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Tidal disruption and tidal coalescence in binary stellar systems

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Summary. Tidal force effects in binary stellar systems are considered. Estimates are made for the rate of increase of the binding energy of a component and the rate of decrease of the energy of the orbital motion of the binary under the simplifying assumption that the motion of the stars in the stellar systems may be neglected in comparison with the orbital motion of the binary system. The time of disruption of a component and the time of coalescence of the pair are obtained as functions of the separation and mass ratio of the binary for distant as well as close overlapping pairs. Applications are made to globular clusters and binary galaxies.

For a pair of identical galaxies the rate of coalescence of the pair is faster than that of the disruption of the components. A study of the tidal effects at the median radius indicates that a pair of contact spherical galaxies moving in circular relative orbit, will merge and form a single system in a time interval less than three periods of revolution. The rate of disruption of the components is six times slower. As the separation decreases, the rate of disruption increases faster than the rate of coalescence.

1 Introduction

The relative motion of two galaxies differs from that of two mass points in an important respect. The stars in the two galaxies are generally accelerated due to tidal effects with the result that the energy of the orbital motion of the two galaxies is decreased. As a result of this phenomenon, two galaxies initially moving in hyperbolic orbits may form a binary system under certain circumstances (Alladin 1965; Alladin, Potdar & Sastry 1975; Yabushita 1977). Computations made by Cox and Toomre (Toomre 1974, 1977; Van Albeda & Van Gorkom 1977) for slow head-on collisions of two galaxies have emphasized that tidal friction in a slow close approach of two galaxies is enormous.

Binary stellar systems, however formed, will revolve around each other with a mean separation that will decrease in time and with disruptive effects on the structures of each other that will increase in time. Many contact and overlapping pairs of galaxies are known (Vorontsov-Velyaminov 1977). Zwicky (1959) had suggested that a problem of much

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interest is to determine how long can overlapping stellar systems co-exist before disrupting each other and losing their identity.

It is the aim of the present paper to make estimates for the rates of disruption and coalescence of binary stellar systems in a preliminary scheme of approximation. The stellar systems are assumed to be spherically symmetric configurations. We compute the increase in the internal energy of a stellar system (that is the energy due to the distribution and motion of its stars) due to the tidal field of another under the simplifying assumption that the motion of the stars in the stellar systems may be neglected in comparison with the orbital motion of the binary. This is the impulsive approximation earlier used by Spitzer (1958), Bouvier & Janin (1970), Spitzer & Chevalier (1973) and Knobloch (1976), to estimate the tidal effects of a passing interstellar cloud on a galactic cluster; by Contopoulos & Bozis (1964); Alladin (1965); Sastry & Alladin (1970); Sastry (1972); Gallagher & Ostriker (1972); Richstone (1975); Biermann & Silk (1976); Toomre (1977), to estimate the tidal effects of pairs of galaxies; by Ostriker, Spitzer & Chevalier (1972) to estimate the tidal effects of the Galaxy on globular clusters. We develop the theory in greater detail than was done earlier and also discuss a preliminary method of studying the tidal effects of close interpenetrating pairs. In order that the impulsive approximation may be valid, we require that the motion of the binary should be fast in comparison with the motion of the stars. In a number of situations, this condition may not be satisfied. The results of Spitzer (1958) and Knobloch (1976) show that if the test stellar system is assumed to retain the same shape throughout the encounter and if the stars are assumed to move in unperturbed orbits, the increase in the internal energy thus obtained is generally less than that obtained in the impulsive approximation. On the other hand, Lauberts (1974) has indicated that the tidal effects re-shape the interacting stellar systems in a way that greatly enhances further interaction which is predominantly in the direction of increasing the internal energy. Thus the two effects work in opposite directions. While detailed computations of the orbits of stars are needed to make accurate estimates for the changes in internal energies of the two stellar systems, it appears that the impulsive approximation would provide a useful first approximation against which the results of detailed numerical work may be compared. Toomre (1977) compares the minimum velocity necessary for non-merging of two galaxies in a head-on collision, deduced from impulsive approximation, with that obtained by Van Albeda & Van Gorkom (1977) from detailed calculations and finds good agreement. This increases our confidence in the impulsive approximation.

2 Theory

We shall first develop the theory for the tidal effects of pairs of stellar systems separated by a distance large compared to their dimensions by expanding the tidal potential in a series in Legendre polynomials. For close pairs which comprise contact and interpenetrating binaries, this method cannot be used. The tidal acceleration in such cases is obtained by performing certain numerical integrations. These will be discussed later.

2.1 DISTANT PAIRS

Consider two spherically symmetric stellar systems of masses M and M_1, revolving around each other in a circular orbit of radius r with angular velocity ω. We shall consider the tidal effects of M_1 on M. We define a fixed coordinate system x, y, z with the origin at the centre of mass of M and the z axis perpendicular to the orbital plane. Let the coordinates of the centre of M_1 be denoted by x, y and z (by definition $z = 0$). Let the coordinates of a representative star in M be x', y' and z' and let its distance from the centre of M be r'. Let r'' be the distance between the centre of M_1 and the representative star in M.

The tidal potential of the star due to M_1 in a rotating frame of reference ξ, η, ζ in which ξ points to the instantaneous position of M_1, η is perpendicular to ξ and in the orbital plane, and ζ is parallel to z, is given by

$$\mathscr{V}_1 = -\frac{GM_1}{r} \sum_{l=2}^{\infty} P_l(\cos\psi)\left(\frac{r'}{r}\right)^l \tag{2.1}$$

where $\cos\psi = \xi/r$ and $P_l(\cos\psi)$ are the Legendre polynomials. We shall consider terms in (2.1) up to $l = 4$.

We obtain the tidal acceleration in the rotating coordinate system from

$$f = -\nabla \ \mathscr{V}_1. \tag{2.2}$$

We then transform this acceleration to the fixed frame by using the equations

$$\xi = x\cos\theta + y\sin\theta \tag{2.3}$$

$$\eta = -x\sin\theta + y\cos\theta$$

$$\zeta = z$$

where $\theta = \omega t$. We obtain

$$f_x = \frac{dv_x}{dt} = \frac{GM_1}{r^3}\,[x'(2-3\sin^2\theta) + 3y'\sin\theta\cos\theta] + \text{higher order terms} \tag{2.4}$$

$$f_y = \frac{dv_y}{dt} = \frac{GM_1}{r^3}\,[3x'\sin\theta\cos\theta + y'(2-3\cos^2\theta)] + \text{higher order terms} \tag{2.5}$$

$$f_z = \frac{dv_z}{dt} = -\frac{GM_1 z'}{r^3} + \text{higher order terms.} \tag{2.6}$$

Assuming that the motions of stars in M can be neglected, we integrate the right-hand sides of these equations over one period P of revolution of the binary system. The terms arising in the tidal potential for $l = 3$, integrate to zero. We obtain up to terms with $l = 4$, the following expression for changes in velocity:

$$(\Delta V_x)_{\rm P} = \frac{GM_1 P}{2r^3}\,x' + \frac{3GM_1 P}{2r^5}\,[3/8\,x'^3 - 3/8\,x'y'^2 - 3/2\,x'z'^2] \tag{2.7}$$

$$(\Delta V_y)_{\rm P} = \frac{GM_1 P}{2r^3}\,y' + \frac{3GM_1 P}{2r^5}\,[3/8\,y'^3 - 3/8\,x'^2y' - 3/2\,y'z'^2] \tag{2.8}$$

$$(\Delta V_z)_{\rm P} = -\frac{GM_1 P}{r^3}\,z' + \frac{3GM_1 P}{2r^5}\,[z'^3 - 3/2\,y'^2z' - 3/2\,x'^2z']. \tag{2.9}$$

The increase in the total energy of M in one period $(\Delta U)_{\rm P}$, is in the impulsive approximation the same as the change in its kinetic energy, and is obtained from

$$(\Delta U)_{\rm P} = 1/2\,M\{\Delta V(R_{\rm h})\}^2 \tag{2.10}$$

where $R_{\rm h}$ is the median radius of M.

From equations (2.7) to (2.10) we obtain

$$(\Delta U)_{\rm P} = \frac{\pi^2 G^2 M_1^2 M R_{\rm h}^2}{(M+M_1)\,r^3}\left\{1 + \frac{1}{3}\left(\frac{R_{\rm h}}{r}\right)^2\right\}. \tag{2.11}$$

We have neglected terms of the order of $(R_h/r)^4$ in the bracket. Even the second term in the bracket is generally quite small. If two homogeneous stellar systems of equal radius touch each other $\frac{1}{3}(R_h/r)^2 = 0.0525$. We shall represent the density distribution in the two stellar systems by those of polytropes of integral indices as described by Limber (1961). The binding energy of M is, by the virial theorem, given by

$$U = \frac{\Omega}{2} = -\frac{\beta_n GM^2}{2R_h} = -\frac{1}{2}\frac{3}{5-n}\frac{GM^2}{R} \tag{2.12}$$

where Ω is the gravitational potential energy, R is the radius, and β_n is a numerical constant which can be readily determined from (2.12) if R_h is specified. $R_h = 0.135R$ for a polytrope of index $n = 4$ which represents fairly well the central concentration of spherical stellar systems. $R_h = 0.794$ for a homogeneous sphere. For polytropes of indices 1, 2 and 3, R_h has the values $0.607R$, $0.439R$ and $0.284R$ respectively.

The fractional increase of the internal energy of M in one period of revolution is given by

$$\frac{(\Delta U)_P}{U} = -\frac{2\pi^2}{\beta_n}\frac{M_1^2}{M(M+M_1)}\left(\frac{R_h}{r}\right)^3\left\{1+\frac{1}{3}\left(\frac{R_h}{r}\right)^2\right\}. \tag{2.13}$$

Spitzer & Chevalier (1973) have defined shock disruption time t_{sh} as

$$t_{sh} = \frac{|U|}{dU/dt} \tag{2.14}$$

and have discussed the utility of this parameter for the overall evolution of the stellar system. We shall write for the shock disruption time, t_d of M:

$$\frac{t_d}{P} = \left\{\frac{(\Delta U)_P}{|U|}\right\}^{-1}. \tag{2.15}$$

Kurth (1957) has given the following formula for making an order of magnitude estimate of the time required for the dissolution of an unstable star cluster due to the tidal field of the Galaxy:

$$t_* = \{G(|2\sigma - \bar{\rho}|)\}^{1/2} \tag{2.16}$$

where $\bar{\rho}$ is the mean density of the cluster and σ is the mean density of a sphere about the galactic centre having the centre of the cluster on its surface, and being uniformly filled with the total mass of the Galaxy. If $\bar{\rho} \geqslant 2\sigma$, the cluster is considered stable. In Section 3, the prediction of equations (2.15) and (2.16) will be compared.

In the foregoing analysis we have assumed the orbit for the relative motion of the two stellar systems to be circular. If we take the orbit to be any conic and neglect the higher order term, we would get:

$$\frac{(\Delta U)_P}{|U|} = \frac{2\pi^2}{\beta_n(1+e)^3}\left(\frac{R_h}{p}\right)^3\frac{M_1^2}{M(M+M_1)} \tag{2.17}$$

where p is the distance of closest approach of the binary and e the eccentricity of the orbit.

It follows from (2.15) and (2.17) that

$$t_d = \frac{\beta_n}{\pi}\frac{a^{4.5}(1-e^2)^3 M(M+M_1)^{1/2}}{G^{1/2}R_h^3 M_1^2} \tag{2.18}$$

where a is the semimajor axis of the orbit.

This may be compared with the less exact equation used by Keenan & Innanen (1975) which, in our notation, is

$$t_d = \frac{\pi}{10} \frac{a^{4.5}(1+e)(1-e)^3 M}{G^{1/2} R_h^3 M_1^{3/2}}. \tag{2.19}$$

In the paper by Keenan & Innanen, the denominator contains $M_1^{2/3}$ by mistake. We have made the correction in writing (2.19). For the value $e = 0.5$ used by Keenan & Innanen, the predictions of (2.18) and (2.19) are in close agreement.

It follows from the law of conservation of energy that the sum of the internal energies of the two stellar systems and the external energy, E, due to the orbital motion of the two systems should be constant. Hence

$$\frac{(\Delta E)_P}{E} = -\frac{1}{E}[(\Delta U)_P + (\Delta U_1)_P] \tag{2.20}$$

where

$$E = -\frac{GMM_1}{2a}. \tag{2.21}$$

Thus as the internal energies increase, the binary would become more tightly bound. We define the time of coalescence as

$$t_c = \frac{E}{dE/dt} = \left\{\frac{(\Delta E)_P}{E}\right\}^{-1} P. \tag{2.22}$$

Since from equation (2.21), $\Delta E/E = -\Delta a/a$, equation (2.22) implies that if we assume that the semimajor axis of the relative orbit of the binary decreases at a uniform rate given by its initial value, the centres of the two systems will merge in time t_c. Using (2.12), (2.17), (2.20) and (2.22), we obtain

$$t_c = \frac{a^{3.5}(1-e^2)^3 (M+M_1)^{1/2}}{\pi G^{1/2}\{M_1 R_h^2 + M(R_h)_1^2\}}. \tag{2.23}$$

If we assume that Ω varies as $M^{3/2}$ as suggested by observations (Fish 1964), and hence M varies as R^2 we obtain from (2.17), (2.20) and (2.21) the relation:

$$\frac{(\Delta E)_P}{E} = \frac{4\pi^2 M_1 R_h^2}{(M+M_1)\, a^2 (1-e^2)^3} \tag{2.24}$$

and (2.23) gives:

$$t_c = \frac{a^{3.5}(1-e^2)^3 (M+M_1)^{1/2}}{2\pi G^{1/2} M_1 R_h^2}. \tag{2.25}$$

In the present preliminary treatment of the problem $(\Delta U)_P/|U|$ and $(\Delta E)_P/E$ are determined by assuming that the effects of escaping stars may be neglected and the binary systems retain the same shape and mass distribution throughout. In $(\Delta U)_P$ we have also included the energy of the stars that were initially a part of the stellar system but have later escaped. Thus the fractional change of energy $(\Delta U)_P/|U|$ of the stellar system would be less than that obtained by the present method if allowance is made for the energy of the stellar system carried away by the escaping stars. The escaping matter from the two galaxies will also lead to a decrease of the magnitude of the orbital energy of the binary system and thus make the binary less tightly bound. Thus the values of t_d and t_c obtained from the present treatment will have to be increased if the effects of the escaping stars are considered. On the other hand, since the tidal effects loosen and flatten the two stellar systems, they become more vulnerable to further changes in internal energy. This effect would decrease t_d and t_c.

2.2 CLOSE PAIRS

In the case of close pairs, the increase in the internal energy of M has been determined by considering the tidal effects due to M_1 on a number of stars of M situated on a sphere of radius R_h. If the stellar systems are close but do not interpenetrate, the tidal force per unit mass on a star in M is obtained from:

$$f_x = +\frac{GM_1}{r''^2}\frac{(x-x')}{r''}+\frac{GM_1}{r^2}\frac{x}{r}$$

$$f_y = +\frac{GM_1}{r''^2}\frac{(y-y')}{r''}+\frac{GM_1}{r^2}\frac{y}{r}. \tag{2.26}$$

$$f_z = -\frac{GM_1}{r''^2}\frac{z'}{r''}.$$

For interpenetrating stellar systems, the modified form of the above equations, is

$$f_x = -\frac{GM_1}{R^2}\left[\frac{d\Phi_1}{ds''}\frac{(x-x')}{r''}-\frac{d}{ds}\left(\frac{\Psi}{s}\right)\frac{x}{r}\right]$$

$$f_y = -\frac{GM_1}{R^2}\left[\frac{d\Phi_1}{ds''}\frac{(y-y')}{r''}-\frac{d}{ds}\left(\frac{\Psi}{s}\right)\frac{y}{r}\right] \tag{2.27}$$

$$f_z = +\frac{GM_1}{R^2}\frac{d\Phi_1}{ds''}\frac{z'}{r''}$$

where $s = r/R$ and the functions $\Phi(n_1, s)$ and $\Psi(n_1, n_2, s)$ are derived from polytrope theory as indicated in Sastry & Alladin (1970). In the study of interpenetrating pairs, we assume that both M and M_1 have the same radius R and that M_1 moves in a circular orbit about M. The change in the velocity of a star with respect to the centre of M in one orbital period of the binary is obtained from

$$(\Delta V_x)_P = -\frac{GM_1P}{2\pi R^2}\left[\int_0^{2\pi}\frac{d\Phi_1}{ds''}\frac{(r\cos\theta - x')}{r''}d\theta - \int_0^{2\pi}\frac{d}{ds}\left(\frac{\Psi}{s}\right)\cos\theta\, d\theta\right]$$

$$(\Delta V_y)_P = -\frac{GM_1P}{2\pi R^2}\left[\int_0^{2\pi}\frac{d\Phi_1}{ds''}\frac{(r\sin\theta - y')}{r''}d\theta - \int_0^{2\pi}\frac{d}{ds}\left(\frac{\Psi}{s}\right)\sin\theta\, d\theta\right] \tag{2.28}$$

$$(\Delta V_z)_P = +\frac{GM_1P}{2\pi R^2}\int_0^{2\pi}\frac{d\Phi_1}{ds''}\frac{z'}{r''}d\theta.$$

The second integrals in ΔV_x and ΔV_y give zero on integrating over a period. Denoting the sum of the squares of the non-zero integrals by I^2, we obtain

$$(\Delta V)^2 = \frac{G^2M_1^2P^2I^2}{4\pi^2R^4}. \tag{2.29}$$

If N stars are taken at the median radius, R_h, we obtain

$$\langle \Delta U(R_h) \rangle = \frac{1}{2N} \sum_{i=1}^{N} \{\Delta V_i(R_h)\}^2 = \frac{G^2 M_1^2 P^2}{8\pi^2 R^4} \langle I^2 \rangle. \tag{2.30}$$

We shall estimate $\langle \Delta U(R_h) \rangle$ with $N = 14$, the stars being chosen on the positive and negative axes of the Cartesian coordinate system and on the centres of the octants of the sphere. It follows from (2.10), (2.12) and (2.30) that

$$\frac{(\Delta U)_P}{|U|} = \frac{5-n}{12\pi^2} \frac{G M_1^2 P^2 \langle I^2 \rangle}{M R^3}. \tag{2.31}$$

The orbital period of the binary is obtained from

$$P^2 = \frac{4\pi^2 r^3}{G(M + M_1)} \left\{ s^2 \frac{d}{ds} \left(\frac{\Psi}{s} \right) \right\}^{-1}. \tag{2.32}$$

Using (2.31) and (2.32) with (2.12), (2.15), (2.20) and (2.22), we obtain $(\Delta E)_P/E$, t_d and t_c for interpenetrating pairs. In this case E, obtained from (2.21), is multiplied by Ψ.

The time of coalescence used in this work takes the extended nature of both the stellar systems into account while the dynamical friction time used by Tremaine (1976a) and others, neglects the extended nature of the smaller stellar system in the sense that ΔU for the smaller system is ignored. If $M \ll M_1$ and if M is regarded as a mass point so that $\Delta U = 0$, t_c may be expressed in the form

$$t_c = \frac{3 V^3 R_1}{2 G^2 M m_1 n_1 r \langle I_1^2 \rangle} \tag{2.33}$$

where V is the relative speed, n_1 is the average number of stars per pc^3 and m_1 the average stellar mass in M_1. If V is in km/s, n_1 in number of stars per pc^3 and m_1 and M in solar masses,

$$t_c = 7.4 \times 10^{10} \frac{V^3}{M m_1 n_1} \frac{R_1}{r \langle I_1^2 \rangle} \text{ yr}. \tag{2.34}$$

The factor $R_1/r\langle I_1^2 \rangle$ decreases as the separation decreases. If the mass distribution in M_1 is that of a polytrope of index $n = 4$, we have:

$$\begin{aligned} t_c &= 6 \times 10^7 \frac{V^3}{M m_1 n_1} \text{ yr} \quad (r = 0.1 R_1) \\ t_c &= 7 \times 10^9 \frac{V^3}{M m_1 n_1} \text{ yr} \quad (r = 0.5 R_1). \end{aligned} \tag{2.35}$$

Equation (2.34) may be compared with the time required for dynamical friction significantly to decelerate a massive object moving with typical velocity V (Larson 1976).

$$T_d \sim 4 \times 10^8 \frac{V^3}{M m_1 n_1} \text{ yr}. \tag{2.36}$$

It follows from the polytrope theory that the escape velocity is given by

$$V_{esc}^2 = \frac{2G(M + M_1)}{r} \Psi(n, n_1, r) \tag{2.37}$$

Table 1. $(\Delta U)_P/|U|$ for distant pairs.

r/R \ M_1/M	0.5	1	2	5	10	50	10^2	10^3	10^4	10^6
3	7.7×10^{-4}	2.2×10^{-3}	5.9×10^{-3}	1.9×10^{-2}	4.0×10^{-2}	2.2×10^{-1}	4.3×10^{-1}	4.3	4.3×10	4.3×10^{3}
4	3.1×10^{-4}	9.1×10^{-3}	2.5×10^{-3}	7.7×10^{-3}	1.7×10^{-2}	9.1×10^{-2}	1.9×10^{-1}	1.8	1.8×10	1.8×10^{3}
5	1.6×10^{-4}	4.8×10^{-4}	1.3×10^{-3}	4.0×10^{-3}	8.3×10^{-3}	4.8×10^{-2}	1.0×10^{-1}	1.0	10.0	1.0×10^{3}
6	9.1×10^{-5}	2.8×10^{-4}	7.1×10^{-4}	2.3×10^{-3}	5.0×10^{-3}	2.7×10^{-2}	5.5×10^{-2}	5.5×10^{-1}	5.6	5.6×10^{2}
7	5.9×10^{-5}	1.8×10^{-4}	4.5×10^{-4}	1.4×10^{-3}	3.2×10^{-3}	1.7×10^{-2}	3.4×10^{-2}	3.4×10^{-1}	3.4	3.4×10^{2}
8	3.8×10^{-5}	1.2×10^{-4}	3.1×10^{-4}	1.0×10^{-3}	2.1×10^{-3}	1.1×10^{-2}	2.3×10^{-2}	2.3×10^{-1}	2.3	2.3×10^{2}
9	2.7×10^{-5}	8.3×10^{-5}	2.2×10^{-4}	6.7×10^{-4}	1.5×10^{-3}	8.3×10^{-3}	1.6×10^{-2}	1.6×10^{-1}	1.6	1.6×10^{2}
10	2.0×10^{-5}	5.9×10^{-5}	1.6×10^{-4}	5.0×10^{-4}	1.1×10^{-3}	5.9×10^{-3}	1.2×10^{-2}	1.2×10^{-1}	1.2	1.2×10^{2}
50	1.6×10^{-7}	4.8×10^{-7}	1.3×10^{-6}	4.0×10^{-6}	8.3×10^{-6}	4.8×10^{-5}	1.0×10^{-4}	1.0×10^{-3}	1.0×10^{-2}	1.0
10^2	2.0×10^{-8}	5.9×10^{-8}	1.6×10^{-7}	5.0×10^{-7}	1.1×10^{-6}	5.9×10^{-6}	1.2×10^{-5}	1.2×10^{-4}	1.2×10^{-4}	1.2×10^{-1}
10^3	2.0×10^{-11}	5.9×10^{-11}	1.6×10^{-10}	5.0×10^{-10}	1.1×10^{-9}	5.9×10^{-9}	1.2×10^{-8}	1.2×10^{-7}	1.2×10^{-6}	1.2×10^{-4}
10^4	2.0×10^{-14}	5.9×10^{-14}	1.6×10^{-13}	5.0×10^{-13}	1.1×10^{-12}	5.9×10^{-12}	1.2×10^{-11}	1.2×10^{-10}	1.2×10^{-9}	1.2×10^{-7}

Table 2. $(\Delta E)_P/E$ for distant pairs.

r/R \ M_1/M	0.5	1	2	5	10	50	10^2	10^3	10^4	10^6
3	2.7×10^{-2}	4.0×10^{-2}	5.3×10^{-2}	6.7×10^{-2}	7.3×10^{-2}	7.8×10^{-2}	7.9×10^{-2}	8.0×10^{-2}	8.0×10^{-2}	8.0×10^{-2}
4	1.5×10^{-2}	2.2×10^{-2}	3.0×10^{-2}	3.7×10^{-2}	4.1×10^{-2}	4.4×10^{-2}	4.4×10^{-2}	4.5×10^{-2}	4.5×10^{-2}	4.5×10^{-2}
5	9.6×10^{-3}	1.4×10^{-2}	1.9×10^{-2}	2.4×10^{-2}	2.6×10^{-2}	2.8×10^{-2}	2.8×10^{-2}	2.9×10^{-2}	2.9×10^{-2}	2.9×10^{-2}
6	6.7×10^{-3}	1.0×10^{-2}	1.3×10^{-2}	1.7×10^{-2}	1.8×10^{-2}	2.0×10^{-2}	2.0×10^{-2}	2.0×10^{-2}	2.0×10^{-2}	2.0×10^{-2}
7	4.9×10^{-3}	7.3×10^{-3}	9.8×10^{-3}	1.2×10^{-2}	1.3×10^{-2}	1.4×10^{-2}	1.5×10^{-2}	1.5×10^{-2}	1.5×10^{-2}	1.5×10^{-2}
8	3.7×10^{-3}	5.6×10^{-3}	7.5×10^{-3}	9.4×10^{-3}	1.0×10^{-2}	1.1×10^{-2}	1.1×10^{-2}	1.1×10^{-2}	1.1×10^{-2}	1.1×10^{-2}
9	3.0×10^{-3}	4.4×10^{-3}	6.0×10^{-3}	7.4×10^{-3}	8.1×10^{-3}	8.7×10^{-3}	8.8×10^{-3}	8.9×10^{-3}	8.9×10^{-3}	8.9×10^{-3}
10	2.4×10^{-3}	3.6×10^{-3}	4.8×10^{-3}	6.0×10^{-3}	6.5×10^{-3}	7.1×10^{-3}	7.1×10^{-3}	7.2×10^{-3}	7.2×10^{-3}	7.2×10^{-3}
50	9.6×10^{-5}	1.4×10^{-4}	1.9×10^{-4}	2.4×10^{-4}	2.6×10^{-4}	2.8×10^{-4}	2.8×10^{-4}	2.9×10^{-4}	2.9×10^{-4}	2.9×10^{-4}
10^2	2.4×10^{-5}	3.6×10^{-5}	4.8×10^{-5}	6.0×10^{-5}	6.5×10^{-5}	7.1×10^{-5}	7.1×10^{-5}	7.2×10^{-5}	7.2×10^{-5}	7.2×10^{-5}
10^3	2.4×10^{-7}	3.6×10^{-7}	4.8×10^{-7}	6.0×10^{-7}	6.5×10^{-7}	7.1×10^{-7}	7.1×10^{-7}	7.2×10^{-7}	7.2×10^{-7}	7.2×10^{-7}
10^4	2.4×10^{-9}	3.6×10^{-9}	4.8×10^{-9}	6.0×10^{-9}	6.5×10^{-9}	7.1×10^{-9}	7.1×10^{-9}	7.2×10^{-9}	7.2×10^{-9}	7.2×10^{-9}

where the function Ψ is given in Alladin (1965). We obtain for identical galaxies moving in circular relative orbit

$$\frac{(\Delta E)_P}{E} = \frac{1}{4\Psi^2}\left(\frac{r}{R}\right)^4\left(\frac{V_{esc}}{V}\right)^4 \langle I^2\rangle. \tag{2.38}$$

This may be compared with the corresponding expression obtained by Toomre (1977, equation 6) for a head-on collision between two identical galaxies. $\Delta E/E$ in our notation is the same as $2\Delta T/(MU^2/4)$ in Toomre's notation.

3 Results

Table 1 gives the values of $(\Delta U)_P/|U|$ for different ratios M_1/M and r/R for distant pairs for circular orbits of the components. The mass distribution in M is assumed to be that of a polytrope of index $n = 4$. For polytropes of indices $n = 3, 2, 1$ and 0, the given values should be increased by factors 8.9, 32, 81 and 173 respectively. Results for any eccentricity in a conic orbit can be obtained from these values by simple scaling using equation (2.17). In particular, it may be noted that $(\Delta U)_P/|U|$ in a parabolic orbit is eight times smaller than that in a circular orbit of radius equal to the distance of closest approach. Table 3 gives the values of $(\Delta U)_P/|U|$ and $(\Delta E)_P/E$ for contact and interpenetrating stellar systems for $n = n_1 = 4$. The reciprocal of $(\Delta U)_P/|U|$ gives the time of disruption in units of the orbital period of the binary. A bold line is drawn in the tables to indicate the limit to the right of which $(\Delta U)_P/|U| \geqslant 1$. For distant pairs and mass distribution in M specified by $n = 4$, this corresponds to the situation $R_t = 0.34R$ where R_t is the tidal radius defined by

$$R_t = r\left(\frac{M}{3M_1}\right)^{1/3} \tag{3.1}$$

(King 1962). In a polytrope $n = 4$, about 95 per cent of the mass is enclosed within $0.34R$. Hence if the tidal radius of M is equal to the radius of the sphere containing 95 per cent of its mass, disruption will occur in about one orbital period of the binary. If M is homogeneous $R_t = 1.9R$ corresponds to $(\Delta U)_P/|U| = 1$ for distant pairs. Thus a homogeneous system gets disrupted in one period of the binary even if its dimension is about half the tidal radius.

Table 2 gives the corresponding values of $(\Delta E)_P/E$ for distant pairs. In Table 2 where a large range of masses is considered, we have assumed $R_1 = R(M_1/M)^{1/2}$. In Table 3 we give $(\Delta E)_P/E$ for close pairs. We have assumed for these cases that $R = R_1$ since a large range in mass ratio has not been considered. With this assumption $(\Delta E)_P/E$ is independent of the mass ratio for a given separation of the components.

The times of disruption and coalescence are shown graphically as functions of separation and mass ratios in Figs 1 and 2 for distant pairs and close pairs, respectively. The units are chosen so that $M = 1$, $R = 1$ and $P' = 1$, where P' is the period of a star in M revolving in a

Table 3. $(\Delta U)_P/|U|$ and $(\Delta E)_P/E$ for close pairs.

r/R \ M_1/M	0.5	1	2	5	10	$(\Delta E)_P/E$
0.1	7.0	2.1×10	5.6×10	1.8×10^2	3.8×10^2	24
0.2	1.5	4.5	1.2×10	3.7×10	8.1×10	6.5
0.5	1.5×10^{-1}	4.5×10^{-1}	1.2	3.8	8.3	1.4
1.0	2.0×10^{-2}	6.0×10^{-2}	1.6×10^{-1}	5.0×10^{-1}	1.1	0.36
2.0	2.5×10^{-3}	7.4×10^{-3}	2.0×10^{-2}	6.0×10^{-2}	1.3×10^{-1}	0.09

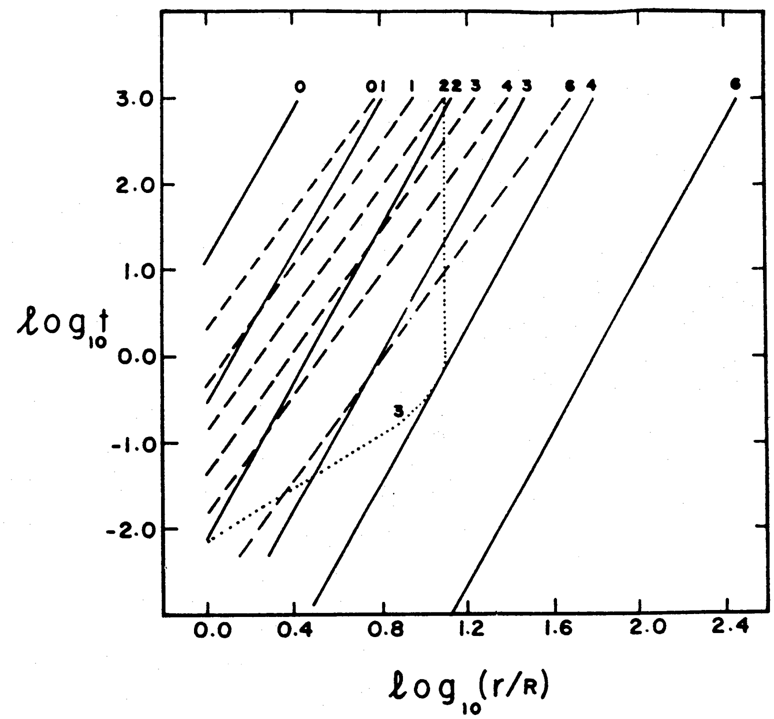

Figure 1. t_d and t_c as a function of the separation for distant pairs. The continuous line denotes t_d, the dashed line denotes t_c and the dotted line is obtained from Kurth's formula. The numbers above the lines denote the mass ratio $\log_{10}(M_1/M)$.

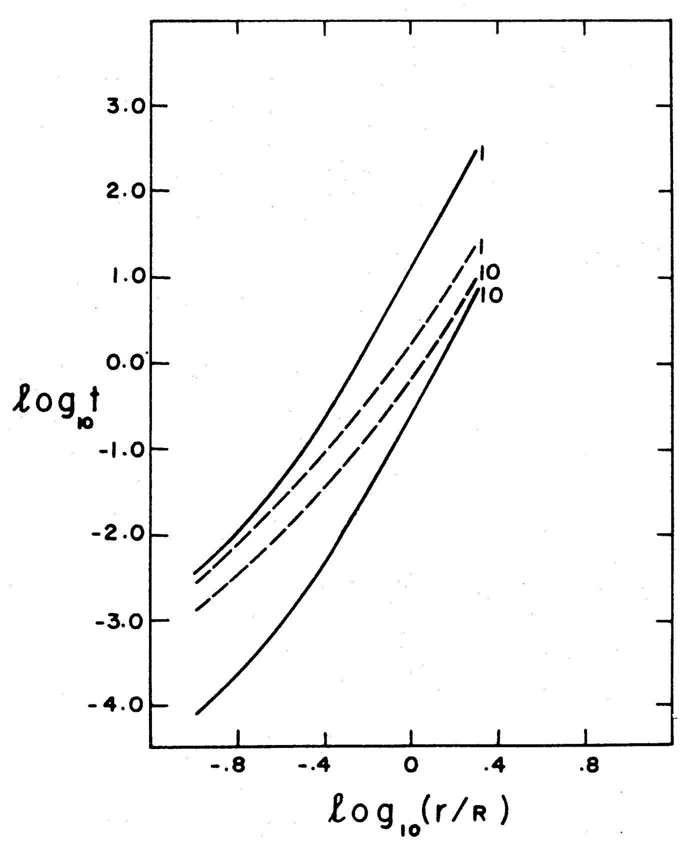

Figure 2. t_d and t_c as a function of the separation for close pairs. The continuous line denotes t_d, the dashed line denotes t_c. The numbers above the lines denote the mass ratio M_1/M.

circular orbit of radius R. The time of disruption obtained from Kurth's formula (2.16) is shown for comparison for the case $M_1/M = 10^3$.

4 Discussion

We apply the theory for distant pairs to evaluate t_d and t_c for a globular cluster moving in a circular orbit at various distances from the centre of the Galaxy. We take the mass of the globular cluster as $10^5 M_\odot$ and represent its mass distribution as that of a polytrope of index $n = 4$, of radius 50 pc so that R_h is about 7 pc. Taking the mass distribution of the Galaxy from Schmidt (1965), we obtain the values of $(\Delta U)_P/|U|$, t_d and t_c given in Table 4. The values for any other mass and radius of the cluster can be got by scaling from equations (2.18) and (2.25). The coalescence rates for globular clusters are small compared to the disruption rates which imply that globular clusters would be disrupted much before their centres merge with the galactic centre. It may be noted that close to the galactic centre the rate of disruption is extremely high. A globular cluster of mass $10^5 M_\odot$ and for which $n = 4$, can survive one period of revolution in a circular orbit at a distance of 320 pc only if its median radius does not exceed 2.5 pc. Tremaine, Ostriker & Spitzer (1975) and Tremaine (1976b) have suggested that nuclei of galaxies can be formed from the tidally-disrupted globular clusters.

In the case of binary galaxies, the disparity between the disruption and coalescence rates is less. Using a modified form of Spitzer's (1958) formula, Keenan & Innanen (1975) have estimated the times of disruption of three galaxies having anomalous surface brightness and truncated halos, namely M32, NGC 4486B and NGC 5846B (Faber 1973) due to the tidal effects of their massive companions, namely M31, M87 (Virgo A) and NGC 5846A, respectively. Assuming $e = 0.5$, Keenan & Innanen obtained the disruption times as 6.9×10^9, 2×10^9 and 8.2×10^8 yr respectively. With the values of M, M_1, R_h, P and e used by Keenan & Innanen and with β_n that of a polytype $n = 4$, we obtain the disruption times for the three galaxies as 6.6×10^9, 1.8×10^9 and 7.8×10^8 yr respectively. These values are in close agreement with those obtained by Keenan & Innanen. Our values for t_c for these pairs are 1.4×10^{10}, 7.2×10^9 and 1.3×10^9 yr respectively.

It is of interest to estimate t_d and t_c for the Large Magellanic Cloud due to tidal interaction with the Galaxy. If the warp of the outer plane of the Galaxy is interpreted as due to tidal distortion caused by the Magellanic Clouds, the closest approach of the Magellanic Clouds to the Galaxy seems to be within about 20 kpc (Toomre 1972). Assuming $p = 20$ kpc, $e = 0.5$, $M = 2 \times 10^{10} M_\odot$, $M_1 = 2 \times 10^{11} M_\odot$, $R = 7$ kpc, $n = 2$, we obtain $(\Delta U)_P/|U| = 0.44$, $t_d = 3.6 \times 10^9$, $t_c = 3.2 \times 10^9$ yr. This implies that in a couple of orbital periods the LMC would disrupt, merging with the Galaxy in the process. On the other hand if we take the perigalactic distance of LMC as 30 kpc as suggested by Fujimato & Sofue (1976) we obtain $(\Delta U)_P/|U| = 0.13$, $t_d = 2.3 \times 10^{10}$ yr, $t_c = 1.3 \times 10^{10}$ yr. In this case the LMC can survive about seven orbital periods. Tremaine's (1976) work which assumes an extended massive

Table 4. t_d and t_c for a globular cluster in the Galaxy.

r (kpc)	M_1 ($10^{11} M_\odot$)	$(\Delta U)_P/\lvert U\rvert$	t_d (yr)	t_c (yr)
0.32	0.036	16	5.4×10^5	5.1×10^8
0.67	0.110	5.6	2.7×10^6	3.9×10^9
3.53	0.349	1.2×10^{-1}	8.9×10^8	7.3×10^{11}
6.18	0.824	5.2×10^{-2}	3.0×10^9	3.4×10^{12}
7.74	1.112	3.6×10^{-2}	5.4×10^9	6.4×10^{12}
8.01	1.185	3.4×10^{-2}	5.7×10^9	7.0×10^{12}
10	1.453	2.2×10^{-2}	1.1×10^{10}	1.4×10^{13}

halo for our Galaxy, indicates that LMC will be disrupted by the Galaxy in $(2\text{–}4)\times 10^9$ yr, increasing the luminosity of the Galaxy by -0.24 mag.

We also make a rough estimate for the tidal effects of the Large Magellanic Cloud on the Small Magellanic Cloud by assuming $p = 10$ kpc, $e = 0.5$, $M = 2\times 10^9 M_\odot$, $M_1 = 2\times 10^{10} M_\odot$, $R = 3$ kpc, $n = 2$. We obtain $(\Delta U)_P/|U| = 0.28$, $t_d = 6.4\times 10^9$ yr, $t_c = 4.8\times 10^9$ yr. Thus in the case of the Galaxy–LMC pair as well as the LMC–SMC pair, the coalescence rate is as important as the disruption rate.

The ratio t_d/t_c for distant pairs is given by

$$\frac{t_d}{t_c} = \frac{6}{5-n}\frac{a}{R}\frac{M}{M_1}. \tag{4.1}$$

It can be seen from this equation that the coalescence process is fastest in comparison with the disruption process if the stellar systems are of the same mass. The relative importance of coalescence also increases with the degree of central concentration of the stellar systems.

The results for close pairs emphasize the vital role of tidal forces in the dynamical evolution of contact and overlapping stellar systems. Contact spherical galaxies of equal mass ($r = R$, $n = n_1 = 4$) will coalesce in less than three orbital periods. The disruption rate is six times slower at this distance. For a separation of the centres equal to $0.5R$ (spheres containing 85 per cent of the mass just touching each other), the time of coalescence is less than a period and the disruption rate is three times slower. As the separation of the pairs decreases, the disruption rate increases faster relative to the coalescence rate.

White (1977) has carried out *N*-body simulations of the interaction between pairs of 250-particle 'galaxies' and his results also show that binary galaxies merge very rapidly if their mass distributions significantly overlap. He finds that for the nearly isothermal ($\rho \sim r^{-2}$) galaxies studied, the condition $r_{peri} \leqslant 3R_h$ was sufficient for the binary to merge completely within one initial orbital time of its first close approach, where r_{peri} is the pericentric distance of the initial orbit and R_h is the half-mass radius of the initial galaxies. For polytropes of index $n = 4$, studied by us, $3R_h = 0.4R$. Our results show that the merging time is equal to half the orbital period at this distance for circular orbits of galaxies. The results of Van Albeda & Van Gorkom (1977) and Toomre (1977) emphasize the vehement nature of the dynamical friction in galaxies approaching each other slowly from a great distance. They also find that the two galaxies soon merge and form a single system.

We have derived the rates of disruption and coalescence from the properties of the stellar systems at the median radius. The tidal force effects will be greater in outer parts than in the interior portions. For contact and overlapping pairs of galaxies, the variation of the disruptive effects over the spatial extent of the galaxies will have important consequences. The outer envelopes of individual galaxies will lose their identity much sooner and form a common outer envelope of the pair while the two nuclei will retain their identity for a considerably longer time. Thus galaxies with double nuclei will be formed in the process of the dynamical evolution of a close pair. The supergiant cD galaxies in rich clusters have extensive envelopes and often exhibit multiplicity of nuclei (Jenner 1974). Merging of galaxies due to tidal force effects appear to be a likely mechanism for the formation of these galaxies as pointed out by several authors (Ostriker & Tremaine 1975; White 1976; Richstone 1976; Marchant & Shapiro 1977).

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