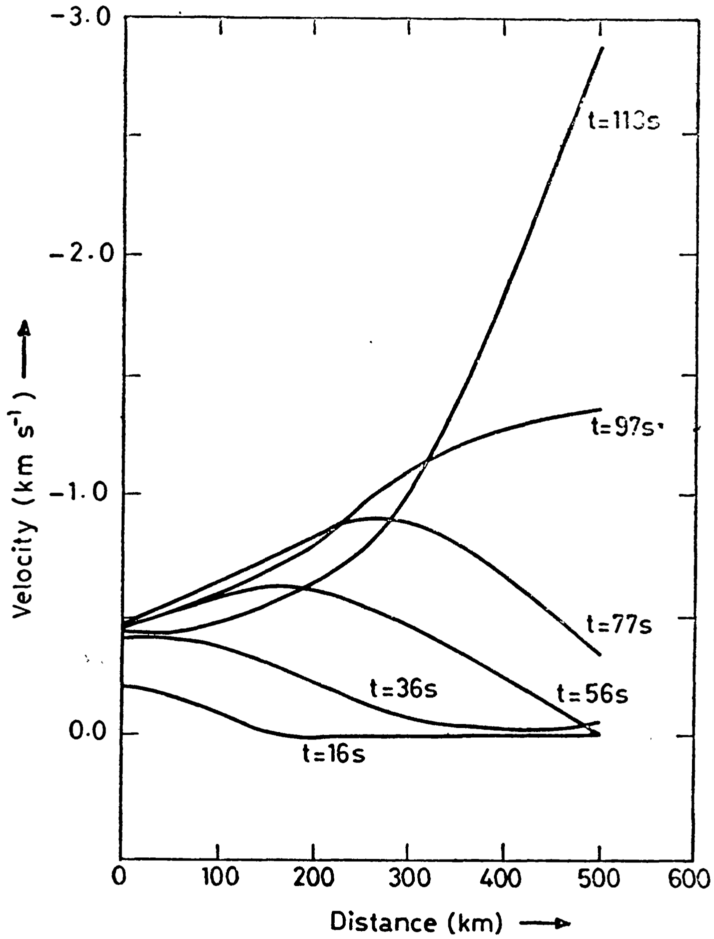

Figure 1. Variation of velocity component in the direction of the magnetic field with distance along the field at five instants of time. The scale height H of the lateral velocity field is 500 km, the half period $T = 120$ s, and the velocity amplitude at the base $V_0 = 500$ m s^{-1}.

particular calculation, we chose $H = 500$ km, $T = 120$ s and $V_0 = 500$ m s^{-1}. The flow begins as a downflow and remains as such throughout the lifetime of the lateral velocity field. The magnitude of the velocity along the field initially decreases with height but at later instants of time, it increases with height. After $t \simeq 100$ s the velocity attains values around 1 to 2 km s^{-1} which are comparable to those observed in isolated flux tubes (Giovanelli and Slaughter 1978; Giovanelli 1980).

So far we have considered the interaction of a single granule with a flux tube. The granular buffeting can last at the most for a lifetime of a granule. Once the buffeting ceases, the downflow relaxes towards hydrostatic equilibrium, in a manner described previously (Hasan and Venkatakrishnan 1980). The relaxation time is approximately equal to the time taken for a sound wave to traverse the length of the field line participating in the lateral motion. Using a typical value of 1000 km for the size of a granule and 10 km s^{-1} for the sound speed in the photosphere, the relaxation time is of the order of 100 s. Therefore, if the tube is acted upon by a new granular velocity field within about 100 s the downflow will never decrease to zero. On the other hand, the downflow will not be stationary but will be modulated by the frequency of granular buffeting. Observations with a coarse time resolution (larger than the relaxation time of $\simeq 100$ s) would only detect the averaged velocity field. Thus, it would be interesting to observe the downflow with a finer time resolution.

Furthermore, downflows caused by granular buffeting could, under appropriate conditions, act as seeds for the convective collapse of flux tubes (Webb and Roberts 1978; Spruit 1979; Spruit and Zweibel 1979). Such a collapse might lead to the strong downflow which is inferred by Deubner (1976) to be present in the dark intergranular lanes.

Finally, it must be mentioned that the interaction of other dominant velocity fields (for example, the five minute oscillations or supergranulation) with the tiny flux tubes would produce negligible internal motions since the horizontal length scales of these fields are much larger than that of the flux tubes. The tubes will have maximum internal response to those scales which are of comparable size. The granular velocity field is the only known dominant field which has dimensions closest to those of the flux tubes.

All computations were performed using the TDC-316 system of the Indian Institute of Astrophysics.

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