EFFECTS OF PARTIAL FREQUENCY REDISTRIBUTION ON THE LEVEL POPULATION DENSITIES IN A RESONANCE LINE

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Abstract. We have obtained a simultaneous solution of the statistical equilibrium equation for a non-LTE two-level atom and the radiative transfer equation in the comoving frames by employing the angle-averaged partial frequency redistribution. R_I with isotropic scattering. In the first iteration we have set the population density of the upper level equal to zero and allow it to be populated in the subsequent iterations. The solution converges within two to four iterations. The process of iteration is terminated when the ratios of population densities in two successive iterations at each radial point, attain an accuracy of 1%. The effects of partial frequency redistribution is to increase the population density of the upper level. Radial gas motions do not seem to have significant effects, although in highly extend geometries, velocity gradients change the population densities considerably.

1. Introduction

In a previous paper (Peraiah, 1980; henceforth referred to as Paper I), we have obtained a simultaneous solution of the radiative transfer equation in the comoving frame and the statistical equilibrium equation for a non-LTE two level atom assuming complete redistribution. We have obtained few interesting results in Paper I. The population densities change dramatically when gas velocities are introduced in a spherically symmetric expanding media. The whole process took 2–4 iterations to converge to a consistent solution. However, in that paper, we have used complete redistribution and it will be interesting to study the effects of partial frequency redistribution.

In this paper, we have considered the angle averaged partial frequency redistribution function for zero natural line width (R_{I-A}) with isotropic scattering. We have considered a law of linear velocity with the geometrical extension $B/A = 3$ and 10 where B and A are the outer and inner radii of the spherically symmetric extended media. The velocity v is set equal to 0 at A, and a maximum velocity at B. The maximum velocities set at B are 0, 5, 10, 30, and 60. In the next section, we present a brief description of the computational procedure and discussion of the results.

2. A Brief Description of Computational Procedure and Discussion of the Results

The procedure has been described in detail in Paper I. We shall give a brief sketch of the method here. To solve the equation of transfer we first assume the

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population densities of the two levels and calculate the absorption coefficient given by

$$K_L(r)=\frac{h\nu_0}{4\pi\Delta\nu}[N_1B_{12}-N_2B_{21}]\,, \tag{1}$$

where ν_0 is the frequency of the hydrogen Lα line, $\Delta\nu$ is a suitable frequency width B_{12} and B_{21} are the Einstein coefficients. N_1 and N_2 are the population densities of the lower and upper levels respectively. The quantities N_1 and N_2 are obtained from the solution of statistical equilibrium equation given by

$$\frac{N_1}{N_2}=\frac{A_{21}+C_{21}+B_{21}\int \mathrm{d}x\,\phi(x)J_x}{C_{12}+B_{12}\int \mathrm{d}x\,\phi(x)J_x}; \tag{2}$$

A_{21} being Einstein's coefficient for spontaneous emission, C_{12} and C_{21} are the rates of the collisional excitation and de-excitation given (Jefferies, 1968) by

$$C_{12}\simeq 2.7\times 10^{-10}\alpha_0^{-1.68}\exp(-\alpha_0)T^{-3/2}A_{21}\frac{g_2}{g_1}(I_H/\chi_0)^2N_e \tag{3}$$

and

$$C_{21}=2.7\times 10^{-10}\alpha_0^{-1.68}T^{-3/2}A_{21}\left(\frac{I_H}{\chi_0}\right)^2N_e\,, \tag{4}$$

where χ_0 is the excitation energy E_{12}, and $\alpha_0=\chi_0/kT_e$ being the temperature I_H is the ionization potential of hydrogen and N_e is the electron density. In equation (2), X is the normalized frequency given by

$$x=(\nu-\nu_0)/\Delta\,; \tag{5}$$

Δ being a standard frequency interval, $\phi(x)$ is the profile function given by

$$\phi(x)=\int_{-\infty}^{+\infty}R_{I-AI}(x,x')\,\mathrm{d}x'\,, \tag{6}$$

where $R_{I-AI}(x,x')$ is the angle averaged redistribution function with isotropic scattering given by

$$R_{I-AI}(x,x')=\tfrac{1}{2}\,\mathrm{erfc}\,(|\bar{x}|)\,, \tag{7}$$

where

$$\mathrm{erfc}\,(x)=\frac{2}{\sqrt{\pi}}\int_x^{\infty}e^{-t^2}\,\mathrm{d}t\,. \tag{8}$$

The symbol J_x in Equation (2) stands for the mean intensity at the normalized frequency x, given by

$$J_x=\frac{1}{2}\int_{-1}^{+1}I(r,\mu,x)\,\mathrm{d}\mu\,; \tag{9}$$

where $I(r, \mu, x)$ is the specific intensity of the ray making an angle $\cos^{-1}\mu$ with the radius at the radial point r. We obtain the specific intensity from the solution of the transfer equation in the comoving frame, given by

$$\mu \frac{\partial I(r,\mu,x)}{\partial r} + \frac{1-\mu^2}{r}\frac{I(r,\mu,x)}{\partial \mu} = K_L(r)[\beta + \phi(x)] + \\ +[S(x,r) - I(r,\mu,x)] + \left\{(1-\mu^2)\frac{V(r)}{r} + \mu^2 \frac{dV(r)}{dr}\right\}\frac{\partial I(r,\mu,x)}{\partial x} \tag{10}$$

and

$$-\mu \frac{\partial I(r,-\mu,x)}{\partial r} - \frac{1-\mu^2}{r}\frac{I(r,-\mu,x)}{\partial \mu} = K_L[\beta + \phi(x)] + \\ +[S(x,r) - I(r,-\mu,x)] + \left\{(1-\mu^2)\frac{v(r)}{r} + \mu^2 \frac{dV(r)}{dr}\right\}\frac{\partial I(r,-\mu,x)}{\partial x}; \tag{11}$$

β being the ratio K_C/K_L of continuum absorption per unit frequency interval to that at the centre of the line, and $V(r)$ is the gas velocity at the radial point r. $S(x, r)$ is the source function given by

$$S(x,r) = \frac{\phi(x)}{\beta + \phi(x)} S_L(r) + \frac{\beta}{\beta + \phi(x)} S_C(r), \tag{12}$$

$S_L(r)$ and $S_C(r)$ are the line and continuum source functions, respectively. The latter is set equal to Planck function. The line source function is written as

$$S_L(x,r) = \frac{1-\epsilon}{\phi(x)} \int R_{I-AI}(x,x')J(x')\,dx' + \epsilon B(r); \tag{13}$$

$B(r)$ being the Planck function; and ϵ, the probability of photon destruction by collisional de-excitation.

The line source function can also be calculated by the relationship

$$S_L(r) = \frac{A_{21}N_2(r)}{B_{12}N_1(r) - B_{21}N_2(r)}. \tag{14}$$

We started the first iteration by assuming $N_2(r) = 0$, so that $N(r) = N_1(r)$, where $N(r)$ is the total number of neutral atoms. After evaluating the profile function with the help of Equations (6), (7), and (8) and assuming the geometrical extension with $N_1(r)$ and $v(r)$, we solve the transfer equation given in Equations (10) and (11). This provides us with the mean intensities J_x. These values of J_x are used to obtain a new set of number densities, $N_2(r)$ and $N_1(r)$ by solving simultaneously the equation of statistical equilibrium equation and the equation

$$N(r) = N_1(r) + N_2(r). \tag{15}$$

The process is continued and is terminated when the ratio of $N_2(r)/N_1(r)$ converges to within 1% in two successive iterations.

We have set $\epsilon = 0$ and $\beta = 0$ and considered an isothermal atmosphere with $T = 30\,000$ K. The stellar radius is taken to be 10^{12} cm and atmospheres with thicknesses 3 and 10 times the stellar radius are considered. The velocity and the density are considered in such a way that they always satisfy the equation of continuity in a spherically symmetric steady state flow. Thus, velocity at $\tau = \tau_{\max}$ is kept equal to 0 and at $\tau = 0$, we have kept it $v = v_{\max}$ and let it vary linearly with radius. The density, therefore, changes as $1/r^3$ for the continuity equation to be satisfied. We have taken the electron density N_e at $\tau = \tau_{\max}$ equal to 10^{12} cm^{-3} (point A is at the inner radius) and $v = 0$ at A, $v_{\max} = 0$, 5, 10, 60 mean thermal units at the point B (outer radius). The results are presented in Figures 1–3.

In Figure 1, we have shown the probability of emission at frequency x for each absorption at frequency x'. Here, the quantity $R_{I-AI}(x, x')/\phi(x')$ is plotted against x, where $\phi(x)$ is obtained from Equation (6). If we compare these profiles with those given in Figure (13-2) of Mihalas (1978; p. 428), the profiles in these two figures are quite similar although different profile functions, $\phi(x)$ are used – the former being taken from Equation (6) and the latter being Doppler

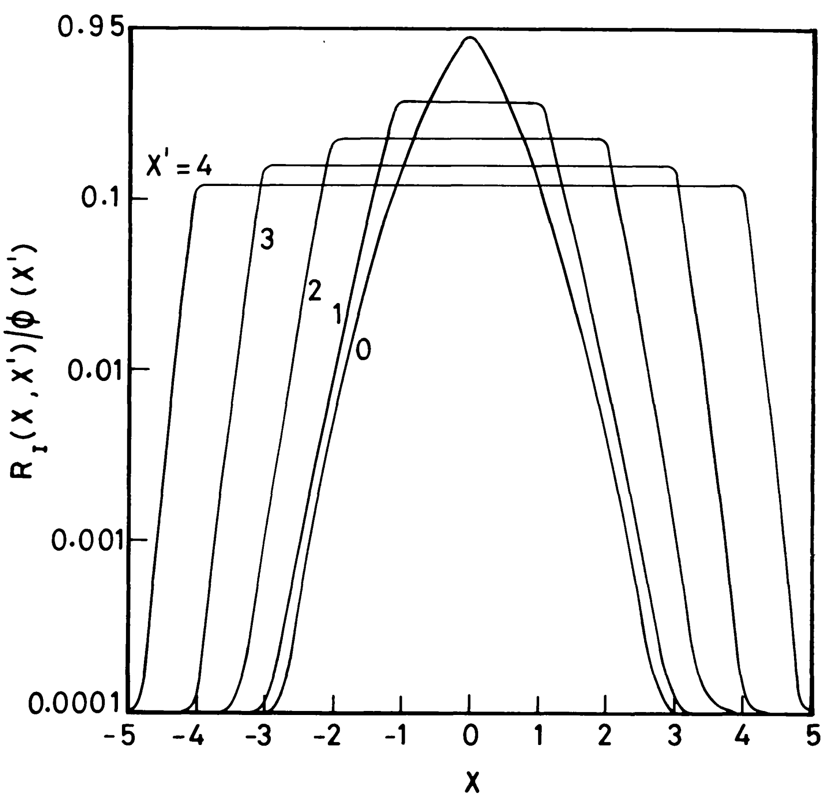

Fig. 1. Probability of emission of photon at frequency x when absorbed at frequency x'. $R_{I-AI}(x, x')/\phi(x')$ are plotted against x.

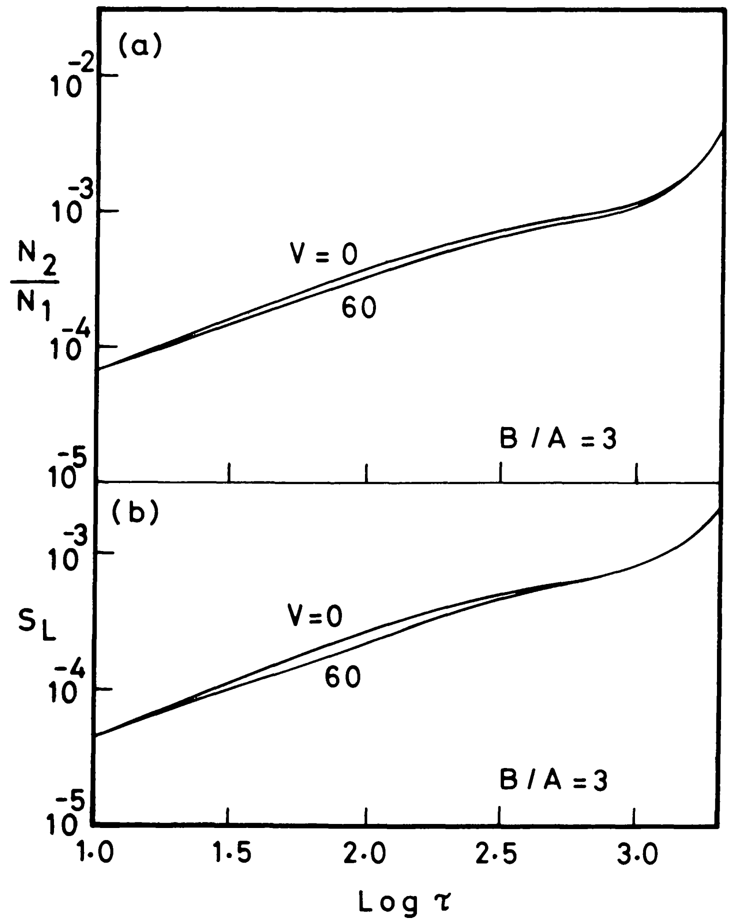

Fig. 2. (a) The ratio of population densities N_2/N_1 are given against τ, the total optical depth, for $B/A = 3$. (b) The line source functions are given for $B/A = 3$.

redistribution. However, the emission profiles given here are broader than those given in Mihalas. In the co-moving frame, it is needed to calculate the redistribution function only once. We have used the profile of the line as given in Equation (6).

In Figure 2(a) and (b), the ratio N_2/N_1 and the line source function are plotted against the total optical depth. The curves are plotted for various velocities for $B/A = 3$. We have plotted only the graphically resolvable curves. It is quite clear from these curves that velocity gradients do not considerably influence the ratio N_2/N_1 and hence the line source function. It is interesting to note that even when we start the iteration with $N_2/N_1 = 0$ the upper level is considerably populated particularly at $\tau = \tau_{\max}$, although it is still less than the equilibrium value.

In Figure 3(a) and (b), we have plotted the N_2/N_1 and S_L for $B/A = 10$. The results are quite similar to those presented in Figure 2(a) and (b). The quantities N_2/N_1 and S_L are reduced more drastically in this case.

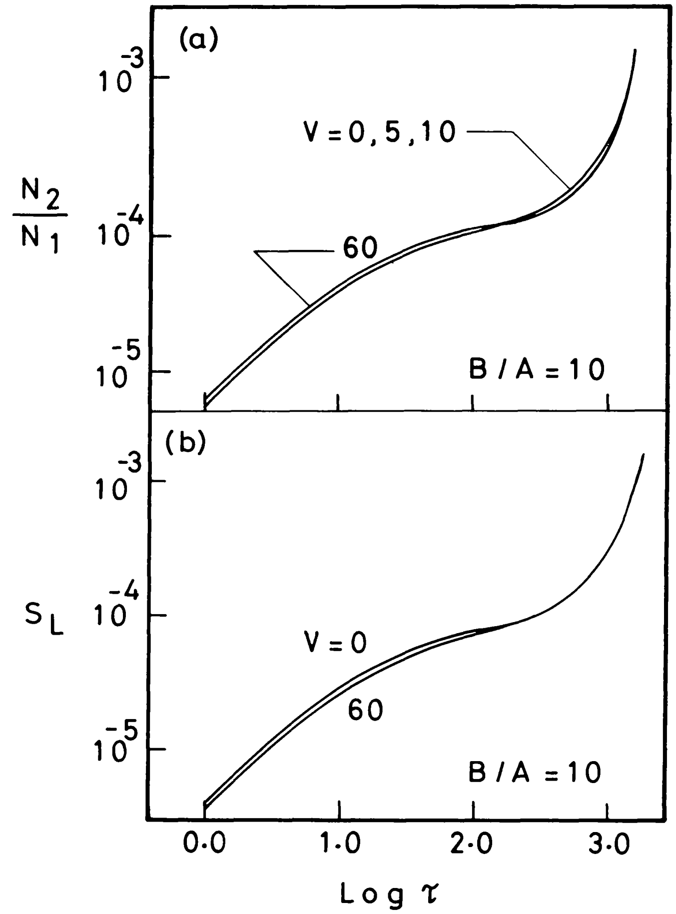

Fig. 3a–b. Same as those given in Figures 2(a) and (b) with $B/A = 10$.

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