Solar Active Region Evolution: Comparing Models with Observations
ASP Conference Series, Vol. 68, 1994
K. S. Balasubramaniam and George W. Simon (eds.)

Production of Magnetic Flux Bundles For Solar Activity By Cascade of Energy in Global Torsional MHD Waves

M. H. Gokhale

Indian Institute of Astrophysics, Bangalore 560 034, India

Abstract. It is suggested that the toroidal magnetic flux bundles, whose emergence above the photosphere produces solar activity, are formed by superposition of global torsional MHD waves whose Legendre-Fourier (ℓ, ν) spectrum is maintained approximately steady by input of energy at $\nu = \nu_* = 1/21.4$ yr^{-1} . Solar activity constitutes 'leakage' of the cascading energy. Flux bundles producing the regularly distributed activity (e.g., sunspot activity) originate in superposition of global oscillations represented by ℓ = 1,3,...,13; $\nu = (1,3,5)\nu_*$. Activity on smaller scales is provided by waves of higher ℓ and ν. (Only a summary of the relevant results and arguments is given here. Details are given in a full paper by Gokhale and Javaraiah which is being submitted elsewhere).

1. Introduction

From Legendre-Fourier (LF) analysis of the magnetic field ('B_{inf}') inferred from sunspot data during 1874-1976 (Gokhale et al. 1992: Paper I) it has been shown that sunspot activity may be originating in the interference of Sun's global magnetic oscillations represented by *sets* of LF terms in B_{inf} having odd degrees 'ℓ' and frequencies close to ν_* ($\sim 1/21.4$ yr^{-1}) and a few odd harmonics of ν_*. In paper II (Gokhale and Javaraiah 1992) it was shown that superposition of the dominant LF terms of frequency ν_*, with average amplitudes and phases during 1874-1976, can reproduce not only the average butterfly diagram, but also the observed canonical behavior of the magnetic field in middle and high latitudes, though the data used for computing the LF terms is only from the low (<35°) latitudes.

In this poster I describe briefly the results obtained and the phenomenology developed in paper III (Gokhale and Javaraiah 1993) for production of activity and maintenance of the LF spectrum, based on the correlations between the variations of the LF phases. The full paper will be submitted elsewhere.

2. Modeling the Variation of the Annual Measure Of Sunspot Activity

2.1. Basic Definitions

In paper III we have defined

$$Q(\theta, t) = \pm P(\theta, t), \tag{1}$$

where

$$\begin{aligned} P(\theta, t) &= \tau_k \, \delta(\mu - \mu_k, t - t_k) \text{ near } (\mu_k, t_k) \\ &= 0 \text{ everywhere else,} \end{aligned} \tag{2}$$

where $\mu = \cos\theta, \tau_k$ is the life span of the spot group 'k', δ represents the Dirac delta function and the signs $\pm$ are chosen according to Hale's laws of magnetic polarities. It can be shown that on time scales larger than the life spans of sunspot groups but smaller than lengths of sunspot cycles, $Q(\theta, t)$ represents 'toroidal magnetic flux emerging at (θ, t), per unit photospheric area per unit time'.

2.2. Superpostion Model : Relation Between LF Terms in $Q(\theta, t)$ and $B_\Phi(\theta, t)$

We assume that the flux emerges fast (e.g., in < 1 y) in the form of flux bundles whenever and wherever critical magnetic buoyancy is reached by superposition of a set of axi-symmetric MHD (eg. torsional) waves described by a set of the values of ℓ and n. Then on time scales ≥ 1 y we must have $Q(\theta, t) = \partial B_\phi(R_c, \theta, t)/\partial t$ where R_c is the radial distance from the origin of the flux bundles.

It can then be shown that the LF amplitudes $q(\ell, n)$, and phases $\delta(\ell, n)$, of $Q(\theta, t)$ are related to amplitudes $b(\ell, n)$, and phases $\varepsilon(\ell, n)$, of $B_\Phi(\theta, t)$ as follows:

$$q(\ell, n) = 2\pi n \nu_* b(\ell, n) f_{\ell,n}(R_c) \tag{3}$$

$$\delta(\ell, n) = \varepsilon(\ell, n) + \pi/2, \tag{4}$$

where $f_{\ell,n}(r)$ are the radial eigenfunctions of the MHD modes.

2.3. The Set of LF Terms in 'Q' Producing Sunspot Activity

By maximizing the correlations of (i) the observed year-to-year variations of the amount of activity during each cycle, (ii) the observed cycle-to-cycle variation of the total amount of activity during each cycle, with those predicted by superposing different *sets* of LF terms in $Q(\theta, t)$, we have identified:

(i): ℓ = 1,3,..13; n = 1,3,5 as the *set* of terms in $Q(\theta, t)$ that would participate in the production of sunspot activity and the *regularly* distributed small scale activity at all latitudes,

(ii): ℓ = 3,5; $n = 1$ as the set of terms (presumably in B_ϕ) in which the initial input of energy seems to take place.

3. Evidence for Cascade and Balance of Energy

3.1. Phase Variations

Temporal variations of $\delta(\ell, n)$, the initial phases of the LF terms ℓ = 1-15 and of n = 1 and 3 in 'Q', as determined from intervals of lengths 11 y successively

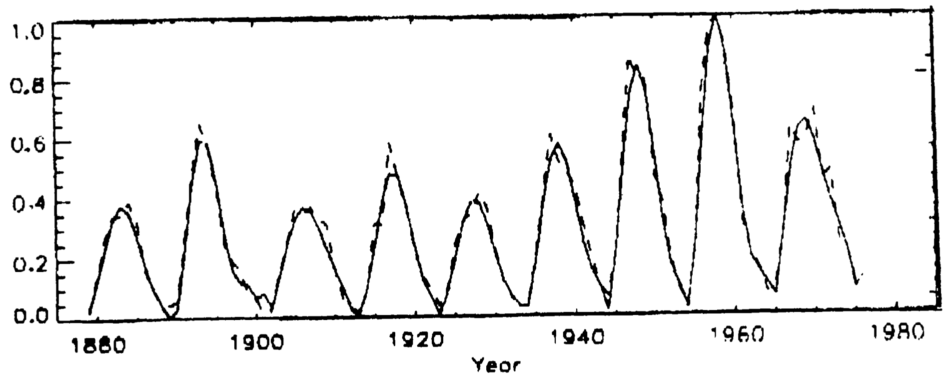


Figure 1. The best correlation between the 'observed' (continuous line) and the 'predicted' (broken line) variation of the annual measure of sunspot activity, given by the set ℓ = 3,5; $n = 1$, is shown for each of the nine sunspot cycles.

displaced by 1 y, are shown in Fig. 2. In view of Eqquation (4), these can also be considered as variations in the initial phases $\varepsilon(\ell, n)$ of LF terms in 'B_ϕ'.

3.2. Cascade of Energy from low 'ℓ' to higher 'ℓ'

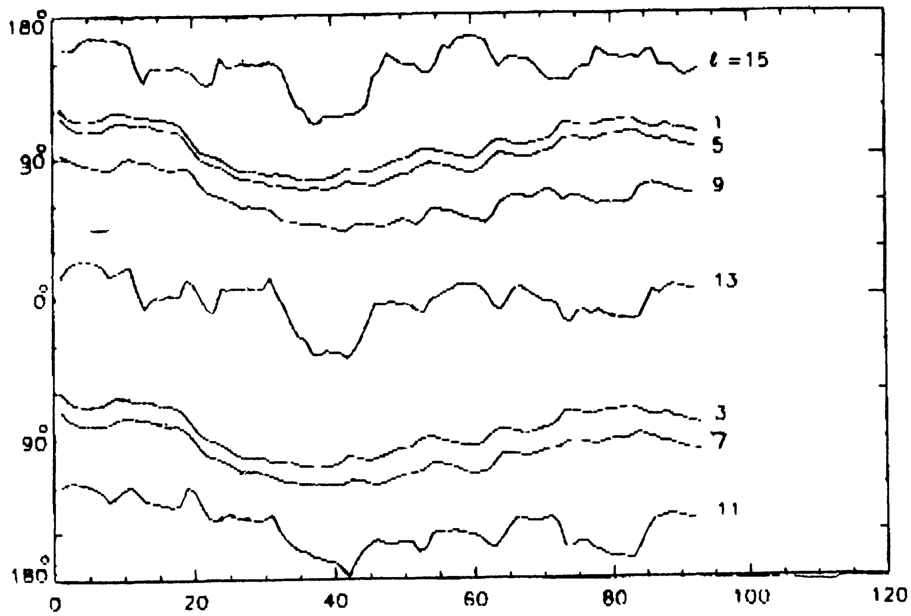
Correlations $\sim$ 90 % exist among the variations of phases of terms with different 'ℓ', suggesting transfer of energy (presumably) from waves of low 'ℓ' to those of higher 'ℓ'. (Such transfer is expected to occur during reflections/refractions of the waves near the center/photosphere).

3.3. Cascade of Energy from ν_* to $3\nu_*$ and $5\nu_*$

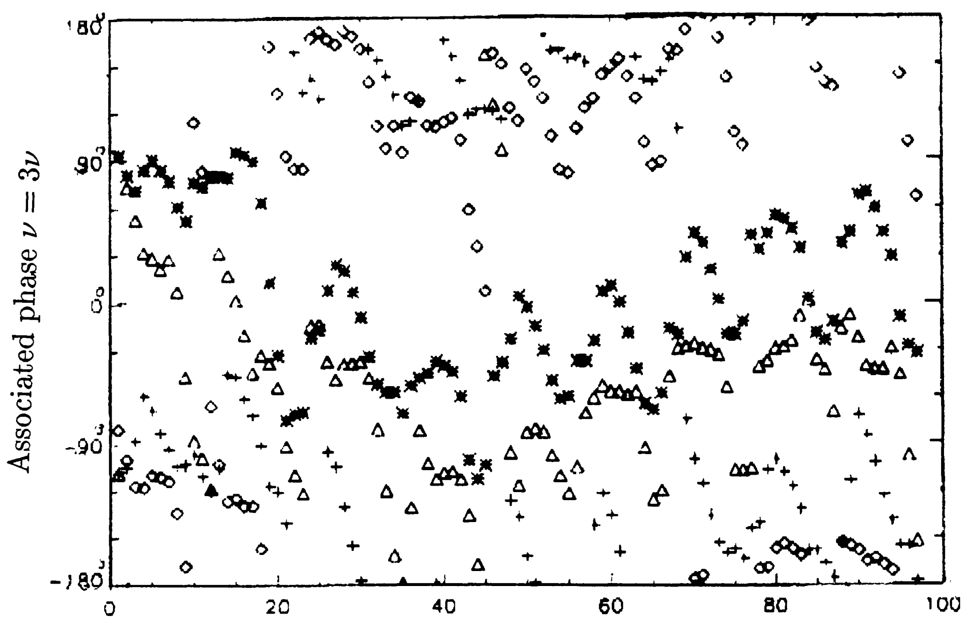
Correlations $\sim$ 85-90 % also exist between LF terms of $n = 1$ and 3 with same ℓ (Gokhale and Javaraiah 1990). Similar correlations are found to exist between phases of $n = 3$ and 5. These correlations suggest transfer of energy (presumably) from terms of frequency ν_* to those of $3\nu_*$ and $5\nu_*$. (Such transfer is expected to result from the accelerations and decelerations in propagations of the waves represented by the phase variations themselves).

3.4. Balance of Input and Output of Energy

Among the correlations of the amplitudes $q(\ell, n = 1)$ during 11y long time intervals (successively displaced by 1 y) with the observed amounts of sunspot activity during those intervals, the best correlation is given by amplitudes of ℓ = 3 and 5 and *not* by the largest two amplitudes (ℓ = 5 and 7). This suggests



a Interval Number



b Interval Number

Figure 2. Temporal variation of $\delta(\ell, n)$ or $\varepsilon(\ell, n)$ represented by values during 11 y long intervals successively displaced by 1 y, for (a) $n = 1$ and (b) $n = 3$. In (b) the signs 'plus', 'star', 'diamond' and 'triangle' correspond to $\ell = 11$, 13, 15 and 17, respectively.

that on time scales > 11 y there is a fairly good balance between the rates of input (according to section 2.3), into the oscillation '$\ell = 3{,}5$; $n = 1$' and the disposal of the cascading energy in the form of activity.

4. Conclusions and Discussion

The foregoing results suggest to us the following tentative phenomenology for (a) production of sunspot activity and (b) maintenance of the 'approximately steady' LF spectrum.

(i) The primary input of fresh energy, into an already existing LF spectrum of torsional MHD waves occurs at $\ell = 3.5$, $\nu = \nu_*$, by an unidentified process.

(ii) Owing to the reflection/refraction near the Sun's center and the surface this energy cascades to the waves of higher spatial and temporal frequencies.

(iii) The cascading energy keeps on leaking out in the form of sunspot activity (and regularly distributed small-scale activity) through buoyant emergence of toroidal flux bundles (wave packets) whenever and wherever waves corresponding to the LF terms $\ell = 1{,}3{,}...13$, $n = 1{,}3{,}5$, interfere constructively.

(iv) The rest of the energy cascades further and leaks out in the form of 'non-sunspot activity' at still higher ℓ and ν.

(v) The input and output remain approximately balanced.

It may be noted that the mode $\ell = 3{,}5$, $n = 1$, in 'B_ϕ ' corresponds to $\ell = 4$, $n = 1$, detected by Labonte and Howard (1985) in photospheric rotation.

Acknowledgments. Support to attend this Workshop was provided by the Air Force Office of Scientific Research (AFOSR) Asian Office of Scientific Research and Development (AOARD). The organizers of the Workshop also provided for partial local hospitality support.

References

Gokhale, M. H., Javaraiah, J., Kutty, K. N., and Verghese, B. A., 1992, Solar Phys. 138, 35.

Gokhale, M. H., and Javaraiah, J., 1990, in 'Basic Plasma Processes on the Sun' (eds. E. R. Priest and V. Krishan), IAU Symp. 142, 119.

Gokhale, M. H. and Javaraiah, J., 1992, Solar Phys. 138, 399.

Gokhale,M.H. and Javaraiah,J., 1993, in preparation.

Labonte, and Howard, R., 1982, Solar Phys. 75, 161.