MODEL OF 'STEADY' PARTS OF ROTATION AND MAGNETIC FIELD IN THE SUN'S CONVECTION ZONE

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ABSTRACT

We present a preliminary model of the 'steady' parts of rotation and magnetic field in the Sun's convective envelope, which vary on diffusion time scales, within the frame work of axisymmetric incompressible hydromagnetics (Ref. 1). We use the boundary conditions for rotation and magnetic field similar to those used by Nakagawa (Ref. 2). The model yields a quadrupole type part in the distribution of the toroidal field whose strength varies from ~ 1 G below the surface to $\sim 10^4$ G near the base of the convection zone. The model also yields isorotation contours similar to those given by helioseismology.

Keywords: Sun's rotation, Sun's toroidal magnetic field.

1. INTRODUCTION

Helioseismology has shown (Ref. 3,4,5,6,7,8,9) that Sun rotates differentially with a weak dependence on radius throughout its convection zone and has a nearly uniform rotation in the radiative core. The differential rotation in the convective envelope has been modeled earlier in the frame work of hydrodynamics by taking into account the meridional circulation, Reynold stresses and viscous stresses (Ref. 10,11,12,13,14,15,16,17). None of these models predict the isorotation contours given by helioseismology. Using Chandrasekhar's equations (Ref. 1), Nakagawa modeled (Ref. 2) the differential rotation in an incompressible spherical shell of infinite electrical conductivity in the presence of an absolutely steady toroidal magnetic field, without any poloidal field. Recently, we (Ref. 18,19,20) have modeled the 'steady' part of the Sun's ploidal magnetic field. As pointed out in the latest of these models, the poloidal part of the field plays an important role in producing solar cycle and activity phenomena and in the evolution of Sun's angular momentum. Hence, it is necessary to modify Nakagawa's model by including 'steady' part of the poloidal field also.

2. MODEL

With reasonable assumptions and approximations in models of the 'steady' part of the Sun's poloidal magnetic field, it has been shown (Ref. 18,19,20) that $\Delta_5 P = 0$ in the convective envelope. Here Δ_5 is the Laplacian operator in 5 dimensional space and P is the scalar function that defines the poloidal field. In the present model, we use similar assumptions and approximations except that eddy viscosity in the convective envelope is assumed to be very large. With these assumptions, the equations given by Nakagawa and Scwartzrauber (Ref. 21) take the form :

$$\Delta_5 T = 0, \tag{1}$$

$$\Delta_5 V = 0, \tag{2}$$

$$\frac{\partial}{\partial z}\left[V^2 - T^2\right] = 0, \tag{3}$$

where T and V are 'steady' parts of the toroidal magnetic field and the angular velocity of rotation respectively. These equations are formulated in the cylindrical coordinates. Solution of the above equations is given as follows

$$T(x,\mu) = \sum_{n=0}^{\infty}\left[b_n x^n + c_n x^{-(n+3)}\right] C_n^{3/2}(\mu), \tag{4}$$

$$V(x,\mu) = \sum_{n=0}^{\infty}\left[v_n x^n + w_n x^{-(n+3)}\right] C_n^{3/2}(\mu), \tag{5}$$

and

$$V^2 = T^2 + f(\varpi). \tag{6}$$

Here v_n, w_n, a_n, b_n, c_n, $(n = 0, 2, 4, ...)$ are constants of integration which have to be determined from the boundary conditions. Similarly $z = x cos\vartheta$, $x = r/R_\odot$, $R_\odot$ is the radius of the Sun, $\mu = cos\vartheta$, ϑ is the co-latitude, $C_n^{3/2}(\mu)$ are the Gegenbaur polynomials of order 3/2, and $f(\varpi)$ is an arbitrary function of $\varpi = x sin\vartheta$ and expressed as

$$f(\varpi) = \sum a_n \varpi^n. \tag{7}$$

Nakagawa (Ref. 2) assumed each of the functions V and T to be a series in Legendre polynomials and applied with appropriate boundary conditions. In our model V and T are taken as solutions (4) and (5) of equations (1) and (2). We use the boundary conditions for rotation and magnetic field similar to those used by Nakagawa, viz., $V = V_{obs}$, $T = 0$ at $x = 1$; V is uniform and T is finite at $x = 0.7$. We solve equations (4)-(7) along with the boundary conditions and determine the coefficiens a_n, b_n and c_n $(n = 0, 2, 4, ...)$.

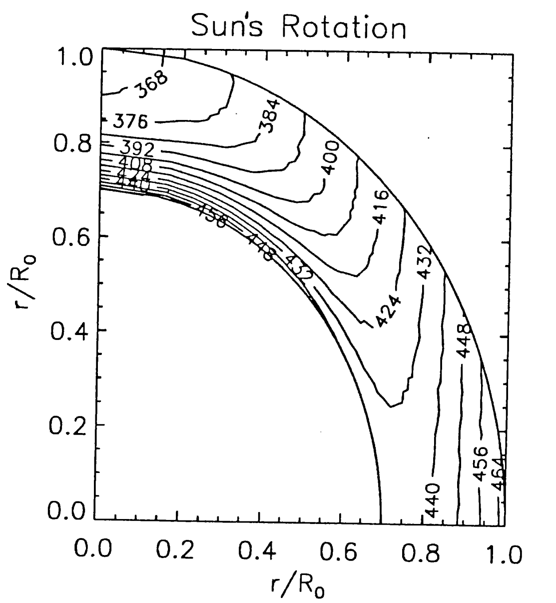

Figure 1: Model of the 'steady' part of the Sun's rotation in the convective envelope. Rotational isocontours are in nano-hertz.

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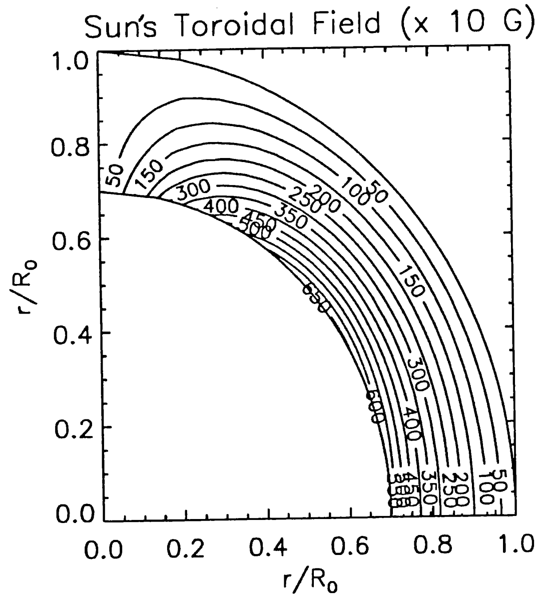

Figure 2: Model of the 'steady' part of the Sun's toroidal magnetic field in the convectve envelope. Isocontours are in gauss.

3. RESULTS AND CONCLUSIONS

With the observed surface rotational boundary conditions, viz., $V_0 = 2.934\ \mu rad/s$ and $V_2 = -0.574\ \mu rad/s$ (Ref. 22), we have computed the rotation profile in the convective envelope, whose isocontours are presened in Fig. 1. Except for small differences in the magnitudes of rotational velocities, our rotational velocity profile is similar to Nakagawa's model and also to that given by helioseismology. However, equatorial rotational rates of our model are closer to the rotational velocities inferred by Gough et.al. (Ref. 9) and are slightly different than the results obtained by Dziembowski et.al (Ref. 4). These differences could be due to time-dependent part of the internal rotation which may be present (Ref. 18,19,20) in the helioseismologically inferred results. Using density values of Spruit's model (Ref. 23) in the convection zone, we have computed the 'steady' part of the toroidal magnetic field whose iso-gauss contours (i.e ϖT) are shown in Fig. 2. We conclude from Fig. 2 that in order to satisfy the rotaional profile presented in Fig 1, the 'steady' part of the toroidal field in the convective envelope should have partly quadrupole type distribution. It is interesting to note that except near the base of the convection zone, the energy of the magnetic field is much less than the rotational energy.

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