EVOLUTION OF THE UNIVERSE THROUGH THE PLANCK EPOCH

C. SIVARAM

Indian Institute of Astrophysics, Bangalore, India

(Received 26 February, 1986)

Abstract. The problem of the Universe emerging out of the Planck epoch is discussed. It is pointed out that an earlier exponential expansion phase well before the onset of the GUTs phase transition is essential. Such an expansion can occur owing to the breaking of scale invariance at Planck energies in a unified theory of gravity with other interactions.

1. Introduction

The presence of microwave black-body radiation with photons of energy 10^{-13} GeV, isotropic and all pervading is taken as strong evidence for a dense hot early phase for the Universe, with the earliest phases perhaps going to 10^{19} GeV (the so-called Planck phase) and beyond!

Further support comes from the observed helium abundance as the Big Bang predicts about 0.25 He as arising naturally from the thermonuclear reactions between neutrons and protons occurring at a few seconds after the start of one expansion and over about hundred seconds later. Very little input is involved (about the only parameters are the neutron–proton mass difference, the neutron life time and the photon to baryon ratio) and no alternative model explains both observations so simply.

However, despite these successes there are several severe theoretical problems with the standard Big Bang especially at the earliest epochs. For instance we have the so-called 'Flatness' and 'Horizon' problems. The 'Flatness' problem arises from the extrapolation that for the Universe to be within an order of magnitude to the closure density (ρ_c) at the present epoch as implied by observations it ought to have been 'fine tuned' to the closure density to one part in 10^{60} at the Planck epoch, i.e. $(\rho - \rho_c)/\rho$ the relative density difference is a function of the epoch, going as T^{-2} (T is the temperature), i.e., proportional to t (the time), implying that for $T \approx 10^{19}$ GeV (one Planck epoch), this ratio is $< 10^{-60}$. Another way of expressing this result is to say that the kinetic energy term $(\dot{R}/R)^2$ and the potential energy term, $8\pi G\rho/3$ in the equation $(\dot{R}/R)^2 = 8\pi G\rho/3$ for the expanding scale factor R in the early universe (radiation dominated) must have been equal and opposite to one part in 10^{60} at the earliest Planck phase, i.e., they must have balanced to an accuracy of some 60 or more decimal places at that epoch ($t_{\rm Pl} \simeq 10^{-43}$ s). Otherwise the Universe would have long since recollapsed.. Even at $t \simeq 1$ s (when the helium was being formed) the two terms should have been equal and opposite to about one part in 10^{18}; i.e., to have a spontaneous Big Bang of this very 'precise' magnitude the Universe should have started out with a total energy of exactly zero (!) which also implies a density exactly equal to the closure density

Astrophysics and Space Science **125** (1986) 189–199.

at all epochs. Equally intriguing is the so-called horizon problem. This comes about as the radiation dominated early universe expands with time t according to $R_U \propto t^{1/2}$, whereas the horizon (range of communication of light signals) expands as $R_{hor} \propto t$, so that $R_{hor}/R_U \propto ct/t^{1/2} \propto t^{1/2} \rightarrow 0$ as $t \rightarrow 0$, i.e., we have smaller and smaller non-commutating regions at early epochs. For $t = 1$ s, for instance, the Universe we observe today occupied a region of space at least 10^{14} km across whereas light could have at most moved 3×10^5 km during the same time, so that at the time $t = 1$ s, the Universe must have been made up of some 10^{27} causally separate regions all of them, however, expanding at the same rate exactly! The high degree of isotropy observed today would be difficult to understand if the early universe were to consist of so many non-commutating regions which could not have exercised any influence on each other. A possible solution to these problems proposed a few years ago is the so-called inflationary model which involves an early exponential expansion phase of the Universe where the scale factor evolves as $R \propto \exp(Ht)$ rather than as $R \propto t^{1/2}$. This is brought about at the epoch when the grand unified theories (GUTs) would predict a unification of the strong, electromagnetic, and weak interactions at energies around 10^{15}–10^{16} GeV ($t \approx 10^{-35}$ s). Some of these theories predict a phase transition at this epoch wherein a large amount of ground state energy (vacuum energy) is released driving the expansion of the Universe. The vacuum energy density gives rise to an effective large Λ (cosmological constant) term which puts the expansion into a temporary de Siter phase with R increasing as $\exp \Lambda^{1/2} t$, with the equation of state (for the empty de Sitter space) as $\rho + P = 0$, i.e., $\rho = -P$, expansion with negative pressure term. In other words, ironically Einstein's discarded Λ-term is effectively rejuvenated into providing a large repulsive force to prevent the Universe from collapsing under its gravity and to drive it into an exponential expansion phase, the vacuum energy $(T_{\mu\nu})_{vac} = \rho_{vac} g_{\mu\nu}$ (= effective $\Lambda g_{\mu\nu}$ term) being converted into thermal energy during the expansion. This exponential expansion increases the size of a causally separate region by a factor of $\exp(Ht) \sim e^{65} \sim 10^{28}$ so that such a single region can expand to the size of the observed Universe thus taking care of the horizon problem. Now in the de Sitter expansion where the density is balanced by the negative pressure, the potential and kinetic energy are precisely balanced (giving a total of zero energy!) wiping out the curvature effects and thus requiring a density exactly equal to ρ_c (i.e., a flat universe) at all epochs.

However, the inflationary model does not account for epochs earlier than the GUTs era (i.e., $< 10^{-35}$ s) and energies $> 10^{16}$ GeV; where the flatness and other problems are even more severe. In particular, it does not deal with the question of how the Universe evolved from the Planck epoch or even earlier epochs! We shall see in the next section that the Universe could not have got out of the Planck epoch without undergoing an earlier exponential expansion phase.

2. The Universe at the Planck Era

At the Planck epoch when $t \approx (\hbar G/c^5)^{1/2} \simeq 10^{-43}$ s the density of the Universe would have been given by $\rho_{Pl} \approx c^5/G^2\hbar \simeq 10^{93}$ g cc^{-1} (trigintillion g per cubic centimetre).

If we assume that it is closed, the mass of the observable Universe is given by $M \simeq 2c^3/3GH_0$. With the Friedmann volume being given by $2\pi^2 R_H^3$, it follows that the Universe we observe today at the Planck epoch (when its density was $\rho_{\rm Pl}$ as given above) would have occupied a region of space whose radius would be given by

$$(R_{\rm U})_{\rm Pl} = \left(\frac{\hbar G}{3\pi^2 c^2 H_0}\right)^{1/3} \simeq 1.4 \times 10^{-13}\ \text{cm}\,. \tag{1}$$

It is quite remarkable (there was no *a priori* reason to expect this) that this length can be identified with the fundamental length characterizing elementary particles and their interactions (cf., e.g., Sivaram, 1982a)

$$\frac{e^2}{2m_e c^2} = \frac{g^2}{2m_p c^2} = \frac{\hbar}{m_\pi c} = \left(\frac{G_{\rm F}}{\hbar c}\right)^{1/2} \frac{g^2}{e^2} = (R_{\rm U})_{\rm Pl} = 1.4 \times 10^{-13}\ \text{cm}\,, \tag{2}$$

where $e^2/\hbar c = \frac{1}{137}$ is the electromagnetic coupling constant, $g^2/\hbar c \simeq 14$ is the strong interaction pion–nucleon coupling, $G_{\rm F}$ is the universal Fermi weak interaction constant $= 1.4 \times 10^{-49}$ erg cm^3; m_e, m_p, and m_π are the electron, proton, and pion rest masses. Thus at the Planck epoch, a region, the size of an elementary particle, contained the whole of the presently observed Universe! In a sense this would be a kind of ultimate bootstrap where each particle can contain all particles and at the same time be part of them, an idea pervading both Eastern and Western mystical thought. Thus, the metaphor of Indra's net illustrated in Avatamsateka Sutra, it is a network of pearls so arranged that if you look at one, you see all others reflected in it – i.e., each object part of every other object or to give an analogy in biology: i.e., that every one of the hundred trillion cells contains a complete genetic instruction to make every part of a complex organism – i.e., the nucleus of each cell carrying five billion bits of information making up the whole organism. Similarly, every region the size of an elementary particle had enough energy content for the creation of the whole Universe!

However, these ideas should not be taken too literally, and used to indraw erroneous conclusions regarding elementary particles or construct untenable models for their internal structure. It is an old notion that each elementary particle – such as an electron – is in itself a closed universe and organised within it are an immense number of smaller structures which form themselves a closed universe and so on to a picture of an infinite regression of Universes nested within each other. There are quantitative models of this sort. For instance, Bussian (1974) has shown that scattering experiments are consistent with a baryon model consisting of Schwarzschild spheres whose mass and radius are of the corresponding Planck mass and radius. Moreover, to match the scattering curves he postulates that there are 10^{20} such black holes per nucleon in the interior of the particle! Markov (1967) has also postulated quantum black holes as the constituent particles of nucleons and the correct magnetic moment is obtained if they are Kerr–Newman black holes! From the remarks above and those following Equations (1) and (2), we may be led to picture an elementary particle as consisting of $\sim 10^{60}$ objects of Planck mass, each constituent, however, having zero total energy, the rest mass energy balanced by its self-gravitational energy. Thus, the total energy of this closed universe is zero – i.e., the total energy of all its constituents (gravitational plus other energies) is zero; a positive 'bare' mass equal to that of the Universe (i.e., total mass of all other particles) is nullified by the negative equally large binding energy.

Thus, in this obviously wrong picture, the 'bare' mass of each particle in the Universe would be equal to the total mass of the Universe (i.e., all other particles) and, therefore, each particle has the potential (energy) content within it for the creation of the Universe (like each cell creating the whole organism). The basic idea behind the hadron bootstrap model according to chew – i.e., 'all hadrons are dynamically composed of one another in a self-consistent manner and in that sense can be said to "contain" one another'. The fundamental length given by Equation (1) determined by only cosmological parameters would correspond to a configuration of zero total energy, the masses of the particles would be determined by what charges are placed on this structure, thus $e^2/(R_{\mathrm{U}})_{\mathrm{Pl}}$ would give the electron mass, $g^2/(R_{\mathrm{U}})_{\mathrm{Pl}}$ the proton mass, etc.

However, one can easily show that a region of size given by Equation (1) would tend to collapse instantaneously under its gravity and would never have led to an explosive Big Bang. Thus, the gravitational self-energy of the system at the Planck epoch would have been ($\sim GM_{\mathrm{U}}^2/(R_{\mathrm{U}})_{\mathrm{Pl}} \sim 10^{119}$ ergs, some 10^{42} times the total rest energy of the constituents. Thus, the Universe would never have emerged out of the Planck epoch; the gravity would simply have been too strong. The only way would be for a large cosmological repulsive term (Λ-term) to have overcome this gravitational attractive force and setting the system into an exponentially-expanding de Sitter phase. We can estimate the magnitude of the vacuum energy term, required as

$$\Lambda (R_{\mathrm{U}})_{\mathrm{Pl}}^2\, c^2 M_{\mathrm{U}} = \text{cosmological vacuum energy term} \simeq \frac{GM_{\mathrm{U}}^2}{(R_{\mathrm{U}})_{\mathrm{Pl}}} ,$$

i.e.,

$$\Lambda \simeq \frac{GM_{\mathrm{U}}}{c^2 (R_{\mathrm{U}})_{\mathrm{Pl}}^3} . \tag{3}$$

A substitution for $(R_{\mathrm{U}})_{\mathrm{Pl}}$ from Equation (1), and for M_{U} ($\approx c^3/GH_0$) gives the value $\approx 10^{66}$ cm^{-2} for Λ. Thus, one would need to involve an effective cosmological repulsive term of this magnitude to have the observable universe emerge out of the Planck era!

Now if we had argued, based on the universality of the relations in Equation (2), that the Universe at its minimum radius should have had a size equal to the fundamental length implied by these relations involving elementary particle interactions only, then we could have used this length for $(R_{\mathrm{U}})_{\min} \simeq (R_{\mathrm{U}})_{\mathrm{Pl}}$ in Equation (3) and arrived at a curvature with $\approx 10^{66}$ cm^{-1}, indeed equal to the Planck curvature without involving the Planck length or Planck density anywhere at all! In Sivaram (1982a) we had shown that it is possible to arrive at the Hubble radius of a closed universe entirely from micro-physical considerations involving fundamental interactions of elementary particles and given an expression for this radius solely in terms of the coupling constants of these interactions – i.e.,

$$R_H = \frac{g^4}{Ge^8}\left(\frac{c^7 G_{\mathrm{F}}^3}{\hbar}\right)^{1/2} .$$

The use of this instead of GM_{U}/c^2 in Equation (3) gives for the necessary Λ at the earliest epoch to be

$$\Lambda \simeq \frac{8g^4}{Ge^{14}}\left(\frac{c^{19} G_{\mathrm{F}}^3}{\hbar}\right)^{1/2} m_e^3 \simeq \frac{8c^5\hbar}{Gg^2e^2} \simeq 10^{66}\ \mathrm{cm}^{-2}, \tag{4}$$

i.e., thus expressing the Planck curvature in terms of interaction coupling constants.

We thus need a large cosmological constant term of curvature $\Lambda \approx 10^{66}\ \mathrm{cm}^{-2}$ at the Planck epoch to enable the Universe to expand into a Big Bang and avoid recollapse to a singularity. The horizon and flatness problems would have been more acute at the Planck epoch, the balance between kinetic and potential energy terms being to one part in 10^{60}. An exponentially expanding inflationary phase at this stage with such a large cosmological term would, therefore, be essential to resolve these problems. In a recent paper it was shown (Sivaram, 1986) that the present value of the cosmological constant may be constrained by cosmology and the uncertainty principle to be

$$\Lambda \simeq \frac{6\hbar H_0 m_e^3 c^2 G}{e^6} \simeq 10^{-57}\ \mathrm{cm}^{-2} . \tag{5}$$

Thus, the present value to its value required at the Planck epoch may be expressed as

$$\frac{\Lambda}{\Lambda_{\mathrm{Pl}}} \simeq \frac{3}{4}\,\frac{H_0 G^2 e^8}{g^4}\left(\frac{\hbar^3}{c^{15} G_{\mathrm{F}}^3}\right)^{1/2} \simeq 10^{-123}, \tag{6}$$

expressing the fact that the present value of the cosmological constant is so close to zero – i.e., only a quadragintillionth of its value at the Planck epoch.

This can be also expressed as the ratio of the closure density given in Sivaram (1982a) to the Planck density: thus

$$\frac{3G^3 e^{16}\hbar^2}{4\pi c^{10} g^8 G_{\mathrm{F}}^3} \simeq 10^{-123} . \tag{6a}$$

We now have to understand how such a large repulsive cosmological terms would have been produced at the time of the Planck era to enable the Universe to emerge into the expanding Big Bang and evolve to its present size and structure. We shall see in the next section that this may be connected with the unification of gravity with other interactions at above Planck energies.

3. Unification of Gravity with Other Interactions at the Planck Energy Scale

At large distances (and low energies), Einstein's general theory of relativity (GR) provides a good description of gravity and is described by the Hilbert action $I_{\mathrm{H}} = (1/16\pi G)R$. The dimensionality of (mass)^2 of the Newtonian coupling constant $(\pi G)^{-1}$ in this action leads, however, to the so-called non-renormalisability of the theory wherein cross-sections and amplitudes for processes involving the interaction of the quanta of the gravitational field with particles of other fields diverge at high energies (E) a dimensionless amplitude of order G^n diverging as $G^n E^{2n}$.

Similar behaviour appears in the case of the Fermi weak interaction theory which provides good description of low-energy beta-decay processes (like decay of neutrons and muons). As the Fermi interaction is also characterized by a dimensional constant G_{F} (like Newtonian G) the cross sections for neutrino processes diverges as $G_{\mathrm{F}}^2 E^2$ with energy, the theory breaking down at $E \approx 1/\sqrt{G_{\mathrm{F}}} \simeq 100$ GeV. Similar thing happens for Einstein gravity at $E \approx 1/\sqrt{G} \simeq 10^{19}$ GeV. However, we now know that the Fermi theory is only a long wavelength (i.e., low-energy) effective theory for the weak interactions. The correct fundamental theory describing the weak interactions (manifesting at high energy) is a renormalizable gauge theory characterized by a *dimensionless* coupling constant which at energies > 100 GeV becomes identical with the electromagnetic dimensionless coupling constant thus uniting these two interactions above this energy. Similarly, we have to consider Einstein's gravity as only an effective long wavelength theory, the correct renormalizable theory at high energies (now $E > 1/\sqrt{G} \sim 10^{19}$ GeV) involving an action with a *dimensionless* coupling constant. To give another example we know that the theory of strong interactions at low energies (< 1 GeV) between pions and nucleons is not a renormalizable theory (it is based on a chiral SU(2) $\times$ SU(2) symmetry). The coupling constant is large (> 1) at these low energies and all perturbative amplitudes are divergent. But we know that this chiral theory like the Fermi theory is only an effective long wavelength theory. The underlying theory of strong interactions manifesting at high energies is quantum chromodynamics which describes the fundamental colour interaction between quarks and gluons with a coupling constant that decreases with increasing energy and the effective low-energy non-renormalizable theory of pions (which are bound states of quarks) with its large coupling (this leads to binding of quarks) emerges from this fundamental well-behaved high-energy theory wit its smaller coupling. Thus, the strong and electroweak interactions are described at high energies by the Yang–Mills field with *scale-invariant* actions *quadratic* in the fields, with *dimensionless* coupling constants tending to zero at highest energies. So, analogously, one would expect gravity at high energies to be also described by a scale invariant quadratic action with a dimensionless coupling constant. The Einstein–Hilbert action $R/16\pi G$ is linear in the curvature, has a dimensional coupling constant and is *not* scale invariant (i.e., not invariant under the transformation $g'_{\mu\nu} \to \lambda^2 g_{\mu\nu}$, where λ is a function of the position $\lambda = \lambda(x)$) but only invariant under the group of general coordinate transformations (GCT). Its dimensional coupling constant causes its bad behaviour at high energies. One can have scale invariance with R

combined with scalar fields like $\frac{1}{6}R\phi^2$ but in such theories, the gravitational constant G is related to ϕ and possesses the dimension of (mass)$^{-2}$. The only possible action for gravity, quadratic in the field strength (equivalently curvature) which is invariant under both general coordinate transformations and local scale transformations (ST) and which has a dimensionless running coupling constant α_G which could, therefore, describe gravity at high energies $> (M_{\text{Pl}})$ is the Weyl-type action

$$I_{\text{W}} \simeq \alpha_G \int \mathrm{d}^4x \sqrt{-g}\, C^{\alpha\beta\gamma\delta}\, C_{\alpha\beta\gamma\delta}\,, \tag{7}$$

invariant under $GCT \times S_{\text{local}}$; or, more generally,

$$I_S \simeq \alpha_G \int \mathrm{d}^4x(C^2 + \beta R^2) \quad (\beta \text{ also dimensionless})\,. \tag{8}$$

This would be the gravity analogue of the QCD action quadratic in the Yang–Mills field. At the appropriate high-energy scale they describe gravity and strong interactions, respectively. They have some remarkable properties in common. For instance in QCD, the colour strong interactions between quarks is linear – such that $V \propto r$ – only systems with zero-total colour have finite energy (leading to confinement of quarks). For the scale-invariant Weyl gravity with quadratic action, the potential between particles also grows linearly with distance as the corresponding Poisson equation is $\nabla^{-4} m\delta^3(r) \sim mr$ (the field equations being of fourth order). This means that for scale in variant gravity only systems with *zero total energy* have finite energy: i.e., energy is confined analogously to colour in QCD. In fact, it can be shown rigorously that all exact classical solutions of the field equations following from the above actions have zero-total energy for $\alpha_G \beta > 0$. Even this would have interesting consequences for the very earliest phases of the Universe when gravity would have been described by such equations; for the possibility exists to picture the initial state of the Universe as a zero-energy configuration; and a fluctuation with zero-total energy is created spontaneously at an epoch earlier than t_{Pl}; thus explaining the equality between kinetic and potential energies to within about one part in 10^{60} at that epoch. The gravitational potential energy at the epoch earlier to t_{Pl} would follow a $\sim Mr$ law, or the gravitational interaction would be vanishing at small distances so that the expansion would be rapid and would follow a $R \propto t^2$ law rather than $R \propto t^{1/2}$, so that there would not be any horizon problem as R_{hor}/R does not now approach zero as $t \to 0$. At energies around M_{Pl} the scale-invariance would be broken by quantum fluctuations and a large cosmological (vacuum energy) constant term $\sim \alpha_0 M_{\text{Pl}}^4$ ($\sim 10^{66}$) along with the linear Hilbert term (which is not scale invariant) κR (with an induced gravitational constant κ) is turned on into the original action. We can make this more explicit by once again drawing analogy with that happens in the case of strong interactions. The QCD Lagrangian has the symmetry $[\mathrm{SU}(n)_L \times \mathrm{SU}(n)_R]_{\text{global}} \times \mathrm{SU}(3)_{\text{colour}}$. At $\lambda_{\text{QCD}} \sim 0.5$ GeV, i.e., at low-energy scales below ~ 0.5 GeV, the colour coupling constant between quarks become strong (i.e., large) and massless scalar-bound states form. In other words, the local colour singlet operator develops a VEV and the global chiral symmetry breaks down (of course the high-energy theory has a small dimensionless coupling constant and is free of the problems plaguing the low-energy theory). However, the low-energy effective theory (i.e., for $E \leq \lambda_{\mathrm{QCD}}$) action must *retain the full original chiral symmetry*. Similar situation can be envisaged for gravity with the group of general coordinate transformations playing the role of global chiral symmetry and the corresponding counterpart of SU(3) colour would be S, the subgroup of conformal transformations. Analogous to the local colour singlet operator developing a VEV at $< \lambda_{\mathrm{QCD}}$ various local S invariant operators would acquire non-zero VEV's at energy scales below E_{Pl}, where α_G becomes large *and* dimensional. A set of second-rank S invariant tensor operators would be given by

$$S_{\mu\nu}(x) = f^{-1/2} f^a_\alpha(x) f^b_\beta(x) \eta_{ab}\,; \qquad f = \det f^a_\alpha\,, \tag{9}$$

where the f's are the Vierbein's gauge fields for the Poincaré subgroup of the conformal group. $S_{\mu\nu}$ can now develop a non-zero VEV, with a simple choice being $\langle S_{\mu\nu}(x) \rangle \sim \eta_{\mu\nu}$. It is to be noted that the metric is no longer a fundamental field; the spinor fields are the basic entities (analogous to the quarks in QCD) and just as below λ_{QCD}, the quarks form scalar-bound states (i.e., the pions which represent small fluctuations about the vacuum); when the QCD coupling becomes large and all the low-energy behaviour of strong interactions follows; below E_{Pl} the expectation values of the product of Vierbeins (analogous to the bound states) generates the metric at energies $\sim M_{\mathrm{Pl}}$ when the gravitational interaction becomes strong (coupling unity) and the coupling *dimensional*, the Hilbert action is turned on and low-energy behaviour of gravity follows. The above VEV does not break S_{local}, it breaks GCT invariance which has been broken down to the Poincaré subgroup and associated with this symmetry breaking there are massless Spin-2 Goldstone fields described by $S_{\mu\nu}$ (i.e., gravitons). But again in analogy with QCD the low-energy effective action must retain the *full original invariance* ($\mathrm{SU}(n)_L \times \mathrm{SU}(n)_R$ in the case of strong interactions and GCT in case of gravity). Thus, the effective low-energy action which in this case must retain the GCT invariance is constrained to be of the form

$$I_{\mathrm{eff}} = \int \mathrm{d}^4x \sqrt{-S}\,(\alpha_0 M^4_{\mathrm{Pl}} + \beta M^2_{\mathrm{Pl}} + \cdots + \gamma R^2 + \delta R^2_{\mu\nu} + $$
$$+ \text{ higher-order terms in curvature}\,. \tag{10}$$

The Einstein term $\beta M^2_{\mathrm{Pl}} R$ and the cosmological constant term αM^4_{Pl} are *not scale-invariant* and are generated as stated above by quantum fluctuations breaking scale invariance (however, GCT invariance is retained) at energies around M_{Pl}. The terms involving higher powers of curvature are suppressed by powers of M^{-1}_{Pl} the terms of order R^n by $(M^{-1}_{\mathrm{Pl}})^{2n-4}$ or $(M_{\mathrm{Pl}})^{4-2n}$. Thus, we see that *only* the $n = 2$ – i.e., the *quadratic* terms in curvature – are characterized by dimensionless coupling constants appropriate for a renormalizable high-energy gravity theory. It turns out that, at very high E, only the quadratic terms dominate; the higher R terms diminishing as E^{-n} (Sivaram, 1985a). Below E_{Pl}, only the induced Hilbert term R with a dimensional induced G survives – apart from the large cosmological (vacuum) term generated, which we shall discuss a little later. We mentioned above that the gravitational coupling becomes dimensional and strong (~ 1) at the Planck energy. This would account for an otherwise paradoxical situation that would arise regarding the coupling constants of gravity and the grand unified theories (GUTs). Most of the GUTs involve the unification of weak, electromagnetic, and strong interactions at emergies around 10^{-3} to $10^{-4}\,M_{\mathrm{Pl}}$. Above these energies ($\sim 10^{15}$ GeV) it is usually stated that these three interactions are characterized by a *single dimensionless* coupling constant (which is estimated variously in a model-dependent way as $\alpha_{\mathrm{GUT}} \sim \frac{1}{40}$ at these energies, in any case $\ll 1$) which, moreover, is expected to decrease logarithmically with increasing energy (i.e., asymptotic freedom with the coupling constant tending to zero at very large E) as appropriate for a non-Abelian gauge theory. Thus, at Planck energies, one would expect α_{GUT} to be $< \frac{1}{40}$ and closer to $\frac{1}{100}$. However, in the meantime the gravitational coupling constant (which is $GE^2/\hbar c$) continues to rise as E^2 (at the GUTs unification energy it is only 10^{-8} as compared to $\alpha_{\mathrm{GUT}} \sim \frac{1}{40}$) and at the Planck energy it becomes ~ 1 – i.e., much higher now than α_{GUT} which continues to decrease. It is also usually supposed that, at the Planck energy, gravity gets unified with the other interactions, such as GUTs. If they are indeed unified at the Planck energy one would expect both gravity and GUTs to be characterized by a single dimensionless coupling constant at E_{Pl}. However, as noted above, if the gravity and GUTs coupling constants continue to behave in an opposite sense as expected between the GUTs unification energy and the Planck energy, they would have vastly different values at the energy E_{Pl}. So, how does one achieve unification of gravity and GUTs at the Planck energy in such a situation, unless there is some drastic discontinuity in either the GUTs or gravity interaction at $E = E_{\mathrm{Pl}}$? We shall suppose that the unification occurs not at E_{Pl} but at energies higher than E_{Pl} when both gravity and GUTs are described by a *single dimensionless* coupling constant α_{U} (which is $\ll 1$) and a quadratic scale invariant and GCT invariant action. Then at $E = E_{\mathrm{Pl}}$, the scale-invariance breaks down; inducing a Hilbert term and an induced dimensional gravitational constant which is strong (~ 1) at this energy and the interactions separate out as gravity and GUTs characterized by differently behaved (with respect to energy) coupling constants at $E < E_{\mathrm{Pl}}$. Of course, the GUTs interaction described by a Yang–Mills field (with a quadratic action in the field strength) continues to remain scale-invariant even at energies $E < E_{\mathrm{Pl}}$ and the GUTs coupling constant continues to remain dimensionless unlike the gravitational coupling constant which is dominated by the Hilbert term with dimensional G at $E < E_{\mathrm{Pl}}$.

An appropriate action for the unified description of gravity and GUTs at energies above E_{Pl} with a single dimensionless α_{U} is given by

$$\int \alpha_{\mathrm{U}}\, \mathrm{d}^4x \sqrt{-g}\,(W^2 - \tfrac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu})\,, \tag{11}$$

where W is the Weyl curvature scalar, related to the Riemann scalar R by $W = R - 6(A^{\mu}{}_{,\mu} - A^{\mu}A_{\mu})$, and A is the Weyl four-vector gauge transforming as $A^a_{\mu} \to A^a_{\mu} + \lambda_{,\mu}$ where λ is the scale parameter. The index a in the Yang–Mills field strength can take values depending on the groups and multiplets considered. The action

(11) is both scale-invariant and GCT invariant. We have the variational principle

$$0 = \alpha_U \int [W^2 \delta \sqrt{-g} + 2W\delta W \sqrt{-g} + \delta(F^a_{\mu\nu} F^{a\mu\nu} \sqrt{-g})] \, d^4x \,, \qquad (12)$$

varying $g_{\mu\nu}$, A_μ independently. To *break* the scale invariance we incorporate a characteristic scale (energy or length) at E_{Pl} where we can set

$$W = \Lambda_{Pl} \,;$$

and if so, the variational principle (12) becomes

$$\delta \int \left[W + \frac{1}{2\Lambda_{Pl}} F^a_{\mu\nu} F^{\alpha\mu\nu} - \frac{1}{2} \Lambda_{Pl} \right] \sqrt{-g} \, d^4x = 0 \,, \qquad (13)$$

substituting the relation connecting W and R, and with the transformation $A'_\mu = (\Lambda_{Pl})^{-1/2} A_\mu$ and $F'_{\mu\nu} = (\Lambda_{Pl})^{-1/2} F_{\mu\nu}$, we have

$$\delta \int \sqrt{-g} [\kappa R + \alpha_{GUT} F'^a_{\mu\nu} F'^{a\mu\nu} - \Lambda_{Pl} (\tfrac{1}{2} + 6A'_\mu A'^\mu)] = 0 \,, \qquad (14)$$

with κ, a *dimensional* constant which is *related* to Λ_{Pl} and α_{GUT} now being induced into the action as a multiplier of the Hilbert term brought into the action by the scale breaking and of course an induced cosmological constant term $\approx \Lambda_{Pl}$ and mass terms for the gauge fields. Thus, below Planck energies, gravity and GUTs separate out, the GUTs interaction being now characterized by a dimensionless α_{GUT} coupling constant (the Yang–Mills part of the action is still scale-invariant), and the gravitational interaction is now described by the Hilbert term with a dimensional coupling κ (which is now large at the Planck energy as $GE^2_{Pl}/\hbar c \sim 1$). We note that α_U which was the single dimensionless constant for both GUTs and gravity at energies above E_{Pl} was much less than 1 and decreasing with energy. After the scale breaking at E_{Pl}, α_{GUT} continues to $\ll 1$ and decreasing with energy just like the unified interactions coupling while gravity below E_{Pl} has a dimensional constant G. At $E_{GUT} \sim 10^{-4} E_{Pl}$, the GUTs interaction further splits into strong and electroweak interactions with different dimensionless couplings while the gravitational interaction coupling decreases with decreasing energy as E^2.

The large cosmological term $\sim \Lambda_{Pl}$ induced at E_{Pl} along the the Hilbert term will cause an exponential expansion of the Universe (the vacuum energy density corresponding to the curvature $\Lambda_{Pl} \sim 10^{66}\ \mathrm{cm}^{-2}$, creates a de Sitter phase with negative pressure $(P = -\rho)$, $\rho \approx 10^{116}$ ergs per cc) enabling the Universe to emerge out of the Planck era and expand by a factor $e^{H_{Pl}t} \sim e^{20} \sim 10^8$ till the GUTs regime when another phase transition releasing more vacuum energy leading to further inflation takes place.

The post-Planck inflation will dilute or wipe out all Planck era relics (like Klein–Kaluza monopoles, tower of massive particles associated with string theories and compactification of dimensions, etc.). The combined Hilbert and quadratic terms $(\sim \alpha R^2 + \kappa R)$ has a long-range solution (corresponding to the modified Poisson equation $\nabla^4\phi + \nabla^2\phi = 0$) give by $\phi = A/r - B/r \exp(-M_{Pl}/r)$, which simply means that we have the usual GR of Einstein and Newtonian gravity at distances $\gg$ Planck length!

Thus, once the Universe has emerged out of the Planck era, the gravity is described by GR as is usually adopted. To understand how such a large cosmological term $\sim \Lambda_{\mathrm{Pl}}$ was reduced to its present value of almost zero as given by Equation (6), we may involve the fact that the ground state of many supergravity (as well as string) theories (which have unification just below Planck energies) is an anti-de Sitter space characterized by a negative cosmological constant of magnitude $\sim (-\Lambda_{\mathrm{Pl}})$. This could cancel out the large $+ve\Lambda_{\mathrm{Pl}}$ induced earlier once the exponential expansion sets in, and matter supergravity multiplets manifest out of the Planckian–de Sitter vacuum. This would be explored in more detail in a later publication.

References

Bussian, A. E.: 1974, *Phys. Rev.* **D9**, 1384.
Markov, M. A.: 1967, *Soviet Phys. JETP* **24**, 584.
Sivaram, C.: 1982a, *Astrophys. Space Sci.* **88**, 507.
Sivaram, C.: 1982b, *Astrophys. Space Sci.* **86**, 501.
Sivaram, C.: 1985, *Gravity Research Foundation Essay May, 1985.*
Sivaram, C.: 1986, *Int. J. Theor. Phys.*, in press.