Astron. Nachr., Bd. 298, H. 1 (1977)

Unusual Extinction Profiles by Rayleigh Scattering

G. A. Shah, Bangalore

Indian Institute of Astrophysics, Bangalore, India

With 3 Figures. (Received 1976 April 20)

The extinction efficiencies for small smooth metallic particles (granules) have been calculated using rigorous Mie theory of scattering. Strong size dependent resonances for spheres composed of sodium, potassium, and calcium have been found. The results seem to have relevance to the problems of interstellar grains especially with reference to structures in the observed extinction curve, diffuse bands and anomaly of cosmic abundances of elements.

Die Wirkungsfaktoren kleiner metallischer Teilchen (Granulen) für Extinktion wurden mit Hilfe der exakten Mieschen Streutheorie berechnet. Dabei wurden kräftige größenabhängige Resonanzen für kugelförmige Teilchen aus Natrium, Kalium und Calcium gefunden. Die Ergebnisse haben für Probleme der interstellaren Staubteilchen Bedeutung, insbesondere für die Strukturen in der beobachteten Extinktionskurve, für die diffusen Banden und für die Anomalie in den kosmischen Häufigkeiten der chemischen Elemente.

The scattering by small particles is of interest in explaining the continued rise of interstellar extinction in the far ultraviolet (Bless and Savage 1972, York et al. 1973 and Nandy et al. 1975). Some of the structures in the observed extinction curves and unidentified diffuse interstellar bands may originate from scattering by dust grains of type hitherto unknown. The anomalous cosmic abundance, i.e., the missing heavy elements in interstellar medium are generally thought to be lost to the formation of grains in (1) interstellar space (Routly and Spitzer 1952; Oort and Van de Hulst 1946; Aannestad 1973), (2) cool stellar atmospheres (Schatzman and Cayrel 1954; Hoyle and Wickramasinghe 1962; Kamijo 1963), and (3) stellar atmosphere or nebulae (Dorschner 1967, 1968, 1970, 1971; Friedemann 1969). The situation regarding the depleted elements has been discussed by Field (1974), Salpeter (1974) and Greenberg (1974). It may as well be that the missing elements provide the initial condensation nuclei which can subsequently grow by further accretion to become single or composite grains of submicron or larger sizes.

The purpose of this communication is to report some unusual theoretical exctinction profiles resulting from Rayleigh scattering by small metallic particles in the size range intermediate between Platt particles and the classical grains. We consider smooth spherical particles (sizes $\lesssim 100$ Å) composed of sodium, potassium and calcium. It may be noted that sodium and potassium have been reported to be depleted in interstellar medium by a factor of 3 to 10 (Lequeux 1975) as compared to solar abundance and they stand good chances to show up in observations by their peculiar scattering properties given here. The ratio of circumference to wavelength, $x = 2\pi a/\lambda$, is typical of Rayleigh scattering domain with $x < 1$ and $mx < 1$; here a is the size of the particle, λ the wavelength of incident radiation and m is the complex index of refraction of the material. The exact Mie theory (Mie 1908; Debye 1909; Van de Hulst 1957) and wavelength dependent indices of refraction (Gray 1963) have been used throughout. These refractive indices refer to appropriate vacuum deposited materials in the form of thin films which are of the same size range as the radii of the interstellar grains. Thus, they are somewhat more realistic in representing the optical properties of the grains compared to the bulk refractive indices. The small particles in the present work are referred to as granules.

On the basis of certain approximation to Mie coefficients corresponding to electric and magnetic dipole and quadrupole modes, Van de Hulst (1964) derived nearly a flat curve of extinction, Q_{ext}, as function of x in the case of hypothetical material with refractive index $m = 0.0 - \sqrt{2}\,i$ and $0 \lesssim x \lesssim 1$. Also, the extinction was shown to have finite value for radius of the particle equal to zero (i.e., $x = 0$). However, further exact calculations (Shah 1973) based on rigorous Mie theory gave some surprising results. This motivated the author to consider particles composed of real materials as well.

The appearance of size dependent resonance in extinction produced by particulate materials considered here can be understood if one recalls the expansions of the MIE coefficients, a_1, a_2, b_1 and b_2 (see, for example VAN DE HULST 1957, 1964) in terms of index of refraction $m = m' - i\,m''$ and x. It is assumed that $x < 1$ and $mx < 1$. It turns out that the main contribution to Q_{ext} comes from the dominant term of MIE coefficient a_1 which has a factor $(m^2 + 2)$ in the denominator. In case of materials with $(m^2 + 2) \cong 0$, a singularity and hence resonance will occur. However, it is not clear why resonance does not appear in case of approximate formulae considered by VAN DE HULST (1964). Some idea can be gained from the exact calculations (SHAH 1973) of MIE coefficients for sodium particles. It has been shown that for nearly pure imaginary m, $m^2 + 2 \cong 0$ and $x \lesssim 1$, the contribution from the electric dipole mode (coefficient a_1) is predominant up to a certain value of x beyond which the electric quadrupole mode (coefficient a_2) is no longer negligible. In fact, the latter may contribute to the extinction efficiency even more than the former. Similar singularities and resonances could be possible for some other MIE coefficients so far not studied in the literature in present context. Therefore, it is always safer to use rigorous theory rather than approximate formulae.

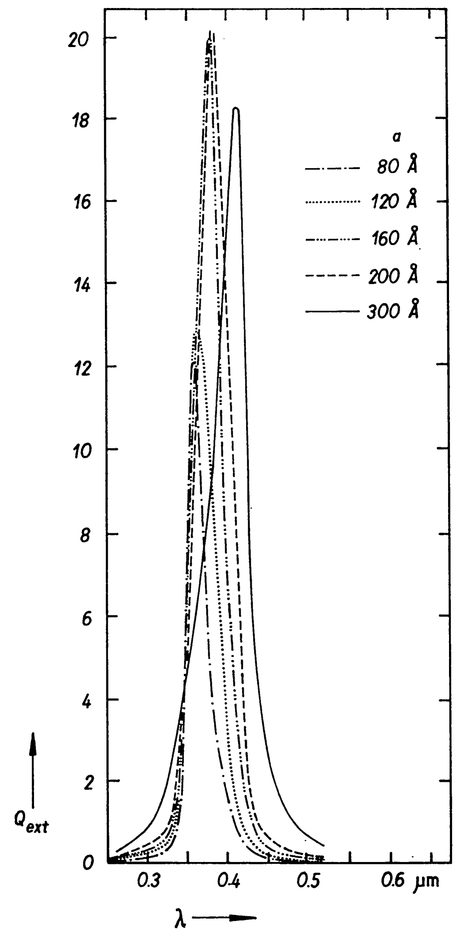

Fig. 1. The size effect on the extinction profile for sodium spheres. Note how the resonance peaks shift towards longer wavelength (λ) as the size (a) increases. The wavelength dependent index of refraction has been used.

The results of exact calculations are set out in Figures 1, 2 and 3 for Na, K and Ca, respectively. The ordinates and abscissae represent extinction efficiency and wavelength in micron, respectively. The parameter in these figures corresponds to the radius of the granule. The resonances in extinction by small spheres of sodium, potassium as well as calcium are strongly size-dependent. It is seen that the values of extinction are much larger compared to the usual same sized dielectric particles at least in the wavelength range of interest here. The peak shifts towards the longer wavelengths as the size increases. Furthermore, the peak value of Q_{ext} for Na in Figure 1, increases with size and attains the maximum value near about $a = 200$ Å. According to SHIFRIN (1968), who has summarized the work by SAVAST'YANOVA (1939 a, b), the scattering coefficient decreases much more rapidly than the ab-

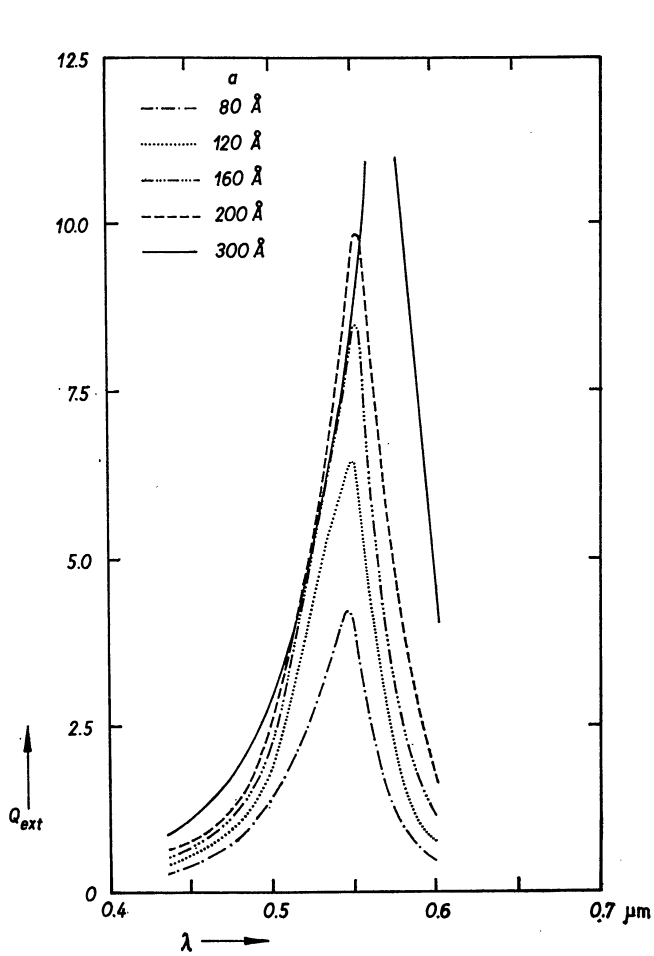

Fig. 2. Same as in Figure 1, but the material is potassium.

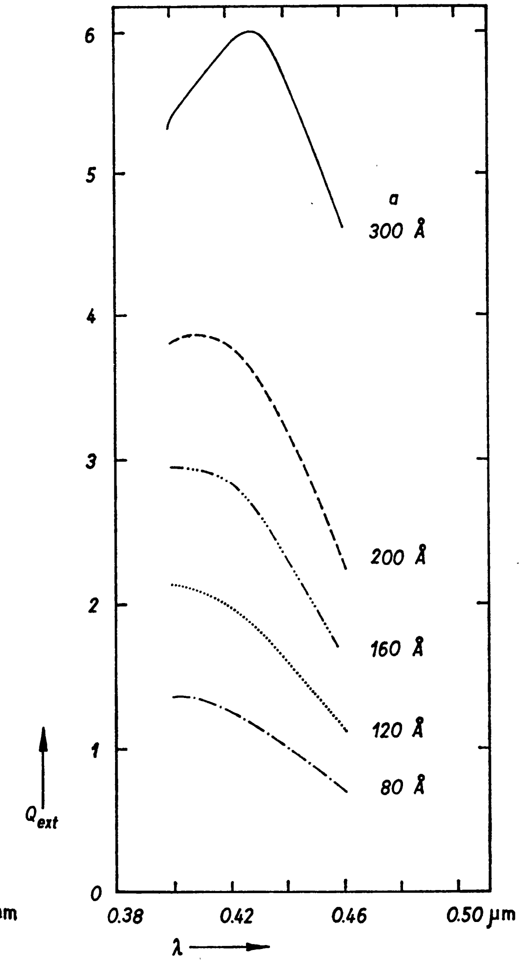

Fig. 3. Same as in Figure 1, but the material is calcium.

sorption coefficient when the particle dimension decreases. However, because of the maximum and minimum in the absorption efficiency (SHAH 1973) such trends cannot be generalized without reference to particular domain of scattering in the $m - x$ plane as described by VAN DE HULST (1957). It is interesting to note that the steep slope in the long wavelength side and the gradual decline towards the short wavelength side mimic qualitatively the observed line profile of the diffuse interstellar band at λ4430.

In the case of Na granules with radius $a = 300$ Å the maximum is reached at 4150 Å. Similarly the peak for potassium granules with $a = 200$ Å (see Figure 2) occurs near 5500 Å but it could reach up to 5700 Å or more with larger sizes. Thus Na and K appear to be promising candidates for the origin of λ4430, λ5780 and λ5795 interstellar diffuse bands. A mechanism for frequency shift may bring the wavelength of the resonance peaks and the line profiles in accord with the observations.

The number density of Na granules can be estimated by assuming that they are of uniform size and distribution in space and that they produce the interstellar diffuse band at λ4430. We shall adopt the maximum observational ratio of the λ4430 extinction to total interstellar extinction to be (see, for example, BROMAGE et al. 1971)

$$\frac{\Delta m(4430)}{\Delta m(\text{grain})} = 0.05 \,. \tag{1}$$

Since extinction in magnitude is proportional to the extinction cross-section, one can write the corresponding theoretical ratio as

$$\frac{\Delta m(\text{granule})}{\Delta m(\text{grain})} = \frac{n\pi a^2 q}{\pi R^2 Q}\,, \tag{2}$$

where
n = number of the Na-granules per grain,
a = radius of the granule = 120 Å,
R = radius of the grain = 0.25 μm,
q = the peak extinction efficiency for Na-granule at λ4430 = 12.5,
Q = the extinction efficiency of the grain at λ4430 = 2.314.

The theoretical values of q and Q are taken from Shah (1973) and Van de Hulst (1946), respectively. The maximum number of Na granules that can be accommodated on a typical grain surface is given by

$$N \cong \pi \left[\frac{R(R+2a)}{a^2} + \frac{4}{3}\right]. \tag{3}$$

From equations (1), (2) and (3), using the values of a, R, q and Q quoted above, one derives $n = 4$ and $N = 1500$. Thus, hardly 0.5 per cent of the total surface of the grain may be occupied by Na granules. Alternatively, if the granules exist outside the grain surface, the projected area of the granules per grain amounts to about one per cent of the geometrical cross-section of a typical grain. Using the line profiles for Na granules with $a_0 = 80$, 200 and 300 Å, the number density of the granules per grain turns out to be 9.36, 0.89, and 0.44, respectively. The total number of Na atoms on this basis is $\cong 5 \cdot 10^{-8}$ to 10^{-7} per cm^3, whereas the cosmic abundance of Na (Cameron 1973) is $2.4 \cdot 10^{-6}$. Thus, although Na granules may account for λ4430, it cannot fully explain the mystery of depleted atoms in interstellar medium unless one considers Na also as an integral part of the grains. However, if the granules acquire dielectric mantles, and the resulting composite grains also contribute to the general extinction curve, much more of the depleted atoms can be accounted for. This problem needs further detailed study.

A critic may question the use of the Mie theory for metals particularly of the alkali group used here. Instead he may favour the use of the photoelectric effect. We refer to the classic work by Hughes and DuBridge (1932) in this connection. It turns out that the photoelectric effect in the case of pure metal spheres as considered here can account for only a small fraction(several per cent at most) of the incident radiation. Most of the light energy can undergo pure absorption and/or scattering. The former gets thermalized in the lattice of the target. Besides, the absorptivity, i.e. imaginary part of index of refraction is obtained from the measured absorption coefficient which accounts for losses due to all processes including photoelectric effect. Therefore, it is clear that the application of Mie theory is quite appropriate.

Finally, it is worthwhile to mention a few applications of the present results in space science. In the study of high altitude atmospheric parameters by rocket discharge of Na vapour, one observes the light coming from the cloud which can have sodium in atomic as well as small solid particulate forms. The latter will be formed immediately after the release of sodium vapour in view of the low prevailing temperature in upper atmosphere at height of $\cong$100 km and above. Therefore, one has to consider the contributions from the resonance radiation from Na atoms as well as the scattered sunlight. The light scattered by the Na grains will be quite significant because the experiments are conducted invariably during twilight hours when the scattering angle will be near 90 degrees. The polarization caused by the solid particles especially for such large scattering angles can provide important information to isolate the two contributions from atoms and grains.

The cometary spectra are known to have part of their energy distribution corresponding to the sunlight scattered by solid particles with size of the order of visual wavelengths. The atomic lines of Na are often the subject of study especially when comet is near the sun. But the simultaneous contribution due to granules composed of Na, K and some other metals can as well be significant because of their pronounced scattering properties.

References

Aannestad, P. A.: 1973, Astrophys. J. Suppl. **25**, 205.
Bless, R. C. and Savage, B. D.: 1972, Astrophys. J. **171**, 293.
Bromage, G. E. Brück, M. T., and Nandy, K.: 1971, Astron. Nachr. **293**, 39.
Cameron, A. G. W.: 1973, Preprint.
Debye, P.: 1909, Ann. Phys. **30**, 59.
Dorschner, J.: 1967, Astron. Nachr. **290**, 171.
Dorschner, J.: 1968, Thesis, University of Jena.
Dorschner, J.: 1970, Astron. Nachr. **292**, 79.
Dorschner, J.: 1971, Astron. Nachr. **293**, 65.
Field, G. B.: 1974, Astrophys. J. **187**, 453.
Friedemann, C.: 1969, Physica **41**, 139.
Gray, D. E. (ed.): 1963, American Institute of Physics Handbook, 2nd ed., p. 6–107.
Greenberg, J. M.: 1974, Astrophys. J. **189**, L81.

HOYLE, F. and WICKRAMASINGHE, N. C.: 1962, Monthly Notices Roy. Astron. Soc. **124**, 417.
HUGHES, A. L. and DUBRIDGE, L. A.: 1932, Photoelectric Phenomena. New York, McGraw-Hill.
KAMIJO, F.: 1963, Publ. Astron. Soc. Japan **15**, 440.
LEQUEUX, J.: 1975, Astron. Astrophys. **39**, 257.
MIE, G.: 1908, Ann. Phys. **25**, 377.
NANDY, K., THOMPSON, G. I., JAMAR, C., MONFILS, A., and WILSON, R.: 1975, Astron. Astrophys. **44**, 195.
OORT, J. H. and VAN DE HULST, H. C.: 1946, Bull. Astron. Inst. Netherlands **10**, 187.
ROUTLY, P. M. and SPITZER, L., Jr.: 1952, Astrophys. J. **115**, 227.
SAVAST'YANOVA, M. V.: 1939a, Usp. Fiz. Nauk **22**, 1.
SAVAST'YANOVA, M. V.: 1939b, Usp. Fiz. Nauk **22**, 129.
SALPETER, E. E.: 1974, Rev. Mod. Phys. **46**, 433.
SCHATZMAN, E. and CAYREL, R.: 1954, Ann. Astrophys. **17**, 555.
SHAH, G. A.: 1973, Proc. Indian Acad. Sci. **77**, 1.
SHIFRIN, K. S.: 1968, Scattering of Light in a Turbid Medium. NASA TTF-477, Washington, D.C.
VAN DE HULST, H. C.: 1957, Light Scattering by Small Particles. John Wiley & Sons, New York.
VAN DE HULST, H. C.: 1964, Publ. Roy. Obs. Edingburgh **4**, 1.
YORK, D., DRAKE, J., JENKINS, E., MORTON, D., ROGERSON, J., and SPITZER, L.: 1973, Astrophys. J. **182**, L1.

Address of the author:

G. A. SHAH
Indian Institute of Astrophysics
St. John Medical College P.O.
Bangalore 560034, India